Other excellent Answers have provided plenty of details, naming many tribes and peoples and dates, so I wanted to offer a few observations from a bit greater distance.

When the Hungarians arrived in Europe, they were called “Turks” by some of the most sophisticated cultures in the region - cultures who had their own emissaries and foreign agents:

the Byzantines regularly called the Hungarians “Turks,” from the 9th century up to the 12th century (when Hungarians started to form the palace guard in Constantinople - see Vardariotai - and could presumably set them straight)

previously, the Byzantines had called the several tribes of horse archery nomads that arrived in Europe “Huns”

the Byzantines referred to others as “Turks” as well during that time, especially the Khazars (“eastern Turks”)

Muslim (mostly Arabic) sources of the same era also sometimes referred to Hungarians as “Turks”

This of course did not mean the Hungarians actually were Turks, no more than that all the horse warrior peoples in Europe were all Huns.

But it does highlight that the Byzantines (and Arabs) weren’t really interested to know the precise cultural identities of these peoples, but rather in their status as formidable horse archery warriors. They wanted to either have them as allies (against other horse warriors or against the Persians), or else set one group against the other so neither would come to Constantinople asking for tribute.

So why do we need to know who the “Hungarians” and the “Turks” were in those times? And what is more important in knowing such things: the fact that members of both groups were formidable horse archer nomads who were allies more often than enemies, had very similar lifestyles and probably mingled a lot? or that one spoke a language of Ugric stock (but 10% of words with Turkic roots) and the other of Altaic (the modern version of which, Turkish, with almost 10% of words with Arabic roots)?

Many early peoples introduced to horses rapidly abandoned their hunter-gatherer or even agricultural roots and took up a horse hunting and warrior lifestyle. This seemed to have included the early Magyars, under the influence of the Turks.

in 1680 the Pueblo tribes successfully threw off their Spanish oppressors and appropriated some 1,800 of their horses; within 20 years nearly all these tribes (and their neighbors) were mounted

the Lakota (Sioux) tribe - originally woodland natives who had been pushed by other tribes westwards onto the plains - first saw horses around 1700. “Almost overnight” they were using horses for waging war, leading to the establishment of their special “warrior societies” (leadership of which was symbolized by owning a white horse)

the Comanche - previously leading a marginal hunter-gatherer existence in some of the harshest terrain in the West - were introduced to horses before the Lakota; within 30 years, they became the most feared and powerful of all plains tribes (“Lords of the Plains”)

early empires in the Sahel (Africa) such as that of Mali and Songhai developed soon after the introduction of the horse - and with it horse warrior culture - in the 14th century (these empires could have expanded further south were it not for the “tsetse fly belt” making it hard for horses to thrive there)

domestication and the creation of what was probably the world’s first horse warrior lifestyle seemed to be what propelled the Yamnaya culture of the Pontic steppe to spread (“explode” is another term that has been used) the Indo-European languages over most of Europe (“Bell-Beaker Culture”), the upper middle east and parts of India around 3,000 BC (non-Indo-European tribes came later)

So it should be no surprise that when the early Magyars - a forest people in the southern Urals - were introduced to the horse in the northern steppe of modern Kazakhstan (? around 100 BC), they likely enthusiastically adopted a horse warrior lifestyle, influenced by the neighboring Turkish tribes (as reflected in their horse-related terminology).

The Goths may have undergone a similar transformation upon entering the Pontic Steppe around 160 AD.

Who are the Turks? One definition is groups of people speaking languages that are mutually related. In the case of Oghuz Turkic, these are remarkably mutually intelligible, from Turkey to Turkmenistan (Kazakh and Uyghur would be much harder).

Who are the Hungarians? Hungarian language speakers are limited to those in Hungary, its borders or its diaspora. Their nearest linguistic kin are the Khanty and Mansi of western Siberia (who are unfortunate enough to live in a region that has half of Russia’s oil). As a Hungarian speaker, I cannot understand these when I hear them spoken, but structured presentation of words (e.g. counting) do sound very similar.

Turkic and Hungarian are both polysyllabic, have a preferred word order in sentences of Subject-Object-Verb (SOV), and avoid consonant clusters, distinguishing them from Sinitic (monosyllabic, SVO) and most Indo-European (polysyllabic, SVO) languages. One can make an argument that both adhere to some extent to “vowel harmony.”

Interestingly, Mongolian, Korean and even Japanese have similar traits.

As for writing, the ancient Hungarian runic system is thought to derive from or be a cousin of Old Turkic script.

There appeared to have been a major transformation on the steppes of Eurasia in the 4th century. Up until then, the steppe horse archery warriors were Indo-European peoples (mostly Iranian), typically thriving on the Pontic or Caspian steppe without posing much threat to Europe.

The Huns were the first non-Indo-European peoples to occupy the steppe, and launched a long series of eastern (mostly Turkic, then Mongol) peoples aggressively pushing their way west and displacing the people before them. These peoples dominated the steppes until modern times.

This may have been related to new technologies such as:

solid (wooden) saddles, with horns (to tie a rope around)

stirrups (the rider can now turn his body backwards to look or shoot an arrow)

reflex, composite bows (short, yet far more powerful than their predecessors)

The Hungarians were just another wave of such peoples from central Asia, distinguished mainly by their unique language and probably not much else.

It has been extremely difficult to distinguish the various tribes that have pushed their way from east to west across the steppe:

while there have been archeological remains found on the steppe, it has been difficult to ascribe these remains to any particular tribe, culture or linguistic group

in the “parade of tribes” riding across the steppe, whether Turkic, Mongol or Magyar-speaking, their lifestyles, customs and even religions have been so similar as to make it hard to distinguish them (hence the ease with which Hungarians and others seem to find “connections” to all sorts of people, from Mongolia to Kazakhstan to Turkey - and vice-versa)

this is true even in the realm of music (descending pentatonic anyone?) and even to some extent dance (movements simulating horse riding, wearing horse boots and making vigorous movements with the legs, arms often held up as if imitating a bird)

many of these peoples have ceased to exist long ago and, being pre-literate, their history is hard to reconstruct

Genetic study of this “parade of tribes” is in its infancy, though growing at a surprising rate. It should not be surprising that those groups who have migrated the farthest have most in common with their neighboring peoples - especially if they have lasted there for a long time (in the case of both Hungarians and Turks in Turkey - 1,000 years). What genetic signal remains of the original conquering groups is now weak and hard to tease out.

This may be helped by study of more than just parental “haplogroups.”

So … “Do Hungarian people have Turkic roots?”

Not in the same sense as the Turkoman or Kazakh or Uyghurs do.

But in their distant past they have shared a geography (the steppe), a lifestyle (horse archery warriors), and many customs (including a strong desire to move and explore), probably drank to excess together countless times in days of old, and even united their families from time to time.

Probably more related than either to the fascinating and warrior Salar people of China.

No. It is a Uralic / Finno-Ugric language, however, Hungarian has a lot of loanword from (unidentified) Turkic languages. Various sources found 300–500 words in the Hungarian language with a Turkic origin. (Origin of many Hungarian words is unknown or disputed.) This is a huge number, probably Turkic was the 3rd most important source of the Hungarian vocabulary. (1st: Uralic 2nd: Slavic). Possible sources of these words:

Onogur (One part of the Onogurs went to Magna Hungaria after the collapse of their empire, they mixed with Hungarians, probably this is the reason, why Magyars were/are called Hungarians / Ungarn / ungherese etc in European languages,)

Hunnic (Based on historical sources and modern genetic researches it is very likely that some Hunnic elements joined the Hungarian tribal alliance somewhere in Eastern Europe, perhaps these were the Onogurs, but other versions are possible as well)

Khazar (The connection between the Khazar Empire and Hungarians is unclear and disputed, but these two ethnicities lived not very far from each other for many centuries.)

Avar (They ruled the Carpathian basin for many centuries, and at least one part of them remained there after their empire collapsed, OR the escaped part joined Hungarians in Eastern Europe and came back.)

Pecheneg (They moved to the Carpathian basin after Hungarians, and were quickly assimilated.)

Cuman (They moved to the Carpathian basin after Hungarians and Pechenegs in the 13th century, and were slowly assimilated.)

Ottoman Turks, who conquered parts of Hungary in 16th century (The loanwords from Ottoman Turks are identified but these are only a very small minority of the Turkic words in Hungarian.)

If we study the above list, it is not really surprising that Hungarian has many Turkic loanwords. (Even if we do not know too much about the older extinct languages in the above list, but probably all of them were Turkic ones.)