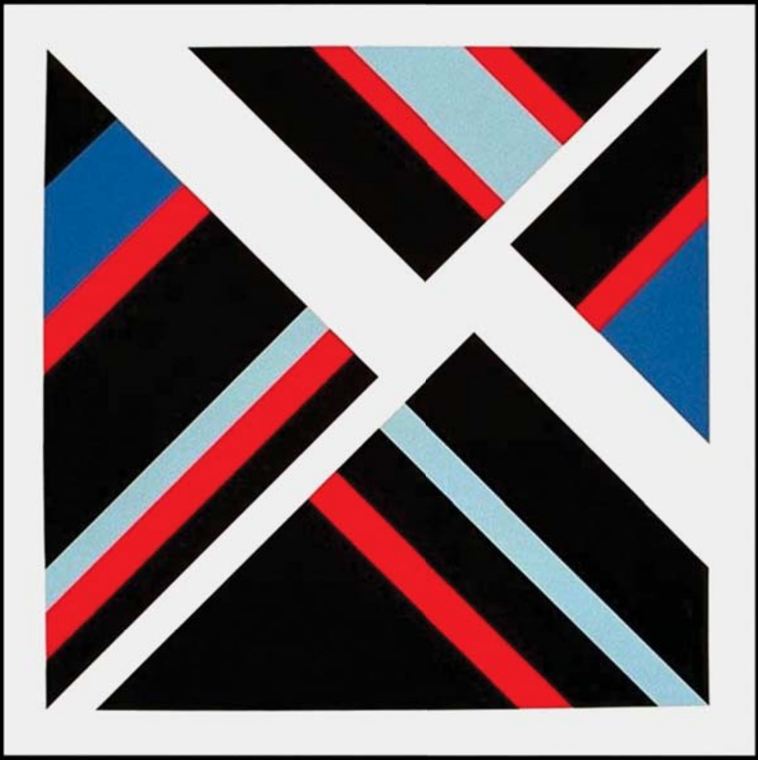
LOGICAL WRITINGS



ERNST MALLY

BERSERKER

BOOKS

ERNST MALLY

LOGISTICAL WRITINGS

BASIC PRINCIPLES OF WILL

Published by

KARL WOLF AND PAUL WEINGARTNER

in collaboration with

OSWALD HUBER, JOHANN MOKRE,
HEINZ AND MONIKA ROTBÜCHER, MARTHA SOBOTKA, FRITZ WENISCH

Publication of 'Grundgesetze des Pollens' was permitted by
Leuschner & Lubensky, Universitâts-Buchliandlunf, Graz, Austria

Library of Congress Catalogue Card Number 73-135106

ISBN-13: 978-94-010-3071-7 **e-ISBN-13:978-94-010-3069-4**
DOI: 10.1007/978-94-010-3069-4

Softcover reprint of the hardcover 1st edition 1971

ERNST MALLY

FOREWORD

As a student of Ernst Mally, I had the good fortune to witness how, around 1930, Mally's philosophy evolved from the rich diversity of Meinong's teachings into its own more cohesive form. Years of intensive development followed until his death in 1944. Due to adverse circumstances, illness and war, Mally's insights did not have a wider impact. It is therefore all the more gratifying that, in the year of the 25th anniversary of his death and his 90th birthday, a group of interested parties came together to publish a posthumous work by Mally, a large logical fragment, once called Schwanberger Logic by Mally, which was written in the last years of the philosopher's life. At the suggestion of Professor Dr. Roderick M. Chisholm (Brown University, Rhode Island), a 'basic work' of deontics, Mally's *Grundgesetze des Sollens (Basic Laws of Ought)* from 1926, is also being republished. This also provides an opportunity to publish a first summary of Mally's philosophy with a list of writings, etc., so that this edition can also serve as a memorial to Mally.

On behalf of co-editor Professor Dr. Paul Weingartner, I would like to thank everyone who contributed to the publication for their invaluable assistance: The Governor of Salzburg, DDr. Dipl. Ing. Hans Lechner, the Salzburg Provincial Government and the Brötje company in Hallein for providing a printing subsidy, Professor Hintikka and the Reidel publishing house for including it in the Synthese Historical Library series, the philosopher's widow, Mrs Else Mally, and her daughters, as well as the Graz-based publishing house Leuschner und Lubensky for granting permission to reprint the *Grundgesetze des Sollens (Basic Laws of Oughtness)*, University Lecturer Dr Martha Sobotka, the heir to the written estate, for making the estate writings available. Dr Gertraut Laurin and mining director DDr Franz Pichler provided personal letters from Mally, which offer valuable insights into the

The development of the philosopher and the creation of the Opus postumum. In addition to the editors, of whom Paul Weingartner was particularly responsible for reviewing the logical content, Dr. Monika and Dr. Heinz Rothbucher from the University of Salzburg also contributed (organising the estate, editing the manuscripts). Dr Fritz Wenisch was the main editor of the "Großer Logik-Fragment" (Great Logic Fragment), while Oswald Huber edited the *Grundgesetze des Sollens (Basic Laws of Ought)*. Prof. Rudolf Szyszkowitz provided a portrait of Mally from 1935 for reproduction. Martha Sobotka provided the photographic portrait of Dr Ferdinand Bilger. We would like to express our sincere thanks to all of them. The editors are confident that this publication will help to restore Ernst Mally's image to honourable memory and encourage renewed interest in his philosophy.

KARL WOLF

Salzburg, December 1969

CONTENTS

WHERE V

INTRODUCTION

KERL WOLF: Ernst Mally's life and philosophical development development 3
JOHANN MO KRs: Theory of objects — Logic — Deontics 16
PAUL WEINGARTNER: Remarks on Mally's later logic 21

Comments on the text 26

A BIG LOGICAL QUESTION

MARTHA SOBOTKA: Foreword 31

1. Preliminary remarks on the use and understanding of language 33
2. Ways of using the word "meaning". The concepts of meaning 37
3. Connection between the meanings of "meaning" 40
4. General discussions about meaning 42
5. Against misunderstandings of meaning arising from blindness to meaning 44
6. The objectivist misinterpretation 54
7. Whether meaning is subjective 63
8. Contextual meaning 67
9. Validity, hierarchy 70
10. Separation of meanings 74
11. Theoretical meanings in general 76
12. The paradoxes. Circular apparent statements and apparent moods statements 78
13. Logical propositions and concepts 82
14. Apparent definitions. "Predicable" 84
15. Decidability. Statement and statement form 86

16. Provisions 91
17. Provisions and classes (content and scope) 93
18. Determinate 98
19. Fulfillability 105
20. Determinate and class 108
21. On the theory of levels 110
22. The general and the specific 115
23. Mathematics: "Illustrative justification" of number theory (Hilbert) 119
24. Finite content, finite justification 125
25. More on finitism 132
26. On the concept of number in "Principia Mathematica" 136
27. Constitution of the concept of "number" 139
28. The "existence" of numbers 143
29. On "existential" logic 150
30. Limits of "existential" logic 156
31. "Material implication", "formal implication", inclusion and consequence 162
32. Structure of logic: implication, inclusion, consequence relationship 174

Formalism I 189
Formalism II 200
Formalism III 208

Letters 219

THE LAW OF INTENTION

Foreword 229
Introduction 231

I. BASIC PRINCIPLES 235

§ 1. Preliminary findings 235
1. Facts as objects of volition 235
2. General object-theoretical prerequisites 238
3. Obligation and demand 241

§ 2. The principles of what ought to be 243
Overview of the principles 250

II. FOLLOWING CONSEQUENCES 252

§ 3. Overview of the consequences 252
§ 4. Conclusions from principles I and II 258
§ 5. Conclusions from Principle III (and I, II) 261
§ 6. Conclusions from Principle IV (and I, II, III) 264
§ 7. Continuation. Contrapositive conclusions from principles IV and V 267

III. THE WILL AND THE FACTS 270

§ 8. The Willing of the Facts 270
1. The essence of the strange conclusions 270
2. The co-will of the implications in general 271
3. "Actual" and "non-actual" implications. The co- of the facts 273
4. The explicit wanting of the facts 275
5. The improper ought 276
§ 9. The intention and certainty of the event 279
1. Desired behaviour in response to uncertain circumstances hold 279
2. Objective certainty in the face of subjective uncertainty 280
3. Possibility in determinations and in certain factual behaviour 281
4. Relationship of volition to events and to deeds matters 283

IV. THE RIGHT WILL 286

§ 10. Laws of material correctness 286
1. General laws of right intention 286
2. A principle of application. Motivation through desire 289
3. Motivation through judgement 291
4. The material Contrary to what ought to be 292
§ 11. The inner rightness of will 294
1. Material and formal correctness 294
2. The judgement-motivated will 295

3. Relative Imperatives 298
4. Formal correctness. The unconditional actual ought 299
5. Primary general requirements as rules 302
6. Degrees of oughtness. The contrary of oughtness 303
§ 12. Willing and value 306
1. Dependence of oughtness on value 306
2. Value and durability 309
3. The objective value 311
4. Goodwill 315

Bibliography **325**
Mally estate in the University Library of Graz 332
Index of names **334**
Subject index 336

INTRODUCTION

KARL WOLF

ERNST MALLY'S LIFE AND PHILOSOPHICAL DEVELOPMENT

Ernst Mally was born on 1 October 1879 in Krainburg (now Kranj, Yugoslavia) to Dr Ignaz Mally, imperial and royal district physician and medical councillor, and his wife Lucia, née Kristof. The family came from Neumarktl (now Tržič) below the Loibl Pass, where his grandfather was a master leatherworker. After Ignaz Mally's death, his widow moved with her children to Laibach (Ljubljana), where Ernst Mally attended grammar school from 1896 to 1898. It was there that he decided to become a philosopher. In 1898, he enrolled at the University of Graz, where he soon became an avid student of Meinong's philosophy. At the same time, he studied German language and literature in his first year, later switching to mathematics and physics, as these subjects were closer to his philosophical aspirations. In 1900/01, Ernst Mally enlisted in the Imperial and Royal Infantry Regiment No. 27 for military service as a one-year volunteer. In 1906, he completed his studies with a teaching qualification in philosophy, mathematics and physics, took up his first position at a secondary school in Graz, and married Else, née Giriczek, with whom he had two daughters. He had already obtained his doctorate in philosophy in 1903. Even as a secondary school teacher, Ernst Mally maintained his connection with the university and, in particular, with Meinong, and qualified as a professor of philosophy in 1913. From 1915 to 1918, Ernst Mally served in the military, ultimately as a captain in the Landsturm. A serious case of rheumatoid arthritis, which he contracted while serving in the field, led to 70 per cent disability, which increasingly restricted his mobility. Later, he was only able to get around with crutches. A portrait by the Graz academic painter Professor Rudolf Szyszkowitz conveys the impression of a thinker confined to his armchair. Nevertheless, the immediate post-war period brought rapid academic advancement. In 1918, Mally was appointed to the educational chair of Eduard

Martinaks entrusted him with this task, which further accentuated the proximity to psychology that was already evident in Meinong and his students. In this context, Mally, for example, created the first Austrian psychological examination centre for professional aptitude and designed "observation sheets for psychological student observation". After Meinong's death in 1920, Mally took over the management of the psychological laboratory, which at that time was distinguished by important experimental psychological work, including the development of tests. In 1921, Mally was awarded the title of associate professor, in 1923 he was appointed associate professor, and in 1925 he became a full professor of philosophy, thus definitively succeeding Meinong in his chair of philosophy, which he held until his retirement in 1942. This was due to his worsening illness, but it nevertheless hit Mally hard. Anyone who has heard Mally speak knows that for him, teaching was not only the imparting of knowledge, but always also a new thought process that advanced the teacher himself. Now this thought process lacked the stimulation of interested listeners. A circle of friends who regularly visited Mally in his flat in Graz made up for this deficiency to some extent, but when the intensifying bombing campaign forced Mally to move to alternative accommodation in Schwanberg in western Styria, the visits became increasingly sparse due to the war. He worked tirelessly on what he jokingly called his "Schwanberg logic" until he was unexpectedly struck down by a feverish illness on 8 March 1944 at the age of 64.

Mally's *work* contains nothing of his suffering. One could speak of three phases in this work: a first phase in which Meinong's basic ideas dominate, although it should be added immediately that Mally contributed many of his own ideas from the outset by confronting the then nascent mathematical logic or logistics with Meinong's theory of objects and, with his *Grundgesetze des Sollens* (Graz 1926), made an original start in deontics. This work, which is being reprinted here, is recognised as a 'basic work' of deontics. The second phase leads Mally away from Meinong's theory of objects in essential points towards his own holistic-dynamistic views on reality and meaning. This is reflected above all in *Erlebnis und Wirklichkeit* (Leipzig 1935) in literary form. The final phase serves to expand and refine the concept presented in *Erlebnis und Wirklichkeit*. For Mally's philosophy of reality, an approach is successfully developed in *Wahrscheinlichkeit und Gesetz* (Probability and Law, Berlin 1938); for the field of logical formalism, there is the large logical fragment with three formalisms, which are published below; the least developed part of Mally's philosophy is the value theory, the basic principles of which can be found in the book *Erlebnis und Wirklichkeit*. In addition, there are notebooks and lecture notes from his estate that could supplement our understanding of Mally's planned theory of values. There are thus three main areas in which Mally operates: 1. Logic and the doctrine of meaning, 2. Theory of reality and experience, 3. Theory of values. In the following, I will outline Mally's development of thought in these three areas. In his lectures, Mally also dealt with topics in psychology, philosophy of language and social philosophy, for which there is some material in his estate. These cannot be discussed in detail here.

I. LOGIC AND THEORY OF MEANING

Apart from notable representatives of transcendentalism and Thomism, Austrian philosophy in the first half of the 20th century was dominated by the antagonism between neopositivism (the 'Vienna Circle') and the Brentano schools. These were based in Innsbruck and Prague (the Brentano school in the narrower sense with Kastil, Strohal, Marty, Kraus), while in Graz, Alexius Meinong (1853–1920) founded the Graz school of object theory, which spread to Italy and Yugoslavia and enjoyed good relations with the Anglo-Saxon countries.Edmund Husserl, also a student of Brentano, based his 'phenomenology' more in western Germany, where he also encountered transcendentalism. The neopositivism of the 'Vienna Circle' (Schlick, Carnap, Reichenbach, Wittgenstein, among others) had a particular impact in England and the USA and gained worldwide significance, partly through its practical applications in cybernetics (N. Wiener).

The 'Graz School', also known as the 'Austrian School', fights against transcendentalism for the subject-independent objectivity of the object, but at the same time also against the reductionism of the neo-positivists, who reduce the objective to sensory data with logical processing. Meinong starts from 'natural' consciousness, which is not denounced as 'naive' or 'pre-scientific'; it requires neither a Copernican revolution nor a restriction to sensory data, but rather a careful analysis of the given in its full breadth. This achieves a certain affinity with common sense philosophy and neorealism.° This basic attitude connects Meinong with his student Ernst Mally, who also opposes "any violent reduction of facts". Meinong starts from Brentano's idea of the intentionality of acts of consciousness and arrives at four main groups of objects:

1. Objects *of* perception, memory and imagination, 2. Objects of judgement and assumption (objective, factual circumstances), 3. Dignitative in feelings, 4. Desiderative (purposes, target behaviour) in desires. However, Meinong's concept of objects goes far beyond the original one, which only refers to what is real. This leads to the formulation:

What is initially an object cannot be defined in terms of form, as there is a lack of genus and differentia: **for** everything **is an object.**

The early Mally clarifies:

The concept **of the object encompasses** everything **and** anything, regardless of whether it exists or **even** is.

The extension of the concept of object beyond the world of things and also, contrary to Kantianism, beyond being thought bears rich fruit: A wealth of object groups come into view, e.g. relations as "objects of a higher order", mathematical objects such as "numbers" and "geometric figures", exact objects whose special nature mathematics has to explore as a "special object theory". In addition, objects are examined whose relationship to reality is particularly precarious: unreal objects such as "golden mountain" and the much-maligned "impossible objects", e.g. "round square", "wooden iron". This leads to the paradoxical statement:

There are objects of which it is true that there **are no** such objects. ®

This paradox troubles Meinong, who seeks to resolve it by distinguishing between different types of being: existence (real objects), existence (ideal objects), and non-existence (e.g. impossible objects). The "round square" is not completely nothing; thought does not grasp at emptiness, but rather at the mode of being of "non-existence," which is at least attributable to all objects as the "utmost remnant of positional character." The possibility that such concepts are mere products of consciousness is ruled out for Meinong (and also for his students, especially Mally), because every act of cognition presupposes its object as a logical prius; the consciousness-independent objectivity of objects is, after all, Meinong's basic axiom – see Meinong's late self-portrayal.[10]

Ernst Mally underwent a transformation of Meinong's views at a very early stage. With regard to the concept of the object, two approaches can be distinguished: the first, which manifested itself very early on, involves a reduction of the "excess of 'objects'", In the second approach, at the beginning of the 1930s, Mally develops a holistic, dynamic theory of reality, which is partly reminiscent of Bergson and the later Whitehead with his doctrine of 'events', and in which the real individual object, the Meinongian object (of ideas), loses its traditional place as a constituent part of the world.

Mally's criticism can be understood on the basis of a simple perceptual judgement. In such a judgement, e.g. "this is a table", three moments can be distinguished: (1) the act, which consists of the implicit conviction of the truth of the perception, (2) the meaning 'table', (3) the intended object, expressed linguistically in 'this'. Mally now states: Every such judgement intends an object and, through its meaning, also referred to by Meinong as an 'inner' object, gains direction towards an object (a real thing or a real event); but only a true judgement also hits an object, whereas a false judgement has a certain meaning, also referred to as a 'form of meaning', but no real object. In primary judgements, an individual reality is intended — a thing, a process — and in the case of true judgements, it is also hit and grasped, whereby a meaning is "penetrated". Only in secondary turns can thinking also make a form of meaning its "target object", for example in the judgement: "'Table' is a general concept". But as can be seen, this quasi-object 'table' is also intended in the grasping of a new, logically superordinate form of meaning. According to Mally, however, an object in the actual, primary sense is only one that can no longer function as a meaning content itself, but is to be intended in the penetration of a first-level meaning content. Brentano's thesis of intentionality remains valid, but contrary to Meinong, it does not mean that every mental act has a (real) object, but only that it always intends an object, namely by means of a form of meaning that does not itself take the position of the object.

The peculiarity of forms of meaning is explained by Mally's theory of determination, which was developed as early as 1912. According to the older theory of objects, a "round square" is both round and square = not round, which contradicts the principle of contradiction. In contrast, Mally says: A form of meaning such as "round square" or "conifer" is determined by its individual definitions, conceptual characteristics, conceptually constituted, but not fulfilled. "Conifer" is not a real conifer, but a botanical term. Meaningful contents must not be objectified or hypostasised by misinterpreting them as carriers or fulfilments of themselves. This also eliminates the logical difficulties with Meinong's "impossible" and "incomplete objects"; they should be replaced by "logically unfulfillable or incompletely determining meaningful contents".

Meaningful contents exist in their own way, independently of thought and independently of their fulfilment in reality. This philosophical "it exists" must not be misunderstood in a conceptually realistic or nominalistic way. Mally combats both Nicolai Hartmann's "ideal objects" and the "incomplete symbols" of *Principia Mathematica* and the "synsemantic expressions" of the late Brentano. The mistake lies in the failure to distinguish between "mean" and "denote". The "perpetuum mobile" does not designate anything because there is no object of this kind, but it is a form of meaning with a specific meaning. Forms of meaning are timeless places in the manifold of the possible 1° or, according to a late formulation (1941):

'There is this determination' **means:** 'the question of its **fulfilment is**
.' 1•

This condemns both the Platonic and the nominalist attempts to trace the category of meaning back to the category of things. The Great Logic Fragment advocates a strict separation of questions of meaning and being and calls for a purification of logic from all 'existential' admixtures.

'Necessity implies actuality' is a false proposition.

In other words: Ab oportere *non* valet consequentia.

II. LEHRE VON DER WIRKLICHKEIT UND DEM ERFAHRUNGSWISSEN

The problem of real things, Meinong's objects of perception, belongs in Mally's teaching "on reality and experiential knowledge," as a lecture in 1933/34 was titled. This brings us to the second step in Mally's destruction of Meinong's "object."

The Meinong generation, as well as many later generations, are dominated by the view that the world is constructed summatively from fixed elements, that there are completely determined individual things and individual processes, the sum of which forms the world.

Christian von Ehrenfels, a student and friend of Meinong, challenged this view by raising the problem of form as early as 1890, which led to a significant shift in thinking. This leads to the following considerations by Mally: In a vivid form, the parts are determined by the whole, e.g. colour parts by their spatio-temporal environment (contrast effect); taken out of context, 'they' are no longer what 'they' were, i.e. they are not completely self-determined objects, not genuine elements. Everything that is vivid is also vaguely defined as a 'whole'; the boundaries between vivid images become more or less blurred; in some cases they stand out more sharply, e.g. in the case of tools, while in others they are less distinct, e.g. clouds. A parallel in art: naturalism separates more sharply than impressionism. According to Mally, however, this applies not only epistemologically, but also ontologically, because everything is interrelated. For Mally, this means that the strict concept of an object is an unfulfilled form of meaning, because there are no unambiguous things, but only (more or less abstract) closed) quasi-things. Here, the proximity to Bergson is clearly evident, but for Bergson, the strictly separate individual things are products of homo faber, who neglects their connection. However, in Mally's work, the meaning of "exact individual thing" should not be interpreted idealistically. It is suggested by the experience of reality: reality is objectively such that it tends to separate individual things and individual processes, but this tendency is only fulfilled approximately; extreme isolation would be an unfulfilled borderline case. In a letter, Mally writes of "new logical things" that he is working on:

These 'things' are ***not*** things at all, **but** a logical continuation of the basic idea of my **work, which,** contrary to the 'thing' prejudice, seeks to do **justice** to **meaning and** to reality that consists of meaningful things, not **finished objects** – **no** primacy of **predetermined** things! Neither in the conception **of** reality nor in the logical formalism that serves this conception "

This world without precise individual things has a dynamic, tendency-like, striving character. Mally draws on *experience and reality* to support this idea: the mythical thinking of early humanity, the experience of children, of artistic people, our own experience when interpreted accurately and without prejudice towards 'things'.

In the primitive conception**,** *forces* permeate and govern **things** in a living way **and** are active in processes, resting alive in **states.** It is not wrong **to see** in the concept of force in physics — insofar as it has not yet been positivistically debunked — **remnants** of this **way of thinking,** which **is** sometimes **called** mystical, sometimes magical, sometimes merely metaphysical, and thus believed to have **already** been refuted.

Mally, of course, does not want to accept these "pre-scientific" views uncritically; he, too, demythologises them, but not through complete negation, rather by working out their true core.

The essence of this is that human experience does not primarily perceive rigid things, but rather tendencies of events. To illustrate: an artistic portrait is not a representation of a fixed here and now, but rather expresses an aspiration, a direction, by continuing the given floating state in a certain direction, or, as one might say, 'idealising' it. A smooth water surface suggests the geometric concept of a plane, although the water surface is only approximately 'flat'.

The essence of every **aspiration is** that **the goal, which** has **not** been achieved, is nevertheless given in terms of its purpose
and directionally given.

In the same way, the so-called laws of nature, e.g. the law of free fall, are exact formulations of tendencies of events that are never exactly realised.

> No longer material elements, **but** probabilities **(i.e. tendencies)** of specifiable forms of occurrence are the last tangible thing that research into reality **encounters.**[°0]

Through this interpretation, as presented in logistical precision in *Wahrscheinlichkeit urtd* Cresefz(1938) in logistical precision, the laws of nature acquire a meaning that strikes a balance between the 'iron' laws of nature of older natural science and the mere recording of facts, as they would have to be understood by a consistent positivism that would reject the concept of a tendency underlying phenomena. According to Mally, the laws of nature do not express how an event actually takes place, but how it would have to take place if other tendencies did not disturb the pure case, which the world, in which everything affects everything else, does not know.

Meinong's more static, diverse world has become fluid in Mally's work, and the plurality of objects has been rearranged by Mally: here we have the striving, holistic reality, the field of the primarily concrete, without the possibility of strictly separate individual things – here we have the diversity of forms of meaning, the refuge of Meinong's objects that exist outside reality; both reality and meaning can only be distinguished conceptually, for reality is meaning "in modo of fulfilment".[°1] But there is only approximate fulfilment of individual meaning, i.e. no exact straight lines, planes, no exact things, persons, life courses, i.e. — seen from the individual's point of view — a "deficient cosmos".[°°]

III• THE "REALM OF DEMANDS AND VALUATIONS"

The "realm of demands and evaluations" emerges relatively late in Mally's publications, first in the publication of Meinong's posthumous work *Zur Grundlegung der allgemeinen Werttheorie (On the Foundation of General Value Theory),* Graz 1922. In 1926, *Grundgesetze des Sollens (*Basic Laws of Oughtness) *and Elemente der Logik des Willens (Elements of the Logic of the Will)* were published, containing remarkable confrontations *between* being and oughtness that are also significant in terms of value theory. In *Erlebnis und Wirklichkeit* (Experience and Reality, 1935) and in the treatise Zur Frage der "objektiven

Truth', there are numerous statements on value theory, and Mally also gave regular lectures on value theory and ethics, for which lecture notes can be found in his estate. There is therefore a wealth of material from which one could form a picture of the planned third part of Mally's philosophy.

Meinong began studying value theory very early on. His *Psychological-Ethical Investigations into Value Theory* were published in 1894, in which he introduced a psychological concept of value based on the economic value theory of the Austrian School (Menger, Wieser, Böhm-Bawerk). Value is something that is constituted by subjective value retention, and secondly, value is a basic concept that cannot be reduced to utility, biological need, sacrifice, cost or labour; rather, these concepts presuppose the concept of value. Meinong's further path always remains close to psychology, but leads away from psychologism to objective 'impersonal' value, which is grasped through 'emotional presentation', without, however, completely abandoning 'personal', i.e. subjective, value. Meinong's students, however, do abandon it, thereby achieving a close proximity to phenomenological value theory, although they avoid separating value from reality. Value is always a quality that is "attached" to reality and can only be isolated in thought.

For Mally, therefore, the concept of subjective value, constituted by feeling or volition, is untenable. In feeling, human beings do justice to the various qualities of value in reality; in volition, they correspond to the ought that lies in 'things'. However, Mally does not fail to recognise the subjective bias of experience, but he gives it a positive interpretation in the sense of a perspective of rank. This means, first, that all individual situations always convey their own perspectives, factual and value-based, on reality. In every situation, humans help things to reveal new qualities. Thus, every temperament, every mood, every age, every zeitgeist is a situation, or, as Meinong would say, a forum that allows reality to take on certain colours, just as the properties of a mineral are determined by applying various test substances. 1st principle: Every finding is (as a finding from a particular perspective) indisputable.°°Its counterpart, however, is the 2nd principle of the ranking order of findings: there are better and worse ones. The better ones distinguish more. In the emotional sphere, and thus also in the realm of judgements: the sensitive person has greater emotional sensitivity to differences. Errors in judgement arise when one prefers a coarser finding to a finer one and when, remaining stuck in a partial meaning, one withdraws from the pursuit of the overall meaning. In his later letters, he writes about this:

> I **cannot** help **but** seek a better one. **That** is the nature of the mind.•4
> **nature of meaning.**•4

It is clear that Ernst Mally intends to combine his philosophy of meaning and reality with value theory, but unfortunately this part is the least developed.

IV. THE IMPACT OF BRNSTALLY'S PHILOSOPHY

The *impact* of Ernst Mally's philosophy was more profound than widespread. He became best known during his period of work on logic and object theory. Theodor Ziehen ranks Mally alongside Couturat and Russell as one of the most important authors on logic· Mally was also entrusted three times with writing *literature reviews on logic and epistemology*. The *basic laws of ought* are considered the 'basic work' of deontics. Symptomatic of Mally's international reputation is his contribution to the commemorative publication for the Swedish philosopher Efraim Liljeqvist, to which he contributed the treatise 'On Subjectivities and their Objective Meaning', which he himself described as his actual 'breakthrough to his own'.° Erich Rothacker writes about *Experience and Reality* (1935):

> For the underpinning of higher **noetic functions through *mythical* and**
> *magical* **experience**, I **would like to** refer to the excellent **book by E.** Mally.

For the later period, which was overshadowed by political events, mention should be made of Mally's unsuccessful attempt to preserve the Austrian peculiarity of philosophical propaedeutics as a subject taught in secondary schools. Mally's Viennese friends, the secondary school teachers Josef Krug and Otto Pommer, had written a textbook on philosophical propaedeutics in the spirit of Mally even before Austria's incorporation into the German Reich (1938), which had a brief but widespread impact, as did Alois Hößer's textbooks spreading Meinong's ideas.

Krug, along with Franz Kröner, Mally's successor to the Chair of Philosophy, Hermann Wendelin, Gertraut Laurin, Lieselotte Kern, and the Bonn philosopher of history Fritz Kern, was one of the few visitors to Mally at his refuge in Schwanberg, where the "Great Logical Fragment" was written. The Philosophical Society at the University of Graz held a memorial service for Ernst Mally, at which Johann Mokre and Karl Wolf spoke about Mally's achievements in various phases of his life.

Salzburg, September 1970.

NOTES

In a letter to Dr. Laurin dated 21 February 1943, Mally refers to 1. the "domain of pure theoretical meaning", 2. the "empirical theoretical domain" and 3. the "realm of demands and evaluations".

° Meinong's student V. Benussi worked in Padua, Franz Weber in Ljubljana. Anglo-Saxon contributions: B. Russell, 'Meinong's Theory of Complexes and Assumptions', *Mind,* N.S. 13 (1904); J. M. Baldwin, *Thought and Things*, London 1906-08, especially Vol. II, p. 423; W. M. Urban, *Valuation,* London 1909; H. O. Eaton, *The Austrian Philosophy of Values*, Norman 1930; J. N. Findlay, *Meinong's Theory of Objects and Values,* Oxford 1963; J. N. Findlay, ‘The Influence of Meinong in Anglo-Saxon Countries’ in *Meinong•Credenkschrift* (Writings of the University of Graz, Vol. I), Graz 1952, pp. 9-19; R.M. Chisholm, *Realism and the Background of Phenomenology,* Rhode Island 1959.

Rudolf Kindinger, the distinguished editor of Meinong's scientific correspondence *(Philosophenbriefe*, Graz 1965) and editor of the third volume of *the Meinong Complete Edition* (Abhandlungen zur Werttheorie, Graz 1968), outlined the relationship between Meinong's theory of perception and common sense philosophy in 'Das Problem der unvollkommenen Erkenntnisleistung' (The Problem of Imperfect Cognition) in *Meinong-Oedenk-schrift*, Graz 1952.

^ A formulation by Mally in his "Vademecum für die Sammlerin" (Vademecum for the Collector), a critical overview of his earlier writings written for Martha Sobotka in early 1933.

^ Cf. Meinong on himself, in *Philosophie der Gegenwart in Selbstdarstellungen* (ed. by Raymund Schmidt), 2nd ed., Leipzig 1923.

^ Ernst Mally, ‘On the Concept of the Object in Meinong's Theory of Objects’ in *Yearbook of the Philosophical Society at the University of Vienna,* Leipzig 1913, p. 14.

° Cf. Franz Kröner, ‘On Meinong’s “impossible objects”’ in *Meinong Memorial Publication* (note 2), pp. 67–80.

® *Untersuchungen zur Gegenstandstheorie und Psychologie* (ed. by Aiexius Meinong), Leipzig 1904, p. 9.

• Ibid., p. 9.

'° Meinong on himself (note 5), p. 144 and elsewhere.

'1 Ernst Mally, *Erlebnis und WirklicMeit*, Leipzig 1935, p. 135. Also noteworthy in this regard are Mally's letters to Franz Pichler dated 5 and 6 January 1934.

'° Ibid., p. 143f.
1• This formulation can be found in the lecture “Gegenstandstheoretische Grundlagen der Philosophie” (Object-Theoretical Foundations of Philosophy) 1929–30.
[14] Lecture “The Nature and Significance of Logic”, 1941, lecture notes in the estate, p. 24.
• Letter to Dr. Laurin dated 14 November 1943.
• Christian v. Ehrenfels, “On Gestalt Qualities,” *Quarterly Journal for Scientific Philosophy,* 1890.
1' Letter to Dr. Laurin dated 4 April 1943.
Ernst Mally, 'Fundamentals of Gestalt', unpublished manuscript (in the estate), p. 73.
'° 'On Reality and Empirical Knowledge', lecture 1934 (in the estate), p. 40.
Ernst Mally, *Wahrscheinlichkeit und Gesetz (Probability and Law),* A Contribution to the Probability-Theoretical Foundation of Natural Science, Berlin 1938, p. 11.
°' According to a note in the estate. More details in the Great Logical Fragment.
•° Kar1 Wolf, *Ethical Contemplation of Nature,* Salzburg 1947, pp. 48ff.
** *Experience and Reality* (note 11), p. 115f.
°4 Letter to Dr. Laurin dated 19 July 1943. More on this in the chapters Religion and Customs and Morality, in *Experience and Reality,* pp. 67ff.
•5 For a comparison of the whole: Karl Wolf, ‘Die Grazer Schule: Gegenstandstheorie und Wertlehre’ in *Philosophie in Österreich,* special issue of *Wissenschaft und Weltbild,* Vienna 1968, pp. 31-56.
°• Theodor Ziehen, *Lehrbuch der Logik auf positivistischer Grundlage (Textbook of Logic on a Positivist Basis),* Bonn 1920, p. 236.
•' See list of publications below, p. 328.
°® See the “Vademecum for Collectors” (see note 4).
•• Erich Rothacker, *Die Schichten der Persönlichkeit (The Layers of Personality),* 5th ed., Bonn 1952, p. 78.
•0 Krug-Pommer, *Textbook for Introductory Philosophy Classes,* Part One: Psychology, Vienna 1933, Part Two: Logic and Theory of Science. Fundamentals of Philosophy, Vienna 1938.
°1 Gert H. Müller, *Das philosophische Werk Franz Kräners,* Basel 1962, p. 34.
•° Funeral service for Ernst Mally at the Philosophical Society at the University of Graz, 16 November 1950; cf. Karl Wolf, ‘Die Spätphilosophie Ernst Mallys’ in *Wissenschaft und Weltbild* 5 (1952) 145-53.

JOHANN MOKRE

GE GEN STAND STH EO RIE — LO GI K — DEO NTI K

The progression of Ernst Mally's thinking, as indicated in the title, will be outlined here, partly with reference to relevant publications and partly based on personal memories from the time I knew Ernst Mally, first as a student and later as a modest colleague. This account does not follow historical chronology, but rather the development of the ideas to be presented.

In this sense, the little-known essay "On the Independence of Objects from Thought" [1], with its treatment of four questions, should be placed at the forefront. Regarding the first: "Can a thought comprehend itself?" it is demonstrated that the statement "a thought comprehends itself" falls under the paradoxes of self-relation discussed by Whitehead and Russell, and is therefore neither true nor false, but meaningless. For a similar reason, the statement "that a thought encounters its object as comprehended by that very thought" is impossible. However, it is also impossible for a thought to encounter an object as grasped by another thought, because this other thought would then be grasped by the first, either as a correlate of the first (which is meaningless) or of a further thought. But the same difficulty arises with this further thought, leading to an infinite regress. Thus, the object of a thought is independent of the grasping thought.

This view forms the basis of Ernst Mally's epistemology, which is a sharp rejection of epistemological idealism and can be described as objectivism, since it is not necessarily limited to realism in the usual sense. What is referred to here as an object does not have to be something real. Later, incidentally, this object is replaced by "the determinate" of a determination and, even later, by the "meaning" intended in a psychological experience, and only the real is regarded as an object. "Object-theoretical consideration"

, however, means quite generally a "consideration of being as such," independent of factual existence.

Against this epistemological background, which was of course already present in Mally as a student of Meinong, stands the earlier work on "Gegenstandstheoretische Grundlagen der Logik und Logistik" (Leipzig 1912). In it, an object logic is developed in a logical manner instead of the thinking logic that was often still the only one in use at the time. The logic of scope is juxtaposed with a logic of content, or rather, class theory with an objective (= factual) theory, through the principle of reciprocity: if *A* implies *B*, then the class *a* defined by *A* falls under the class *b* defined by *B*. In the same way, the individual case of a general fact is also juxtaposed with the individual thing. Already in this work, there are hints in the "Applications" about a theory of possibility and similarity.

This appears in completed form in 1922. Here, the exact

Definition of two important terms. One is the "implicative community," the other is the "determining element." The implicative community generally establishes similarity, and in the specific case of the implicative community with facts, possibility. Determining elements, however, allow a determination to be broken down into its elements and made accessible to a kind of quantification. In this way, degrees of possibility and similarity become understandable as an augmentable relationship. There is implication between similar things and thus also the possibility of mutual substitutability. This transition from one case to another similar case is also fundamental to the conclusions of analogy and induction. One can see how far such seemingly purely formal considerations can carry us.

The interpretation of possibility becomes of decisive importance in the final phase of Mally's thinking to be presented here, in the development of a doctrine of ought and value· The starting point here is that we relate to the facts of the world in different ways: thinking, by grasping them, willing, by trying to shape them. We call thinking correct when it achieves its goal, when it grasps the facts; we call willing correct when it achieves its goal, when it realises the desired state of affairs. But there is just as little to be desired in reality as there is to be changed in it.

because their cases are completely determined. However, the facts to which the will is directed are not understood as completely determined facts of reality, but rather in incomplete determinations that give them a certain possibility. Actually, the meaning of volition is that this possible state of affairs should become reality. Of course, this depends not only on the perceived determinations and the possibility given with them, but also on many other circumstances of reality. One of these is my own volition as a piece of reality. That is why we often say not only: I want to do something, but I will do it; of course, only with a certain probability that depends on the two circumstances mentioned above. Mally expresses it this way: those who think clearly and want strongly know most about the future.

Just as the laws of facts, as represented by (objective) logic, are conditions for correct thinking, the laws of ought are conditions for correct willing. To believe that willing first constitutes ought would be, in the realm of willing, what epistemological idealism is in the realm of thinking. Just as logic begins with axiomatic fundamental laws of facts, **Mally** also believes he can specify axiomatic laws of oughtness. He names five of them, one of which will be mentioned here, namely the principle of co-requirement or consistency: what is implied by a required fact is itself required. It is — as axioms so often are — basically self-evident. The fact implied by the demanded fact is, after all, a necessary condition of the demanded fact, and the demand for a fact would be meaningless if the necessary condition for it were not also demanded. Applied subjectively: when we desire something, we also desire its necessary conditions. This has far-reaching significance. Since the facts are implied in any given circumstance, they are always also demanded, i.e. required, which is also implied when a strong desirer expresses...

expressly wants under any circumstances, come what may.

Just as a meaningful logic of being is only possible if there is actual being, so a meaningful logic of ought is only possible if there is actual ought, at least *one* actually ought-to-be fact. It can be shown that this and the facts are equivalent in terms of being and also equivalent in terms of ought.

The question of correct volition (or oughtness) is also linked to the question of objectively correct value because of the correlation between oughtness and value. A judgement is called materially correct if it corresponds to the facts, even if purely by chance. The laws of logic provide the necessary – but not sufficient – conditions for this. Similarly, a desire that achieves its goal is said to be materially correct, and again, the necessary but not sufficient conditions for this are the axioms of oughtness. This includes, for example, that all individual desires, but also all their implications, come together to form a consistent desire.

Since material correctness depends on many unpredictable determinations of reality, it cannot be demanded, neither in judgement nor in volition; but it can be striven for, taking into account the available partial aspect of reality. Formally correct is judgement with probability (assumption) relative to the available partial aspect and also volition relative to it. Both may not prove themselves, but they have the probability of proving themselves and are provable.

However, this probability of realisation (the possibility) is clearly not the only factor in the choice of goals, but also the value of these goals. We should act not only to the best of our knowledge, but also to the best of our conscience. What we really go by is the expected value, composed of possibility (m) and value; the latter *(w:* value of the individual goal) measured against the total value for me (iF), i.e. approximately: *w|W*. This is the measure of feasibility: *b —-m- (w|W)*. In this composite probability, *m* can now be interpreted as "external possibility", w|iP as the possibility that, in proportion to the individual value, the greatest value iP corresponding to my overall will also becomes reality.

This also provides access to a theory of objective value, because this overall will is formally correct if, as formally correct, it is free of contradictions; objectively, however, this corresponds to a consistent system of determinations that are supposed to be, which are correlated with objective value. Of course, this is only an intellectual equivalent for value. Its essence is done justice in the sense of value. It is rather the case that the ethical sense of value predominantly guarantees the probability of realisation, rather than the other way around, which is why this theory cannot be called a utilitarian ethic. This idea that the good

Ultimately, this corresponds not only to the classical doctrine of the convergence of being and value from Aristotle to Leibniz, but also to the pre-scientific view as expressed in phrases such as "honesty is the best policy," etc.

This connection between value and possibility also establishes a link to the next phase of Mally's thinking, to the doctrine of potential aspirations.

Graz, September 1970

NOTES

! *Zeitschrift für Philosophie und philosophische Kritik* 155 (1914) 37-52.

• Cf. Bertrand Russell and Alfred North Whitehead, *Introduction to Mathematical Logic* (translated into German by J. Mokre), Munich 1932, p. 55f.

° 'Studies on the Theory of Possibility and Similarity, General Theory of the Relationship between Objective Determinations', *Academy of Sciences in Vienna,* Philosophical-Historical Class, Proceedings 194, Volume I, Treatise, Vienna 1922.

* *Fundamental Laws of Oughtness, Elements of the Logic of Willing,* Graz 1926. Ibid. p. 48.

PAUL WEINGARTNER

COMMENTS ON MALYS' LATER LOGIC

The title 'Mally's Late Logic' does not refer to a unified work or system. This is already clear from the first part of the introduction, from the 'Additions and Changes' to Formalism **III**, and finally from the overview of the estate (at the end of this volume). Rather, Mally's late logic consists of many fragments and parts that can only be partially arranged in a complete order because they deal with many different metalogical topics concerning the philosophical foundations and prerequisites of logic. In this respect, the following remarks refer only to some aspects of the uniform and systematised parts of Mally's later logic, namely Formalisms I, II and **III.**

In Formalism I, Mally deals with propositional logic. Mally's system is based on Hilbert and Ackermann's propositional calculus.[1] However, Mally makes some interesting additions. First, he attempts to introduce a more general terminology with regard to propositional variables (than is commonly used). He speaks of *figures* and *formulas:* some figures are formulas (= correct formulas), some of the correct formulas are *basic formulas* (= axioms). Every formula is a figure, but not vice versa. (The term 'figure' seems to correspond to the so-called *well-formed formula*.) *'X', Y'* are *basic figures* and have the meaning of variables (placeholders); the quotation marks are not part of the basic figure. Basic figures are not formulas. — A second point that goes beyond the usual formal requirements for an axiomatically constructed propositional calculus is the negation rule. Mally's negation rule (§') states: Not every figure of formalism I is a correct figure. This rule is not included in the Hilbert-Ackermann system. According to Mally, the negation rule should be added to the substitution and derivation rules of a system.

The reason Mally gives for this is that this rule is intended to exclude from the outset — or rather, that a system of logic should be structured in such a way from the outset — that not every figure of formalism is a correct figure (cf. Formalism I 5.0).

In Formalism II, Mally describes a predicate calculus. The propositional calculus is taken from Formalism I. The newly added axioms are the principle of universal substitution and that of existential generalisation. However, it should be noted that Mally understands this predicate calculus in a way that differs somewhat from the usual one. The main point is that he wants to establish a predicate calculus that does not make the existential assumptions that are made in today's standard calculi, especially in *Principia Mathematica*. According to Mally, propositions that say that something exists are *foreign to logic.* For this reason, he speaks of a *calculus of the fulfilment of determinations.* To indicate that the quantified propositions (z) *Fx* and *(Ex) Fx* are to be understood without existential import, he writes them with square brackets as *{x} Fx* and *{Ex} Fx* and proposes the following interpretation: *'{x} Fx'* means "*'Fx'* applies regardless of fulfilment" or "*'Fx'* applies unconditionally". *'[Ex} Fx'* means "*'Fx'* is fulfilable" or "*'Fx'* allows for a correcting fulfilment". The individual variables 'x', *'y'* ... serve only as "placeholders" *@laceholders).* Such an interpretation, as proposed by Mally, therefore makes no assumption about whether or not there are objects to which the symbols of formalism refer; and, according to Mally, an interpretation of formalism must say nothing at all about such *facts*. According to him, the logical system of *Principia Mathematica* is not a logic in the pure sense, but an existential logic; and he refers to a concession made by Russell himself (which he made in 1919, i.e. nine years after the first publication of *Principia Mathematica*):

> The primitive propositions in *Principia Mathematics* are such as to allow the inference that at least one individual exists. **But** I now view this **as a defect in** logical purity.

Formalism **III** is an extension of the pure predicate calculus (calculus of *the fulfilment* of determinations) of Formalism II to a predicate calculus of *the fulfilment* of determinations (which contains Formalism II). It introduces *'(Ex) Fx'* as a primitive term (a term that is not explicitly defined), which, however, is defined by a kind of implicit definition.

is defined: *‘(Ex) Fx’* means “there are objects (values of x) that satisfy Fx” or “there is at least one satisfaction of *Fx”*. The sentence *‘(x)Fx’* is then introduced by an explicit definition (D5) that says “every value of x, and there are such values, satisfies *Fx”*. Mally uses *'(Ex) Fx» Ex} Fx'* and *'{x} Fx (Ex)Fx'* or the better version *[EF}* (x) *Fx (Ex) Fx)'* as axioms (see correction); important theorems in this context are *'(x)Fx» (Ex) Fx'* (the converse does not apply), *'(x)Fxx(Ex)Fx'* and '[x) *Fx» (Ex) Fx'*. Formalism III also contains a type of definite labelling (description), whereby 'z' is used as a symbol for individual objects.

The ideas underlying Mally's Formalismen II and III for a logic free of existential presuppositions have been espoused in a similar way by a number of philosophers and logicians since that time (roughly since 1940). The progressive dismantling of existential presuppositions in logic begins — if one disregards scholasticism, especially late scholasticism and certain approaches in Leibniz — first vaguely in Bolzano and then explicitly in Brentano; here, of course, we are still dealing with the beginning of the dismantling of the existential presuppositions of Aristotelian syllogistics: Bolzano assumes, like Aristotle, that the universal proposition "All *A* are *B*" presupposes the existence of a domain of things (with at least one element) with property *A*, but on the other hand he speaks of "objectless ideas", so that Łukasiewicz's axiom "Some *A* are *A*" (which he uses to interpret Aristotelian syllogistics) does not apply according to Bolzano. [4] Brentano was the first to explicitly reject these assumptions of Aristotelian syllogistics, which is why he only recognises 15 of the 19 Aristotelian forms of inference. In the classic works of modern logic, in Frege's *Begriffsschrift* (1879), Hilbert and Bernays' *Principia Mathematica (I* 910) and in the *Grundlagen der Mathematik* (1934), this dismantling of the existential premises of Aristotelian syllogistics has already been accomplished.The first attempts at a further reduction of the existential premises can be traced back to both the Graz School and the Polish School of Logic. In the Graz School, Mally's first, as yet unexplicit attempts can be traced back to his work 'Gegenstandstheoretische Grundlagen der Logik und Logistik' (1912), in which he

He puts forward arguments against extensional logic, whose symbols must always denote things that exist in some way or corresponding classes of things; rather, he believes that logic deals with the meanings (intensions) of symbols and not with the things denoted (extensions). Meinong praises this view and Mally's criticism and speaks of a tendency towards *the internalisation* of logic (as opposed to the *externalisation* of scope logic).[8] In 1935, Mally had already developed the idea of logic as a system of pure forms (determinations).[9] Formalism II and III are then an attempt to establish such a system of pure determinations. In the Polish school of logic (at least since around 1929), Lesniewski in particular has dealt extensively with highly differentiated calculi that make fewer existential assumptions than the usual predicate calculi of modern logic.[10] Since 1940, many philosophers and logicians have addressed the issues surrounding the problem of the existential presuppositions of logic and their dismantling, and proposed solutions. Quine, Hintikka, Barcan-Marcus and others, in particular, have dealt with this in detail.[11]

This brief and incomplete overview already shows that Mally, together with Lesniewski, occupies an important place in the history of the development of this problem area with his fundamental ideas of a logic purified of existential presuppositions: If Brentano was one of the first philosophers to call for the removal of the existential presuppositions of Aristotelian logic, Mally was one of the first philosophers to propose the even more radical purification of modern logic from existential presuppositions.

A few months before his death, Mally changed several points in his Formalism III; in particular, he used different definitions and axioms. A selection of these changes and additions are included in this volume as an appendix to Formalism III. The notes at the end of the volume, in which Mally explains some of his fundamental ideas concerning Formalisms II and III and specifically addresses questions of the conditions of existence, may contribute to a better understanding.

Salzburg, September 1970

NOTES

G. D. Hilbert-W. Ackermann, *Grundzüge der theoretischen Logik (Fundamentals of Theoretical Logic),* 2nd edition, Berlin 1938.

° B. Russell, *Introduction to Mathematical Philosophy,* London 1919, p. 203, note. For a discussion, cf. P. Weingartner, 'Der Begriff der Existenz in Russells Theorie der Deskription' in *Deskription, Analytizität und Existenz* (ed. by P. Weingartner), Salzburg 1966, pp. 69–86, pp. 78ff.

• Cf. B. Bolzano, *Wissenschaftslehre,* II, Sulzbach 1837, pp. 24f. and 328ff.

• Cf. B. Bolzano, *Wissenschaftslehre,* I, Sulzbach 1837, pp. 304ff. For a discussion cf. I. Dapunt (=pseudonym for E. Morscher), ‘Zwei Typen von Systemen der traditionellen Logik,’ *Archiv für Geschichte der Philosophie* 50 (1968) 275-81.

• F. Brentano, *Psychologie vom empirischen Standpunkt,* Leipzig 1874, Hamburg 1955, vol. II, chap. VII, §15. Cf. A. N. Prior, *Formal Logic,* Oxford 1962, p. 166f.

^ Cf. D. Hilbert and P. Bemays, *Grundlagen der Mathematik,* Vol. I, Berlin 1934, p. l05f. '

E. Mally, ‘Gegenstandstheoretische Grundlagen der Logik und Logistik’, supplement to Vol. 148 of *the Zeitschrift für Philosophie und Philosophische Kritik,* Leipzig 1912.

^ *Philosophie der Gegenwart in Selbstdarstellungen* (ed. by R. Schmidt), Leipzig 1923, Vol. I, pp. 113 and 153.

° Cf. the following passage: "... since pure form (as determination) ... is not introduced as reality", in E. Mally, *Erlebnis und Wirklichkeit,* Leipzig 1935, p. 134, cf.

p. 123. The author is indebted to Prof. Wolf for the reference to the passages indicated in notes 7, 8 and 9.

10 Lesniewski was probably the first logician to establish systems of logic and ontology that not only made limited assumptions about existence (in contrast to the systems of modern logic), but also allowed both existing and non-existing things to be assigned to the value ranges of variables.

S. Lesniewski, ‘On the Foundations of Ontology’, *Compies Rendus des Séances de la Société des Sciences et des Lettres de Varsovie* 23 (1930) 111–32. For a discussion, cf. C. Lejewski, ‘On Lesniewski’s Ontology’, *Ratio* (1 */) 56–78; J. Slupecki, ‘S. Lesniewski's Calculus of Names’, *Studia Logica* OI (1955) 7-71 ; G. Küng, *Oncology and Logistical Analysis of Language,* Vienna 1963, chap. 8; P. Weingartner, ‘Ontologische Fragen zur klassischen Wahrheitsdefinition’ in *Grundfragen der Wissenschaften und lhre Wurzeln in der Metaphysik* (ed. by P. Weingartner), Salzburg 1967, pp. 37—64, esp. pp. 56ff.

* Cf. W. ¥*. 0.* Quine, ‘Notes on Existence and Necessity’, *The Journal of Philosophy 40* (1943) 113-27; *From a Logical Point of View,* Cambridge, Mass. 1961 (and numerous essays); J. Hintikka, ‘Existential Presuppositions and Existential Commitments’, *The Journal of Philosophy* 56 (1959) 125-37; R. Barcan-Marcus, ‘Interpreting Quantification’, *Inquiry* 5 (1962) 252-59.

COMMENTS ON THE **TEXT**

I. TEXT DOCUMENTS

In addition to Meinong's estate, which is curated by Rudolf Kindinger and kept in safekeeping by Martha Sobotka, the Graz University Library also holds the written estate of Ernst Mally. This includes a large collection of Mally's handwritten notes with the collective title "O.M." (Opus Magnum), Mally's jocular name for his planned multi-volume magnum opus. A 21-page manuscript written in pencil was probably typed during Mally's lifetime. There are two copies of this transcription, which contain annotations by various hands, probably those of Franz Kröner and Hermann Wendelin, professors at the University of Graz, who wanted to prepare the fragment for publication with Martha Sobotka after 1945. However, the plan could not be realised at that time.

This edition draws on Mally's aforementioned manuscript *(Grofles Logik-Fragment)*, adding the handwritten *Formalismen* I, II and **III** with *letters* to Dr Gertraud Laurin, which provide insight into the overall plan of the "Schwanberger Logik" (Schwanberg Logic), as Mally called this first part of his planned magnum opus among his circle of friends.

II. TEXT DESIGN

The edition adheres closely to the original, particularly with regard to the use of quotation marks and Mally's habit of emphasising expressions and phrases by placing them between commas. Their function is to encourage the reader to read more thoroughly. The small print after a colon has also been retained because the rules are unclear in this case. In clear cases, the punctuation has been supplemented in accordance with the applicable rules. The § symbol in chapter headings and notes has been omitted in accordance with the above-mentioned transcript and in line with Mally's wishes. Where individual words have been added or changed to make sense, this is noted in the editors' comments marked with •. Mally's notes (without *) are reproduced verbatim, even where they are linguistically incomplete, for example. Only references have been supplemented and updated where possible. Only those parts of the letters that are relevant to the discussion have been selected. In the *Basic Laws of Ought*, some logical symbols have been replaced by those now in common use; instead of = there is now =', instead of '.' there is '/\'. The table of contents of *the Basic Laws of Ought* has been moved to the front.

II I. D A T I O N

Only a few of the manuscripts are dated, but for the most part their creation can be traced through Mally's letters to Dr Gertraud Laurin. The earliest mention is found in a letter dated 24 August 1941: "My work is progressing *slowly*." 1 April 1942: "Worked diligently, namely on the Opus." 15 July 1942: "The beginnings of the O.M. are lying next to me." 23 August 1942: "I often worry whether I will ever finish this work, which in a deeper sense is the work of a lifetime – perhaps at the end of my life." 10 September 1942: "On Monday, I reviewed what I had written. I had already lost track of the overview, and I would have liked to start rewriting the inadequate parts right away and then continue. But I can't do that yet; first, I have to read a few things that I need to take into account ...". 27 September 1942: Mally "writes philosophical notes, which will probably soon 'condense' into solid pages of the O.M.". 8 February 1943: "Progress is being made, but there are still some setbacks, so that today I have to discard what I wrote yesterday and the day before." 10 March 1943: "I now know what to say about the concept of numbers and the associated logical questions." 24 May 1943: "Now I am dealing with 'illustrative' ... geometry in my work." 14 June 1943: "The constitution of the concept of number must be followed by a little more mathematics. In particular, 'infinite sets' must be considered ... Then comes the matter of the 'infinite conjunction or disjunction of individual cases' and everything that runs counter to existential logic

To say is... and this is followed by... 'logic without the axiom of existence', which is thus completely without axioms and contains only analytical propositions. Finally, what is logical and what is logic, to conclude the formal part". 19 July 1943: "... the number §, after breaking it down into three, written down ...". 27 September 1943: "... Autumn should bring me the systematic structure of logic ... ; now I hope to establish the calculus in a few formulas. First, the content had to be put in order; for formalism, I can use much of what others have already accomplished. What I have to do and hope to do is to shape it in such a way that the meaning of logic is best brought to bear." 3 October 1943: "... at the moment it is the theory of analytical propositions, to which those of logic belong, i.e. the logic of logic ...". 11 December 1943: "... The logic I am developing is exactly what I need for the theory of reality. The transition is already mapped out ...". 12 February 1944: “Wen-delin, who studied logic with me at the time and has since always combined logic with mathematics, his field of expertise, including in terms of forms and formal methods, is finding great interest and appeal with a series of lectures on logic before a growing circle of close and distant colleagues; he wants to serve up a chapter of Schwanberg's logic to these people soon. I want to give it to him and gain at least a clean typescript in the process ...". 27 February 1944: "... just want to finish the fair copy of Formalism I, ... only a few pages are missing ...". 3 March 1944: "In gurgite vasto, something has come of these terrible nights. Corrections, improvements, more clarity. Thank God".

The transcription of the fragments presented was therefore carried out by From 1941 until the first days of March 1944, Mally began working on formalism in the autumn of 1943.

ERNST MALLY

GRAND LOGIC F RAGMENT

Martha Sobotka

FOREWORD

On 8 March 1944, Ernst Mally passed away in Schwanberg, where he had fled from the bombing raids of the Second World War, almost paralysed as a result of an injury sustained while serving at the front in the First World War. In deep solitude, he worked until the end on creating the foundations of a logic that was to form the first part of a three-volume work, a philosophy of the natural world view. In his last will and testament, Mally bequeathed his literary estate to university lecturer Dr Martha Sobotka, "because she played a significant role in my intellectual life and work during crucial years".

I painstakingly sorted and examined this estate, which consisted of hundreds of jumbled loose sheets of paper—the result of looting in 1945—and, in accordance with a testamentary disposition, handed it over to university lecturer Dr. Franz Kröner, who was to advise me on its utilisation. Dr. Kröner and university professor Dr. Herrmann Wendelin, friends of Ernst Mally, focused primarily on the "logical fragments," but saw no possibility of publication in the post-war period.

I therefore took back the posthumous writings and published a lecture by Mally on "The Nature of Natural Law" in the Vienna *Journal of Philosophy, Psychology and Education* (1948).

Since all further efforts to make the estate accessible to the scientific world failed, I entrusted the writings to the library of the Karl-Franzens-University in Graz, on condition that they be edited by an expert.

On the 25th anniversary of **Ernst** Mally's death, University Professor Dr. Karl Wolf, the only one of his students on whom his teaching had a lasting influence, as numerous treatises attest, and I...

decided to commemorate our great teacher and friend in this book with gratitude and reverence. I would like to thank Professor Wolf, his colleagues and the sponsors for ensuring that a significant part of his estate has taken shape as a contribution to contemporary philosophy.

Graz, 12 September 1970.

1

NOTE ON THE USE AND UNDERSTANDING OF LANGUAGE

Many have warned of the dangers of language, and it has been much maligned for its imperfections. Its inaccuracy is perhaps the least of the faults attributed to it; but its worst is said to be that, as an expression of a fundamentally erroneous primitive conception, it repeatedly repeats the venerable but ultimately annoying images of mythical thinking, instilling them in the minds of people of a new era from childhood and making it immeasurably difficult for them to think impartially and critically. It does not allow people to approach the pure facts at all, always offering them concepts that have already been formed and forcing even the most independent thinkers to use these forms of expression and, when they communicate, to take upon themselves the risk of being misunderstood.

Certainly, these difficulties and dangers exist. They are still there, albeit to a lesser extent, even when we succeed in using a new language in a subfield of science that seems to be free of all these archaisms. To introduce an artificial language similar to mathematical sign systems, one needs a pre-existing and, ultimately, natural language, as the only one that, insofar as pointing and all cooperation with the "situation" belong to its "natural" means, does not require any other pre-existing language in order to introduce itself and develop further. Moreover, no new sign prohibits those who need it, even if they otherwise need it correctly, from assigning it a meaning determined by that mythical thinking. Thus, someone may calculate correctly but have the strangest ideas about the so-called nature of numbers.

A language can be very useful for certain purposes, and it can be assumed that it will increasingly adapt to these purposes, albeit not without deviations and detours. Natural languages have a wide, almost seemingly unlimited variety of such purposes.

However, practical matters of "everyday life" tend to prevail. Communication is precise and reliable enough to guide the actions and behaviour of individuals in the context of life, insofar as it cannot be organised without words: just as understanding mathematical sign language among experts ensures such consistent usage that there is hardly any dispute about the solution to an arithmetic problem, for example. However, this does not guarantee a similar unanimity, even among experts, for example with regard to the "nature of numbers" or, to put it in more modern and less demanding terms, in fundamental questions of arithmetic and other branches of mathematics. Everywhere, the forms of expression of a language fulfil their mediating function, largely independently of the burden of thought and perception with which they are laden. What matters initially is only the manner of use in a particular area of application: the functional concepts that belong to these areas. The areas shift and expand constantly with the increasing diversity of experiences and activities; new means of expression emerge, old ones change their functional meaning.

Everywhere, it seems that the only thing that matters is the service that a language performs in the language community; according to a widespread view, this consists in controlling its externally perceptible behaviour, which can not only be recognised in certain observable features of this behaviour, but is also exhausted in them; of course, the linguistic utterances themselves also belong to the behaviour in question.

The question of whether this view can be implemented is not to be discussed here. In any case, a statement such as "someone thinks this or that when using a certain expression" also has a meaning in itself. It refers not to a mental experience, but to speech and all the physical, and in some cases internal, behaviour that has otherwise been described as the physical counterpart or accompanying phenomenon of the experience of thinking. The fact that someone "thinks something" when they use an expression, and what they think, is taken into account by the language theorist in every conception of language, speech and hearing: without prejudging final interpretations, and in order to be complete, they can always deal with the meaning of language. It can only be what someone thinks when they use an expression, be it a word or a phrase, this conscious meaning...

content, for a specific use and a whole range of applications, may be of little or no importance. But there will always be uses of the term for which the conscious content of thought is decisive. In arithmetic and in all internal mathematical use of numerical names and their combinations, what one thinks of as the "essence of numbers" is not noticeable; but he can make statements in which he expresses himself precisely on this point, and if he is superstitious, this mathematically irrelevant factor may even determine his actions. But we can disregard this; there are certainly meaningful metamathematical statements; in them, definitions of mathematical terms come into play that play at most only a hidden role in mathematical usage.

However, the dangers of language are like many others: they can be avoided if one is aware of them. They are particularly *common* in discussions that are referred to as philosophical. As has often been noted, misunderstandings, misinterpretations and difficulties arise particularly from the fact that the usual form of our declarative sentences seems to presuppose a "sentence object", a "subject object", and perhaps also one or more "objects"; that we speak objectively of **space,** time, activity, property and relationship. The verb makes the "predicate" appear active or passive, and in the end we speak of language and how it misleads us through its nominalisations, still using the dangerous forms to warn against them, to render the dangers of error harmless. This succeeds, provided we agree on a flawless meaning of their use; but it is not without difficulties. The old, apparently deeply rooted **tendency to** objectify, which does not cause any serious disturbance in countless applications of language, has been fatal to many philosophical trains of thought. If anywhere, the requirement of presuppositionlessness applies to fundamental reflection, and first of all in the sense that no objects are to be assumed as given.

We must also make use of language in fundamental reflection. In order to understand its statements without the prejudice that is not permitted here, it is necessary to abandon the habitual attitude in which one — in the sense of a genuine prejudice — considers an object to be given in the sentence and regards this as an assessment of the "subject of the sentence", as a speech act.

and thinking about it. Instead of looking at something that is being said about, one should rather pay attention to what is being said; and that is not to be regarded as an object; it is not an object to be judged, but rather the content of the statement to be judged. No one — except in a poetic context — would take the sentence "The day is rising" as a statement about an object or a living being "the day"; it is understood as the sentence "It is dawn" or "Day is breaking". But in a fundamental reflection, we must understand every statement as a subjectless sentence. If it contains names of objects that are being judged, they must be thought of as introduced by the judgements°* of the objects. To accomplish this for every justified statement without distorting the meaning in a finite process will be the task of a fundamental reflection.

In this context, it is less important to focus on the sentences and more important to focus on the meaning. Philosophy, which is fundamental reflection, does not have to formulate sentences about sentences. It pursues meaning everywhere and seeks the greatest, most comprehensive meaning. Language is, like other forms of knowledge, primarily a means to an end; first, in order to use it reliably, and then, in the course of its individual investigations, for its own sake, it examines its peculiar relationship to meaning. Above all, it must be concerned with making its language a clear expression of meaning.

Now that I am undertaking a general consideration of meaning in the following section, I should no longer have to fear the misunderstanding that meaning is assumed here to be an "ideal object" and considered in order to be judged in statements "about it". Despite all linguistic appearances to the contrary, meaning is not to be judged, described and discussed, but only ever to be assessed and stated, expressed. What appears as a statement about a meaning is to be understood as a statement and explanation, a clarifying unfolding of the meaning itself. Understood in this way, statements "about" meaning and meanings also have their own meaning.

USE OF THE WORD "MEANING" THE CONCEPTS OF MEANING3*

After the preliminary remarks on language usage, it will be clear that the question "What is meaning?" is only permissible to a limited extent. It is inadmissible if it requires the identification of an object called "meaning". But it is justified if it requires a definition of the intended use of the word "meaning". Such a definition should be based on the usage that is customary in language and must clarify in what ways it follows this usage and in what ways it deviates from it. Grimm's dictionary lists a large number of uses of the word. An attempt to bring them into a clear and consistent context leads to the following

OVERVIEW OF THE MAIN MEANINGS

1. An older meaning, almost obsolete, in which the word *Sinn* stands for *the subject of acts of meaning*; one could say *Sinn, which ponders. "Des Menschen Sinn" (the human mind); a firm, constant, unwavering mind;* in a narrower sense, this includes *having something in mind.*

2. Related to 1, but not obsolete: *sense* as *the ability to feel, perceive, comprehend or understand. The five senses; sixth sense; to have a sense for something.*

3. *Meaning of mental acts, acts of meaning,* namely a) *meaning of a thought,* b) *of a demand, a requirement,* c) *of a judgement, an evaluation,* also called *content, meaning* of such an act.

4. Meaning of the expression of a mental act, i.e. *expressive meaning,* and in particular the meaning of sentences: a) of the declarative sentence, in short, the *statement,* b) of *the imperative sentence,* c) of *the evaluative sentence.* In the context of the sentence, parts, words and phrases that are not sentences have their *meaning,* i.e. they signify something, have *significance.*

Signs that do not belong to a verbal language also have *meaning* as *significance.*

belong, insofar as they appear as *an expression of meaning*; they can be translated into spoken language, but perhaps not completely, according to the indicative gesture. Here, a distinction must be made between the *expressed meaning* and the *meaning of expression*, of setting expression. In particular, the meaning of talking about the meaning of speech (what is the meaning of you saying that here and now? — The meaning of the speech may be beyond question). This case belongs to the next meaning:

5. *meaning of an action*, of doing something, more generally: *of behaviour*. First attributed to human behaviour, then animal behaviour and finally also plant behaviour; related to purposefulness, in accordance with the "determination of life", probably derived from the meaning of behaviour, especially the meaning of action.

6. *Meaning of an institution*, which encompasses modes of behaviour and usually also associated objects; e.g. meaning of state institutions; of organs, organisations. This also includes the *meaning of a work* as the result of an action, a deed.

7. In the broadest sense, one speaks of *the meaning* (*of* any) *event*, process and, in connection with this, an "institution" of material reality — whereby, unless one thinks of the actions of a creator, the "figurative" application of the term is felt.

In applications 3 to 7, *meaning* usually has the distinctive (concise) meaning *of good sense*. It is important to note that there is a non-distinctive meaning of the word:

8. *Meaning* as *a manner*, *certainty*, especially *"inner" certainty*. The meaning of a movement is initially its *direction* ("in the direction of the arrow"), in the case of rotations, the *direction of rotation*, the *direction of circulation* (these applications are missing in Grimm). One speaks of *the* general *direction* of a process. This is now the meaning of an event in general, value-free and not bound to purposefulness or "determination".

One is referred to the meaning that the dictionary gives as the oldest recognisable one and associates with *journey*, *path*. Here, there is directionality even apart from intention.

Determination, but understood in the truest sense as "inner", as *self-determination*, is the *meaning of human beings*, their *nature* (meaning) as spiritual and moral "beings". Just as the general and overall course of an event can allow and encompass changes of direction, phase changes and the reversal of a partial meaning into its opposite, so too can "human meaning" and "changes of meaning"; This only shows that the word *meaning*, in both value-free and value-laden usage, can have a more comprehensive, more holistic, and at the same time "deeper" or more partial, and at the same time less profound meaning.

3

COMBINATION OF THE MEANINGS OF "SIN N"

The connection between the different uses of the word *"meaning"* can be understood from the meaning *of mental acts*. In acts of meaning, we become aware of every meaning, and in it, everything we can know. From the meaning of an act, reflection proceeds backwards, as it were, to the meaning-setting act, the *"meaning that thinks,"* but forwards, in the meaning and direction of the act, always to *the fulfilment* of the meaning. A being that sets or is capable of setting meaning acts actually *behaves;* it is the subject of *meaningful* activity or attitude; its *meaning* is *good or bad, depending* on whether it fits into a larger, more comprehensive meaning. The mental acts of meaning-setting are meaningful behaviours in two ways: in addition to the meaning that is their content, which they "set", they have the meaning of action or behaviour, the meaning that it has in each case to set precisely this meaning, to deal with it. When one attributes meaningful behaviour to an animal or plant, one takes the living being in a broader **sense** as a subject, as a being that, if not setting its own **meaning**, at least lives it; in relation to it, a behaviour or an organ has its "good" or "less good" meaning. "Human meaning," one's spiritual and moral "essence," is seen in a relationship of superiority to one's creaturely meaning; countless questions and tasks are connected to this: the *"meaningful shaping of life"* of the individual life in the community, in the "world." Here, behaviour and action are in the true sense of the word, action is an activity that springs from its own meaning; here, works and "institutions" have a meaning that is "laid within them" and a meaning of their own that is not clearly determined by the creator's intention; communities have a "spiritual" meaning that transcends the biological meaning of animal and plant associations. The meaning of a naturally and spiritually formed "institution" of human community, which is initially a tool-like meaning, is connected by linguistic and other signs with that of normally — according to the norm of the "language" to which they belong — the *meaning expressed,* which is the content **of** intellectual **positing**. The act of cognition and the statement that expresses it have a value-free *factual and factual meaning*. In the sense of a statement of fact, we take note of everything and everyone about whom we acquire knowledge; all the content of a theoretical representation is understood as the meaning of statements, and it must include every kind of meaning and everything that can be dealt with at all.

4

GENERAL DISCUSSIONS ON THE MEANING 4*

This fact is decisive for the company's fundamental and comprehensive reflection. An overview of the uses of the word "meaning" has more than just linguistic significance; by indicating the areas of meaning, it points to the tasks at hand. Every pursuit of knowledge moves within theoretical meanings; the philosophical has the peculiarity of aiming at the whole — not just at a whole, as every theory does — but not seeking totality and completeness in the accumulation of individual insights, which does not bring one closer to such a goal, nor in roaming through all fields, which are innumerable and unlimited, but in seeking a dominant centre. Of course, this cannot be a supreme proposition or a finite structure of propositions from which all truth can be derived, but only a mental attitude, a basic conception and way of thinking which, open to all meaning, has access to all **meaning**; which is capable of incorporating every meaning. The task is theoretical, a task of cognition, but since there is non-theoretical meaning in demands and evaluations, which must be experienced in *its own* way in order to be understood, in order to be grasped at all, it is not merely a task of the intellect.

To do justice to meaning, any kind of meaning, even theoretically, requires a rich and varied experience and activity. But then there is the need for deepening. While all knowledge takes place in meaningful content, philosophical knowledge seeks to grasp meaning in all content of meaning. Obviously, there is everywhere a difference between weaker, smaller, more limited meaning and stronger, greater, more essential meaning; it is important to reflect on this, for philosophy is fundamental reflection.

That is why it is always assigned the task of giving meaning, interpreting and clarifying. It soon took on this task in the manner of an all-encompassing science — in the most natural way, though not of natural origin, but in separation from myth and religion — as long as that apart from it, there was no science yet, soon as science above the sciences, finally, more modestly, as science about the sciences – as scientific theory, fundamental research and, again and again, in more recent times mostly in the theoretical field, but also in the practical field, with the hidden or open claim to proclaim "values", to "re-evaluate" them, that is, to actually give them meaning. Whereby it happens, especially in enlightened, "scientific" times, that valuing becomes devaluation, the denial of value or the "reduction" of higher value, i.e. higher meaning, to lower value: an encroachment of causal explanation and reduction into the realm of meaning and an expression of a profound misunderstanding of meaning and questions of meaning. All naturalism is such a fundamental misunderstanding, a blindness to meaning, whether it be materialism or, in a subfield, utilitarianism or psychologism, biologism or physicalism. However, metaphysical doctrines of the spirit, as well as the non-metaphysical or hidden metaphysical doctrines of recent and modern times, are also in danger of misunderstanding and misinterpreting meaning; even their concepts of spirit tend to be strange mixtures of meaning and reality. The most important requirement is to grasp meaning thoroughly and purely. Thoroughly: not superficially, not one-sidedly, not, for example, only as theoretical meaning and not only as value-determined meaning; purely: not in reification, not in subjectification, free from the imposition of determinations alien to meaning. Then the endeavour to find a satisfactory answer to the main philosophical question, the question of the relationship between meaning and reality, may become promising.

5

AGAINST MISUNDERSTANDINGS ARISING FROM CONFUSION

To grasp meaning purely, one must first distinguish and separate pure questions of meaning from questions of being. Most and the most disastrous distortions of the concept of meaning come from failing to make this necessary distinction. To explicitly identify such ambiguities is already to create clarity; it is necessary criticism.

(l) The terms *thought, concept, sentence, statement, judgement and demand* do not always mean the same thing. A thought can "pop into one's head", then "fade away" or "disappear"; a concept "arises", is "conceived", "formed", "changes", "develops" and has its own history; judgements are formed and changed, and sentences — statements as well as demands and evaluations — arise at a certain time, are established and overturned. All these expressions are applicable to acts of judgement as mental and spiritual experiences or to mental attitudes, behaviours, "dispositions" and their temporal occurrence among people; but they are not applicable to their meanings. It is not a new insight that meanings are timeless or supra-temporal, just as they are, of course, supra-spatial; but when one calls them that, one should not associate them with the idea of eternal duration or any other metaphysical distinction: temporal determinations — duration as well as emergence and passing away — are simply inapplicable to a meaning, and if one attempts such an application in words, one ends up with sentences without any clear and comprehensible meaning. When a new concept emerges, as the concept of irrational numbers once did in the history of science, the question of the time and circumstances of its emergence is meaningful and justified if it refers to when and by whom, for example, and on what occasion the concept was introduced into scientific thinking; it concerns the thinking of a specific meaning, a real event. However, it does not make sense to ask whether the logical content of the concept on that occasion whether it came into being or whether it already existed before, whether it will still exist once there is no one left to think it. One question of meaning is whether, with the defining determination of the "irrational number", its representability by a "finite" decimal fraction, i.e. one that ends after a finite number of digits, or a periodic decimal fraction, is compatible or not: what is contained in a meaning, what is not contained in it, what is compatible with it, what is incompatible with it, these are questions of meaning. As permissible questions, they concern meaning itself. The question of the existence and occurrence of a thought or another proposition concerns the process of positing, the habit of such positing, the ability to do so: it is a question of being and has nothing directly to do with meaning and meaning content (what a meaning — in terms of partial meanings — contains). What the meaning contains is completely independent of all reality, including the reality of its positing. It goes without saying that "positing" meaning does not mean "putting it into the world," "creating" it, or "bringing it forth," because "positing" meaning in this sense means nothing at all. The change and duration of concepts are the change and duration of attitudes of mind, especially in connection with certain expressions and in relation to certain "objects of judgement". If, for example, the concept of acid has changed in chemistry, this means that the same word is now associated with a different idea, not that the same idea now contains something different than before — especially since it makes no sense to say that a meaning contains a meaning "now" or at any other time: it is not a temporal or real relationship.

It should not be necessary to dwell on such observations, but it is necessary. They are closely related to questions of validity and truth, questions that are once again controversial today, and allow for clear decisions.

(2) It makes sense to ask what the meaning of an action, an institution, or a statement is, or in what sense a process has taken place, but it makes little sense to ask whether meaning exists at all. For the assumption of a negative answer is contrary to meaning. "There is no meaning" is, without any restrictions, a sentence of rather questionable meaning: if it is to be taken seriously, it claims a meaning for itself — even a valid one — but without such a claim it is nothing that could be described as true or false.

Because every assertion has a claim to meaning, even without acting on that meaning, the fundamental denial of all meaning is meaningless, not contradictory in words, but in deeds. That this statement is a refutation, and one of the strongest kind, is demonstrated by a demand that cannot be justified: the demand for good meaning, which applies to every act of meaning. The fact that meaning exists at all cannot be denied without being absurd, and therefore cannot be doubted, and therefore there is no need to question it, and so the assertion (that it exists) takes on a special position on the edge of the meaningful; it may only be appropriate as a defence against general doubt or a blind hostility to meaning. However, its positive content remains unassailable. It would not be reasonable to ask about the "mode of being" of meaning; it would be a question of being in the pure realm of meaning, where only questions of meaning, not of being, are relevant. One immediately moves onto safe ground when, instead of saying "there is meaning", one states, for example, "there are meaningful things, things that have meaning, of such and such a kind (e.g. the acts of meaning of ascertaining, demanding, evaluating)". In detail, however, the question of meaning, in connection with realities, will be the question of what meaning a proposition, utterance, action or institution has; and if one asks whether it has any meaning at all, this means whether it is meaningful: it is not the question of the existence of an object "meaning of this reality", but a question of the kind "How is this reality?". However, whether a case of reality has any meaning at all, in the broadest sense of the meaning of events, facts or circumstances, cannot be asked at all, but only what this meaning is.

(3) “Naturalism”, the endeavour to comprehend and interpret all reality in terms of natural science

is a fundamental misunderstanding of meaning. In its crudest form, it appears as materialism, today "purified of metaphysics" as "methodical materialism" or physicalism, which replaces the assertion "Everything is physical" is replaced by the demand or stipulation that only statements that appear "in physical language" are to be accepted as "factual" (logical and mathematical propositions are not considered factual, but are partly stipulations of language usage and partly tautological conclusions, transformations of the stipulative propositions). Statements of psychological content are considered factual only if they can be translated into physical language: "Otto is sad" means "The

Leib, whose name is Otto, has such and such externally observable and such and such internal states" — some of which may not yet be sufficiently specific — or it means nothing at all. The objection to this is that such a description, apart from its vagueness, which contrasts with the specificity of the sentence's content, does not convey what he means. The objection is familiar to the proponents of physicalism; it has probably been raised many times, but as far as I know, the response to it is that it is based on a lack of understanding and means nothing. Nevertheless, it seems essential to me. Ultimately, the person who makes a statement must know best what they mean by it. I certainly do not mean by a statement of the kind in the example what the proposed translation indicates. Even though Otto's "sad" expression and posture come to mind, I understand it as an expression of an experience that is not merely an internal state, but rather the experience of a state and, more than mere conditionality, is meaningful. The sad person — whether they are sad about something specific or simply in a sad mood — is not only in a meaningless state, like the pressure and temperature of a gas, but in an inner and outer *attitude* whose inner aspect is still very much part of its "outer" aspect, whose "inner" aspect is a life of meaning. It does not matter whether this statement convinces the physicist; what matters is whether it is right in opposition to physicalism.

If one explains that factual statements are only sentences that can be expressed in physical language, one will have to specify what a language is, what the expression — the use and understanding — of sentences is, and one will have to say this in physical language. But that seems to be an impossible task. One can say that language is a system of signs, i.e. of physical phenomena of a certain kind, which tend to occur as accompanying phenomena of certain states within a majority of people and to be accompanied by certain external and internal states and processes in these people (which they "trigger"); and that this interplay is based on "convention", i.e. on habit and tradition. These statements can certainly be supplemented and refined to a great extent in a physical sense; but their essential content will always be the description of a general rule, one that not exact, legality of events among people who are referred to as members of a language community. If one calls this event human behaviour, then an expression that is not purely physical language has already been introduced – one speaks of the "behaviour" of solid or liquid bodies under certain conditions in a clearly different sense – and it will not be possible to replace it with a purely physical one. Behaviour such as, here in the realm of language and speech, addressing, listening, communicating, understanding, commanding, obeying, is in its external appearance entirely expressive of meaning, always an expression of experienced and, beyond that, established meaning, and this peculiarity cannot be grasped physically. One may reply that this is not surprising; that not everything about "behaviour" seems physically comprehensible is only because we are accustomed to assuming, instead of the still unexplored internal bodily processes, something that cannot be determined physically at all, which we call "meaningful-spiritual" (or similar); but this "assumption" is again speech accompanied by corresponding internal bodily states.

Assuming this to be the case: the physicalist representation of a herr-
If this description of language use, of speaking and understanding among people, is accurate — and I have no doubt that it could be given better and more comprehensively than it has been here — then it must always be examined whether this description itself and its understanding are something that can be described in physical language. It is a structure of sentences that can be put into the form *of a* general sentence, an "there is" statement in connection with a general if-then sentence of the imprecise kind that is usually distinguished from the merely approximate rules of exact laws. Since physicalist doctrine requires that factual propositions be expressible "in physical language," it must explain what "propositions in physical language" are — which has not yet been discussed here — and must specify what it means by "language": in any case, it will not be able to make its demand or "determination" without formulating general statements — if it should otherwise be able to depart from general statements, which is not to be assumed. She will therefore have to acknowledge, by her actions, that there is such a thing as understanding general sentences, because she claims it for herself; but it is precisely this fact, the understanding of general sentences, that cannot be grasped in physical

language (nor that of understanding of any kind,
e.g. from individual statements, but it is most clearly seen in the case of universal statements). In physical language, it can be expressed that there are certain states and processes in humans that are (approximately) regularly connected with the phenomena of linguistic "signs", that, for example, a human (or a dog) tends to jump up when given the sign "Up!" if he is sitting or lying down. Someone with a physicalist view might take this set of individual cases of behaviour to be an "understanding of the sign 'Up!'". But by declaring "This set of cases — a certain 'reaction to a sign' — is what is called understanding the sign," he has uttered a sentence of such a kind that whoever understands it "means" in *a* single act the whole set of individual cases that is being discussed, "grasps" them, "encounters" them, without, of course, "visualising" each one individually. The general statement refers to a set or class of cases and is made in a certain way, or the statement is not understood. No "description in physical language" does justice to this understanding, which is a grasping of a generality.°* The physicist does, however, speak in general terms about the occurrence of individual cases, but his general statement is, as a case of physical occurrence — the "speech movement" — and described in physical terms, a single event and nothing more, a meaningless process — "meaningless" means having no meaning except factual or factual meaning — and by no means the comprehension or representation of a set of cases.

What is physically tangible in the case of understanding a linguistic expression is, at best, a certain type of observable behaviour of a human being — or an animal — in a class of cases. If such behaviour is observed in a sufficient number of cases, one will say, in the physicalist or "behaviourist" sense, that the human being or animal "understands" the expression. But genuine understanding is already present in the individual case of the expression's application; it does not consist in that set of "correct reactions" to it. I understand a general instruction to always behave in a certain way under certain conditions, not by behaving accordingly every time, but even before such cases occur and independently of them, by grasping the meaning of the instruction. What this means cannot be expressed in physical language, but it certainly means something: I can very well distinguish when I have understood an expression and when I have not.

(4) Here, theoretical reflection on psychological experience is referred to concepts of a non-physicalistic psychology. This too is often pursued in a naturalistic sense and then easily degenerates into psychologism that misinterprets meaning. The "experiences" of this psychology are states and processes, only, unlike physical ones, they are described as "psychic" or "mental," as "internally perceptible" and non-spatial. This "mental" aspect is basically meaningless, but it is expected to have meaning, especially that of cognition. Although this psychology displays a variety of directions, it corresponds to understanding the "soul" as a collection of mental experiences, i.e. dissolving it completely into a multiplicity of such "elements". They come from outside, as sensory perceptions – or these perceptions are simply there at the time – are accompanied by a "feeling tone" or evoke feelings, no matter what, merge into perceptions, disappear, are "reproduced" as representations. If the perceptions or their elements, the sensations, come from outside, aroused by stimuli, then, in the older conception, they are images of the objects and processes of the outside world, later signs for them; a representation then takes place only in the sense of assignment, element by element, without any similarity between the object and the assigned mental element.

The fundamental characteristic of this theory of representation, in both versions, is that it empties spiritual experiences of meaning and objectifies them: the external object corresponds to an internal state as an object of a different kind and just as meaningless as the former – the fact that the meaning of events and things is also a meaning remains fundamentally unnoticed. Of the countless difficulties and shortcomings of this theory, only one need be mentioned here: that the existence of the mental image contributes nothing to the recognition of what is depicted if one does not know the association – neither its law nor the mere fact of the association. But "knowledge" has no place in this doctrine of elements; it does not occur among the mental elements, and how that which is commonly called knowledge comes about should be specified. The acts of meaning of knowledge and recognition are not determined by a relationship—of an objective nature—between "inner" and

“external” objectivity. Thus, philosophising psychologists have not yet completely abandoned the attempt to develop a theory that remains within the realm of mental experiences – genuine psychologism. Only "the psychological," whether element or whole, is considered "given" or "immediately given" here, and some of this psychological content is, under conditions to be specified in more detail, "projected outward" and thus becomes an (external) "object." Such projection affects not only complexes of sensation, but also feelings, especially in primitive conditions, and thus "values" and a world of things of value, things laden with value, arise, such as the mythical-religious world. It is difficult to imagine what the figurative expression "to project" is meant to convey. In order to "move something outside," one must have something "inside" and an "outside" to which it can be moved. The image seems to presuppose both; after all, it arose from the common conception that somewhere "inside," namely in the body, which stands in space, experiences take place and sensations, ideas and feelings reside. To this is now added the subsequent interpretation: the "inner states," which are solely "contents of consciousness," are "transferred outwards." The theory that is this interpretation already presupposes what it wants to "explain". But if it avoids this error, it says, taken literally, nothing more than: there are mental experiences, and among them also the experience of the opinion that some of these experiences are not
— or not only — an experience, but something entirely different, something non-spiritual. An opinion so obviously wrong that it cannot be expected of natural thinking, and, what is essential, an opinion, an act of judgement or perception, and precisely that should not exist among the mental elements; it should be traced back to the "simpler" experiences of sensation and imagination. — The theory must surely be understood differently. — Experiences, as inner states, are, according to plausible findings of developmental psychology, by no means "given" in the first place; what is the primary object of psychological consideration — that older "elemental psychology" — is by no means the primary phenomenon of natural experience; The distinction between the mental and the non-mental, the "outer world", is a later achievement, as is the objectifying consideration of it that is presupposed for it.

Experience. No one will take the experience of seeing red or hearing thunder as an external object, but rather red, the appearance of red, and thunder will be understood as follows: in the experience of seeing or hearing, light and sound are perceived (and, in a more mature understanding, are assumed to be "external" events). Light, colour, brightness, sound, which we "experience" or "perceive", are not our experience or perception at any stage of our development, and our perception is never misinterpreted as a process of our spatial environment. But in the perception of light, in hearing, a sense is directly experienced, which we can express linguistically at the appropriate level of development in a subjectless sentence: in the "one-word sentence" "Light!", in "It resounds". If meanings were not experienced, they could not be expressed. Experiences are not meaningless states or processes, "psychological" counterparts of meaningless external events, but are meaningful. A theory that seeks to reduce everything to meaningless elements — including Mach's doctrine, although it does not claim to be psychology or physicalism — is incapable of accounting for itself and its propositions. These propositions are meaningful expressions of acts of meaning for which neither one of the meaningless elements nor a quantity, sequence or combination of them can arise. These theories are too modest; they forget themselves and that they too are something that should be grasped with their concepts; their thinking does not go that far.

(5) A particular area of naturalistic misunderstandings has long been that of values. If, in a physicalist manner, one connects meaningless events of speech sounds with meaningless events in humans — one cannot really say: human behaviour — through a regular connection between their occurrence, or, in a psychological-naturalistic manner, connects meaningless "mental" states with any "external" things and processes, one always only grasps meaningless facts: one encounters only the external, factual, material meaning of the event one wishes to grasp, and completely misses and misjudges its essence, which is its inner meaning, which presupposes an *experience of meaning* — on the part of the speaker, the judge, the demander, the evaluator, the actor. Thus it can happen that instead of addressing the meaning of a theory, a belief, a moral attitude, or an artistic movement, one explains or attempts to explain it in psychological or biological terms

and thus "refute" it. The result is shallow, mindless platitudes. But even Nietzsche's spirit, at times when it was dominated by positivism, was not spared from this sin against the spirit.

Until now, meaning has been defended against misinterpretations arising from blindness to meaning, blindness to meaning of varying degrees of severity. It does not matter what errors have been explicitly advocated, for example in literature. What matters are the fundamental opinions and views, *the basic principles. If they are strictly pursued, then it becomes clear where they lead.* -

Now we shall discuss misinterpretations of meaning that are not misjudgements arising from blindness to meaning, but rather overinterpretations and exaggerations. They introduce into meaning what is not in it. The main types are: the *objectivist* interpretation, the subjectivist interpretation, and also the "transcendental" amalgamation of the two: Husserl.

6

OBJECTIVIST INTERPRETATION

In its purest, simplest form, the theory of representation recognises only meaningless "psychological states", such as "sensations" and "sensory complexes", and meaningless objects associated with them; in this version, it is completely blind to meaning. This theory of association now finds its counterpart in linguistic theory: here it is a "sound complex" that is assigned to an object, be it an individual one — as a proper name — or any one of a certain type, and "designates" it by virtue of this assignment. The association is made "arbitrarily", "by convention", and it alone turns the meaningless sound complex into a word; the meaning does not lie in the word, so to speak, but is replaced by the "association", which is a factual relationship between facts, mediated by the behavioural habits of a language community. A word that has no assigned object — in the broadest sense: it could also be a process, a state, a relationship — does not designate anything; it is not actually a word, but nothing more than a combination of **sounds** — or letters, a "meaningless sign", "meaningless" in the expression, e.g., of *Principia Mathematica.*® *But there is a difference between an expression such as "perfectly rigid body", "perpetuum mobile", or even "round square" and a meaningless combination of sounds or letters. There is indeed nothing concrete to which an expression of the first kind could be applied, which it *would denote,* but each of these expressions means something, it *has* meaning, i.e. it has sense. Being meaningful, having meaning, does not yet mean denoting something. The fact that we can say: "There is no perfectly rigid body," "no perpetual motion machine," and "There can be no round square" makes it clear that the subject names of these sentences each have a specific meaning: each of them expresses a meaning, but one that cannot be fulfilled, and the last name expresses a meaning that cannot be fulfilled, that is unfulfillable precisely because of the nature of its meaning; each means something without designating anything. For the sign meaning of the word under consideration, it is not the designation that is essential, but rather the meaning, which is the expression of a sense. But because even unfulfilled meaning "means something" — that is its essence — some people were led to believe that every meaningful expression must have an object that it designates, and if this object could not be found in the realm of reality or reasonably assumed to exist, then it was demanded. This gives rise to the doctrines of "mental non-existence" and the "immanent object" of "consciousness," which means, insofar as it has not been psychologically vulgarised, the content of meaning. In Meinong's "object theory," these ideas are thought through to their logical conclusion with the greatest consistency, a consistency that also takes on the logical contradiction. The doctrine emerged from a strongly rationalistic psychology and psychological epistemology and, according to its origin, is an "overcoming" of psychologism, ending in complete "objectivism". At the beginning of this rationalistic psychology is the statement that every mental experience is either an idea or has ideas as its "psychological prerequisite". But, it is further explained, whoever imagines always imagines something: the idea has its object. According to later doctrine, the idea "presents" an object to comprehension, which is only achieved through an act of judgement or acceptance. In every act of comprehension, an object is meant. This meaning of the act of perception is now turned into an object, and since meaning does not always correspond to an "existing" object, there are cases where it has a "non-existing" object. This leads to the paradoxical statement that there are objects that do not exist, which of course can only be ventured if one understands "exists" differently each time. A distinction is made between different types of being: first, in the assertion that, apart from the things and events of reality that "exist", there are objects such as difference, similarity, equality, numbers and other "ideals" that certainly do not exist as real things in time, but certainly "exist" outside of time. Only such an ideal object can even exist with necessity, such as the equality between 2 times 2 and 4 or the difference between red and **green**, while we do not know of any necessary existence. If there were a completely rigid body, it would exist if there were

If there were equality between red and green, it would exist. The reason we can make both statements lies in what we understand by "completely rigid body" on the one hand and "equality between red and green" on the other, that is, it is concluded, in the being of one object and in the being of the other. The being (of an object) is independent of the existence (of the same object). The object is, according to its being, i.e. according to its objecthood, "predetermined" as a "pure object", its existence — or non-existence. The statement that a completely rigid body does not exist and the other that a round square is impossible does not necessarily exist, one thinks, is only possible and meaningful insofar as the predicate of non-existence is predetermined by its object; the non-existence of the round square is precisely the non-existence of this object and of nothing else. Thus, the non-existent is attributed with a, so to speak, weakest form of being under the name of "extra-being," and in general it would be said: Every object, as a pure object, is extra-being; according to its pure object nature, it stands outside of (actual) being and non-being, as a pure and mere extra-being. Only when a pure object, either in the case of an ideal object, necessarily on **the basis** of its being, or, in the case of a real object, merely actually, is added to it a being or non-being that "belongs" to it, quite actually, as an objectivity of its own kind, does a being (existing or existing) or non-being object result.

Even for me, having grown up with the "object theory," it is difficult to patiently reflect on these ideas, and it is easy to blame "language," an excessive belief in words, for them. In fact, every word here is made into an object, and just as words are strung together in a sentence, objects are supposed to come together to establish the "facts" that we recognise and express as peculiar "objects of a higher order", outside of time. But the attempt to think in this way is significant in its relentless consistency and cannot be dismissed entirely with a reference to the peculiarities of "our language". A language is not a given and is by no means the last or first thing that can be invoked to explain the peculiarities of thought. It always *becomes* and *is* an expression of, among other things, thought.

Meaningful content is conceived. The fact that, when we think about the meaning of a determination, we imagine something specific, regardless of whether it exists or not, lies in the nature of the meaning we think about. And when we determine that something specific does not exist, the idea of the "non-existent object", indeed "this specific non-existent thing", is already there. We repeatedly speak of "objects that do not exist". Here it is necessary to decisively free ourselves from language, namely from a way of thinking that is closely associated with this language. "There is no such thing as a completely rigid body" is a sentence whose content can be expressed by the other sentence "The determination to be completely rigid remains unfulfilled". This is a peculiarity of the determination, of the meaning, to remain unfulfilled in reality, and at the same time of reality, not to fulfil the meaning. And so it is a peculiarity of the determination "to be round and square" or of the determination "$x^2= 2$ and x is a rational number" to be unfulfillable, and that "for every reality", i.e. regardless of any reality.

In these statements, the only thing that interferes with precise thinking is the fact that meanings are spoken of as if they were objects about which something could be said. In fact, the "object theory", and not it alone, has understood them in this way. Its misinterpretation of meaning is a double objectification: it assumes a "pure object" as the independent carrier of every determination, and it regards the determination itself, like every meaning, as an "ideal" object, that is, if one wants to enforce the object conception, again as the carrier of a determination, namely the determination 'to be this and that determination' — which is, of course, a conception that involves a circle. But this conception, which always and everywhere aims at "bearers of determinations," is the essence of what I call objectification. It is the peculiar way of thinking of object theory. It, and not — or only secondarily — the idea that the "object" stands "opposite" the perceiver — *Meinong* always rejected such subjective determinations in his mature object theory. From this point of view, object theory cannot be refuted, as *N. Hartmann* wants. The inappropriateness of this way of thinking precedes any theory of cognition. It carries questions of being into pure questions of meaning and misjudges the meanings.

objectifying them. The same mistake, at least the first one, is made by those who declare meanings to be ("ideal") *entities*. This is done by *N. Hartmann,* among others.

The first assumption, of a mode of being outside of being, leads in the case of unfulfillable determination to a violation of the law of contradiction — it does not help, with *Meinong,* to "restrict" the validity of this law to the realm of the existing or the possible, because there is no other realm; the contradiction lies in the assumption that there is a being outside of all being and a bearer of non-being. But even in the case of a fulfilable, non-contradictory determination, the result is logically untenable. The concept of the Euclidean plane triangle leaves the side ratios undefined; it defines something as a "conceptual object" that, in the sense of object theory, is a triangle in the sense of object theory, without being equilateral or non-equilateral; according *to Meinong,* it is neither, but has both the possibility of being equilateral and that of not being equilateral. Such "incomplete objects" are said to be the carriers of mere possibilities, without the actuality of a possible determination. The untenable theory
The question of "possibility and probability"13t, which is based on this premise, is not considered here. It suffices to note that what applies to no triangle, no x, of which it is actually true that x *is a triangle*, is attributed to "*the* triangle (per se)": the "incomplete object" is the fulfilment of incomplete determination without fulfilling it, for it can only be fulfilled in some particularisation. It violates the law of excluded middle and is consequently counted among the "impossible objects," which leads to the problem that the incomplete object with compatible constitutive determinations, which as a representative of "possible" particular objects should also be "possible," , is just as impossible as the "impossible object" constituted by contradictory determinations.[14]

The misinterpretation of linguistic expression that exists here is eliminated if one formulates it more precisely in accordance with modern logic. Sentences such as "The flat Euclidean triangle has an angle sum of two right angles" and "The round square is round" do not violate linguistic usage and are also understood as logically correct, as tautologies. But understood in this way, they are an expression of implications between determinations: "If x is a Euclidean plane triangle, then the sum of the angles in x is two right angles," "If x is round and square, then *x* is round (and square)." This is, of course, correct and self-evident, but despite their form, these are not statements about objects that could be called "the Euclidean plane triangle" or "the round square": such objects do not exist. What does exist, in a certain sense, is in any case — and it cannot be said without contradiction that it does not exist — the assumed definition: "x is (be) a Euclidean plane triangle", "x is (be) a round quadrangle". A determination such as "z is a triangle" ("being a triangle") is incomplete, which, together with linguistic usage, gave rise to the concept of the "incomplete object". There is also the unfulfilled, even the unfulfillable determination: a determination is, according to its content, independent of fulfilment. This independence, together with misleading language usage, gave rise to the assertion of the "independence of being from existence" and, in connection with the first error, to the assertion of the "independence of the (thus defined) pure object from existence".

If determinations and other meanings of all kinds appear in object theory and outside of it as "ideal objects,"15* then there would be no objection to the designation, were it not an expression of a view that does not do justice to the meaning. When I speak of the highest peak in the Alps, I mean one object: the object that fulfils the determination of being the highest peak in the Alps, but the determination in which I mean it and "encounter" it is not meant and encountered as an object; it is not thought of, it is posited, it is the content of the positing, not a targeted object; in a certain sense, it is the positing itself. The same applies to the meanings of other, arbitrary positions. It is in the nature of a meaning to be posable in an act of meaning, to be able to appear as the content of an act of positing. It is inherent in the meaning itself that it cannot be known at all without an act of positing, of which it is the content; anyone who wants to know what "the relationship of being greater" is must have made assertions of the type "x is greater than *y*"; it is in such an assertion, as its content, that "being greater" first comes to their consciousness. If they now speak, at least in words, of "being greater" as an object they are thinking of, then

Therefore, what is meant remains what it is, a meaning, a sense that is known as the content of a proposition. However, when talking about "being greater", one usually spares oneself the executive thinking of the meaning in a proposition *"*x is greater than *y"*, but one is, as it were, ready for such execution, which would be necessary to realise the content of the statement "about being greater". It is a curious fact that we make progress in this way in "indicated operations", even more progress than we could make in executed operations, if they could be carried out to completion. When we speak, we always make use of this and accept the risk of some misinterpretation; the most common is the objectification of meaning. What is actually and essentially conscious only in the proposition is now regarded as an object in a second act that is now possible and, at least in terms of linguistic expression, judged. The naming, the nominal form of the verb, the "substantivisation" — which turns *"x* is greater than *y"* into "being greater", "it is raining" into "rain", a demanding "Thou shalt not kill" into a "prohibition of killing", a "shall" into a "should", and a judgemental "Beautiful!" "beauty", is the beginning of the belief in "ideal objects", the misrecognition of the thoroughly non-objective nature of meaning. But by "making the meaning to be posited the object of my consideration", I only grasp, in a second positing, a more comprehensive meaning, in which the former appears as a partial meaning. Of course, the statement just made could not avoid speaking of meaning as if it were an object, but it can only be understood insofar as, instead of actual acts of positing particular meanings, something like a representative indication occurs in a "general act of positing" wherever the word meaning appears. A content is not an object and does not become an object by giving it a name that allows one to say "about it" and "from it".

In the case of the "object theory," what led to the objectification of meaning was initially an effort to make clear distinctions. It began with the concept of representation; the concept of a tree was correctly distinguished as a mental experience from the tree itself, which is not a mental but a physical object; however, Meinong described the content of this concept as that "constituent part" of the representation by virtue of which it is precisely the representation of a tree and differs from any representation of another object. This content is, as part of a mental event, itself mental: a mental entity that "corresponds" to the object in a peculiar way, is "assigned" to it, so that it can "present" it (to comprehension), "show" it. The "content" understood in this way is, of course, not a meaning, but a "psychically real" thing that has the peculiarity of standing not for itself but for something else — although it can also present itself to "inner perception". One can see that this "content" of the representation

- to which later judgements, assumptions, feelings and desires are added — is a (mental) state or a "moment" in such a state: an object of its own kind. The world of the "real" is neatly divided into "physical" and "psychological objects". The act of meaning — the mental content of the (tree) idea can be approached by an act of judgement or mere acceptance, as a "being-meaning", thus "taking possession" of it grasping what it merely presents, the tree — the act of meaning is also psychologically real, it has what it posits as its next object: in the case of grasping the tree, this next object, the "proper object" of the positing, is the being of the tree, and in it the tree, as the "appropriated object" of the positing, is also grasped. One has the act of judgement, expressible in the words "the tree is", on the mental side, **the tree on** the physical side, two real objects, and one has the being of the tree grasped in the act of judgement as a third, but "ideal" object. It is assigned a kind of mediating role between those two real objects, but it too is an object. Being — as well as being-in-itself, relation, value-being, ought-to-be — is always beyond the mental experience of "comprehension" and, in a non-spatial sense, opposite it: Meaning is interpreted as an object, and its essential peculiarity, as meaning-content, of appearing *in* the positing and becoming conscious without being a mental entity and part of the mental-real, is lost sight of: the act of meaning is actually forgotten. What remains is the incomprehensible relationship between the mental and its opposite, between object and object. Thus, the end of the objectification of meaning is a misrecognition of meaning. It is not only present in object theory, but everywhere where the existence of "ideal objects" is asserted; even the *" essences"* that are grasped in phenomenological essence-gazing are objectively interpreted meaning®; in the execution of an act of meaning, meaning is grasped neither as an object nor as an essence.

That the act is meaningful, an act of meaning, means that it has a meaning of its own that cannot be understood as a relationship to meaning as an object without being compelled to attribute a content to the act, namely a meaning (not a Meynongian "psychic content") that establishes the relationship. The meaning of an act is never its object; it belongs more intimately to it. Figuratively speaking, it is not aimed at the act of meaning, but rather at the direction of its aim (namely, fulfilment); non-figuratively speaking, it is its meaning.

Since meaning is not an object, it is also not a partial object, a "moment in the object," not to be read from it, as the theory of abstraction would have it. When one speaks of concrete determinations as "moments in the object," of a "distinction" between, say, the common features of a majority of objects, and the like, such statements are only possible because of their vagueness. They are meant figuratively, and it is not said how: the meaning that the image is supposed to express is not specified so precisely that one can clearly recognise whether it is right or wrong. If one attempts to derive a specific meaning from the image, one immediately recognises its untenability: it is that of "ideal objects". — Added to this is the opinion that a "determination on the object" is a part of the object, "in some sense" — how, is inconceivable. A large part of philosophy, at least philosophical writing and lecturing, thrives on such vagueness and ambiguity. Along with all that is worthless, they do have a certain driving force for some people — otherwise they could not exist and should not be allowed to.

7

BE OBJECTIVE

One can always invoke the freedom to call everything and anything that can be spoken of an object, and one speaks of meaning and content. There is nothing to object to this freedom except, in the present case, the inappropriateness of its use. Anyone who speaks of meanings as objects must specify the particularity that distinguishes them from other objects, such as houses, ships and mountains, and the essential difference is precisely their non-objectivity. This becomes clear in the way we become aware of meaning: it is not found, like a thing that we encounter, that confronts us, or that we confront, that we encounter. The relationship is much more intimate: before I can think about and speak of meaning, I must establish it, it itself — thinking, demanding, evaluating — and always, even when I speak of it, I do so with meaning only insofar as I establish it. It does not stand opposite my positing as something I aim at and encounter; in the act of positing, it is its direction, that is, its meaning. If one calls it an object because one speaks of it, the reason is very external; one can only think of meaning insofar as one thinks, at least in some suggestive way, *i/in*, or, as one might say no less correctly, thinks in it *(thinking* here, as the old *cogitare* in Descartes and elsewhere, used for *positing* in general). This essential relationship, this immediate possession of meaning or standing in meaning, is already obscured by the objectifying naming and is, so to speak, fundamentally forgotten and theoretically obscured by the concept of the "ideal object". The object of a thought is understood to be something that satisfies the determinations that are the content of the thought, a "carrier of these determinations." The determination itself and every meaning is not a carrier of determinations, unless one "makes it so (linguistically)" by, for example, "determining" it.

"states". If we are already misled and almost compelled by linguistic usage to speak of "objects that do not exist", we should not turn this linguistic evil into a theory that elevates the nonsensical to a principle.

Given the non-objectivity of meaning, or to put it more cautiously, since it is not correct to speak of meanings as objects, it is easy to arrive at the opinion that meaning is subjective. Consciously or unconsciously, the rule applies: "What is not an object and yet is spoken of is subjective," meaning that it belongs to a subject, occurring only in the consciousness or experience of the subject. But a subject is a spiritual experience, in the narrowest and most literal sense a meaning-setter, in the broadest sense something that not only fulfils the meaning of objects and events, but also actively carries out its own meaning, a living being. Thus, the concept of the subject means a special relationship between something real and meaning; insofar as something real is a subject, it is not to be regarded as an object — the bearer of meaningful determinations — but, *because of* its relationship to **meaning,** only to be grasped in a co-experiencing or post-experiencing fulfilment of meaning, "understanding". And I myself become aware of my subject nature, my selfhood, in the setting of meaning, as a meaning-setter (initially, probably in acts that contain the meaning of demand in unclear, value-laden, illustrative fulfilments), since the fulfilment of the set meaning as an object and the setting self clearly diverge. In the distinction between ego and object, subject and object, which one habitually 'handles' by using words for 'objects', it is all too easy for the subject to become completely objectified, appearing in a kind of observation "from the outside and from the side" opposite the object, itself an object, only with the special property of having mental states and, within them, a "representation" of the object. In doing so, meaning, insofar as it is not overlooked, is now thrown onto one "side" or the other: it is considered an object or it is considered "mental content". Both are wrong. Meaning is neither an object nor is it "psychological" and, in *this* sense, "subjective". Even if a subject, in psychological experience, assigns meaning, it is nevertheless completely independent of the subject and experience: independent in terms of meaning, because it contains nothing of its being assigned by the subject, and dependence in terms of meaning does not come into play at all.

Consider, because the question of the existence of meaning does not apply, because it is not a question of meaning.

Of course, I am only aware of meaning in context; I only know meaning that I am aware of, i.e. that I know. This is tautological and not particularly surprising: it does not imply anything that is not tautological (the principle of consciousness). Above all, it does not follow that it is essential to meaning1®* to be consciously or knowingly the "content of consciousness"; *the meaning contains nothing of this.* From x *is divisible by 4* follows x *is divisible by 2* and *x is a whole number,* but nothing about a knowing or not knowing of this meaning. But from the fact that I posit *"x is (or be) divisible by 4"* in an act of consciousness — which is in no way contained in or implied by the meaning I posit — it follows, however, that *"x is divisible by 4"* is a posable meaning; but this is again not a special "peculiarity of the meaning" and cannot be claimed for a "theory of the object of meaning" and a "theory of ideal objects", but only means that there is (at least one) positing of "this meaning", i.e. a **positing of** "x *is divisible by 4"*. We can determine such "positivity" for every meaning that comes into consideration — in the precise sense of this phrase. It does not mean (meaningful or existential) binding of the meaning l°* to positing and the positing subject; it means only that for every meaning *A* there is the meaning "positing of *A"* and the meaning *"A* positing" as an always meaningful conceptual supplement. Such *a concept* of the "meaning poser" is probably the content, the meaning, that is expressed in the discourse on "consciousness in general" and, albeit not without careless actualisation, in the discourse on "subjective spirit". Much profundity and little clear thinking has been devoted to the relationship between subject, object and meaning. It must first be clarified in a simple way in terms of meaning and sense. Every meaning "goes towards fulfilment", "means" objective fulfilment, and every meaning is "thinkable", i.e. it can be posited: in this respect, the concepts of the objective, the "object" in the broadest sense, and the "subject", in an "intellectualistic" sense, of "consciousness", *are meaningful additions* to the concept of **"meaning",** "meaning content", for which "meaning in general" or simply *meaning* is to be substituted here — for in the idea of meaning there is indeed meaning,

Meaning par excellence, not merely intended, but expressed. This fact in no way justifies the "idealistic" view that all reality should be subsumed in it as a pure relationship of meaning, thereby unnoticedly accepting the relationship of meaning as a kind of reality, since it is supposed to replace common reality, albeit in a refined form. In fact, only meanings remain for logical idealism – "concepts" of objects and subject or subjects – and those who think in this way overlook and want to overlook the fact that they themselves think only as a real being and that they mean the real fulfilment of meanings; for them, only the meaning of meaning is noteworthy, the meaning of objectivity and selfhood; they misjudge the object and the self, just as they misjudge the meaning itself, which they actually carry out. The Cartesian *cogito, ergo* sum means real positing and a real positor. Whether there is a right to assert this reality rather than that of an objective fulfilment remains to be examined. In this way, of course, he actually withdraws this meaning, invalidates it. But that will be dealt with later. (Standing in the reality of meaning, I become aware of all reality, both objective and my own. This does not require any "reference" to the reality of the act, as the act is not the subject of a statement but a conscious execution; nor does it require any conclusion from the reality of the act to the reality of the executor.)

If meaning is called "subjective" because it can be posited by subjects, such "subjectivity" is no reason against the "objectivity" of meaning; it does not imply any attachment to a subject, any dependence on it. However, since "subjectivity" usually stands for such dependence and subject-boundness, and since, on the other hand, "objectivity" is understood not only as the negation of this, but as objectivity — namely, being an object, not just pointing to objects, meaning an object — it is probably better not to apply either of these terms to meaning.°0*

8

SINNZUSAMMENHANG

All meaning-finding and meaning-fulfilling is incomplete. Every meaning achieved points beyond itself: the solution of one task always brings with it new tasks, both in practical terms and in terms of knowledge. We know of no meaning that is absolutely complete, no isolated meaning; each points to meaningful additions; it is not possible to continuously (“discursively”) visualise a meaning in everything it encompasses, in everything it points to. Nevertheless, one can call the content of a proposition a “whole” if one attaches importance to such an inconclusive designation: it is always, of course, a unit of meaning. We only know partial wholes. A conceptual content is a determination "of something", "for something" or a relationship between something else to be determined; in statements that are purely logical or mathematical formulaic sentences, the meanings of conceptual determinations are unfolded, and the complementarity and need for complementation arising from the relationality of these determinations are incorporated into the statements. The sentence "twice two is four" applies with its infinite set of consequences,
e.g. the greater-than and less-than relationships of "twice two" to members of the natural number series and the relationships to numbers of an arbitrarily extended range; and it only applies to certain meanings of the words, defined or definable in an axiomatic system of natural numbers — whereby these definitions, like all definitions, presuppose certain meanings of words and symbols, and ultimately those that are understood without explicit agreement, without therefore being "pre-given" or independent; the theorem applies simultaneously with every true theorem, with every special prerequisite to be introduced, and applies with the incalculable variety of possible applications, their theoretical and practical meanings. What one means to grasp and express immediately as the meaning of the theorem is nowhere clearly distinguished from the multitude of things that are implied in

is meant in a narrower or broader connection to this immediate meaning.

By applying concepts, a statement about reality certainly captures
The "given" fulfilment of provisions (or presupposes them) can never exhaust what it assumes to be a universally determined reality. A demand is a demand for fulfilment, which it grasps only in an incomplete determination, one-sidedly, and assumes to be universally determined. The demand "that it be so" requires that it be so, regardless of the facts — which are established — and in the context of the whole of reality.°1* It takes into account, in a meaningful way, circumstances and consequences, both desirable and undesirable — without, of course, being equally responsible for all of them — and is bound to good sense, to everything that should be, so that it fits in with it.

An evaluation is an evaluation of the fulfilment or modes of fulfilment of meaningful determinations and, in its own way, participates in their contexts of meaning. In the sense of the proposition, there always remains a reference to meaning that is not explicitly stated but is to be stated. An all-encompassing, unfinished connection emerges; every meaning can be connected to every other in manifold ways, and each of these connections is itself a meaning — except that for countless meaningless or far-fetched ones, there are a small number that are actually significant, "meaningful." An infinite field of possibilities **for** intellectual **activity.**

In various versions, we find the idea of a "realm of meanings"; since every context of meaning is itself meaning, this would be an all-encompassing, supreme meaning, attributed to something like the divine spirit as a postulate. This supreme and total meaning would have to place every meaning, sense and nonsense, in the right relationship to every other.

The idea has logical difficulties. It also usually falls prey to the error of objectification from the outset, even if it does not literally posit an all-encompassing order of "ideal objects". One already objectifies when speaking of the all-encompassing infinity of the supposed overall meaning, anticipating "the unfinishable" as if it were finished. But this misguided idea gives rise to a completely legitimate concept that we always need when we speak of meaning and meaning content in general –

without talking about a totality of all meanings, which would be absurd. When we say, for example, that every meaning, of any proposition, points beyond itself, that it is relative, this statement *about* arbitrary meaning derives its meaning from the exemplary proposition of some specific content or from an attitude towards proposition which, itself a proposition, is expressed in the use of the word 'meaning (in general)'. This is an act of relating a readiness to move from any proposition to propositions of more comprehensive meaning, in any direction. In this unlimited potentiality of further positing, which is experienced in the thinking of meaning in general, there lies a direction, that is, a meaning of which the talk of an all-encompassing structure of meaning is an expression – an inadmissible one, insofar as it sets a never-attainable, seemingly objective goal instead of a direction.

Such turns of phrase must be used with caution, only in the ever-vigilant readiness to reinterpret them from the seemingly objective to the meaningful and substantive, through the execution of positing and attitude, which lies in the positing. Given this, it will be permissible to say that "the direction towards the structure of meaning" lies in the meaning of every positing as the meaning of meaning.

9

VALIDITY, ORDER

To assign meaning is to assert it. An assertion and the statement that the assertion is valid are synonymous; a demand and the claim that the demand should be valid, an evaluation and the claim that it is valid, are equivalent. In this sense, it can be said that a meaning contains the claim to validity. This also applies to the meanings of theoretical propositions of a free and non-binding nature, which, in contrast to judgements (assertions or assumptions), are called "mere assumptions" °°*. "Assuming there is a perpetual motion machine", "x is an integer" and "assuming that there is a perpetual motion machine", "it is true that x is an integer" — the assumption of the validity of a "fact" or a determination has no other meaning than the assumption of the fact or the determination; the non-binding nature of "mere assumption" is only a peculiarity of the behaviour, the statement, of the person making the assumption: he does not commit himself to the meaning by making the assumption, as he would do in the case of a "fact" by asserting or even merely supposing it; However, in the case of a provision ("x is an integer"), one cannot, according to its meaning, advocate it without judgement; here, there is only free, accepting positing. The claim to validity, however, lies in the content of the positing.

Although every meaning contains a claim to validity, not every claim is valid. A negative proposition contains the claim to validity of the invalidity of a meaning that could be the content of an "affirmative" proposition. Here we have two contradictory claims; only one can be justified: there are also unjustified claims to validity, "bad" meanings. It is precisely in its claim to validity that its unjustification lies, its violation of validity, that is, of valid meaning.

The claim to validity that lies in a meaning is at the same time

Entitlement to validity in every valid sense, entitlement to validity in the context of meaning. Herein lies a semantic advantage of the valid over the opposite. In purely semantic terms, they do not stand in the same way opposite each other, even if each is the exact negation of the other. The "forward pointing" of meaning applies differently to what is valid than to what is not valid: if the meaning is not valid, it "points" to non-valid content, which it includes as consequences, but always also in the direction of valid meaning through its claim to be valid with everything that is valid. This is the basic direction, *the* meaning in every meaning. In setting a meaning, whether it may be valid or not, we always relate the basic attitude to validity, that is, in the valid sense, even in a false attitude; that is why the false is inconsistent and absurd in itself; and that is why the setting of acts of meaning, despite all aberrations, will generally have the direction towards the valid. What lies in this direction, based on a meaning that has been achieved, is a better meaning.

Once a decision has been made, once an attitude has been adopted – each of which is meaningful – then, insofar as this condition is concerned, the more correct behaviour is the more probable one, as opposed to a less meaningful one that can be specified just as clearly, but not as opposed to the vast and obvious variety of possible deviations from it. This results in the probability of an accumulation of behavioural cases close to the correct, good meaning. Since in every attitude we relate to the basic attitude "towards what is valid", i.e. towards the valid meaning, it is "natural" for us to follow the better meaning, but not necessary. What the better meaning is in a particular case may, of course, be unclear; but because the claim to validity lies in every meaning, the demand for the better meaning always applies. To dispute this is not possible without absurdity; but the demonstration of absurdity is the most decisive refutation. That this is so does not require further justification. The remark that every proposition contains the claim to the validity of its meaning, and that the absurd therefore contradicts its own claim, does not contribute anything to the justification of the unjustifiable; it only makes clearer what lies in the meaning of every proposition. Certainly, someone can "place themselves outside of meaning" by declaring, when made aware of an absurdity in their speech, demands or actions, that they do not care, and by continuing with their

Absurdity persists. It persists in a partial sense, in a negative sense, and cannot be refuted because it refutes itself and has pronounced the judgement that condemns it. Recognition of the demand for meaning cannot be enforced. There are always those who are incorrigible, whom the demonstration of absurdity only causes to flee into the darkness of inadequate thinking, confused feeling or blind will, and those who seek the highest and deepest meaning fundamentally beyond what is sensibly justifiable; for them, antinomy is the hallmark of philosophical spirit, instead of being an invitation to examine, clarify and correct their concepts. Here, since the demand for good meaning is not recognised, there is no understanding. Recognition cannot be forced logically, because it is a prerequisite for all reasoning and explanation. For the same reason, it is impossible to prove that there is such a thing as valid meaning. The assertion that there is no valid meaning is either a claim that itself lays claim to validity, or it is not meant seriously; in either case, it lies outside the realm of reasonable discussion. Although in all positing the basic attitude is towards what is valid and every valid meaning is posited meaningfully, every new positing is nevertheless a new decision, both in the theoretical and in the practical realm, and the commitment to that basic attitude, the commitment to meaning, can therefore be called a free intellectual attitude.

The drive towards unfolding is unmistakable in spiritual life. This
It would be misleading to describe events and their "dynamics" as the self-unfolding of meaning. Meaning does nothing and drives nothing, because it is not reality. But the reality of positing and its basic attitude is determined by meaning and therefore, although it by no means proceeds as a constant progression, is an unfolding of meaning. Instead of the timeless reality of a realm of ideal objects, the basic attitude unfolds not into an objective whole of meaning, but into the meaning of meaning, into real positing. This observation and the basic view expressed in it avoids statements about being and becoming in questions of meaning where they are inadmissible, and limits them to the reality of meaning-determined life. Nor is any "deduction" of the fact of spiritual development undertaken. Facts can only be experienced. However, from the experience of setting meaning and the basic attitude related to it, through reflection on

the meaning more comprehensible, that life unfolds meaning and pursues the better meaning within it, even if only in pure abimingen.

No one knows where these developments began; there is no point in asking about a state that could be considered the goal, nor is there a uniform path, even in the sense advocated by older ethnic psychology. We find peoples and ethnic communities and, in their "worldviews," certain ways of perceiving and evaluating things, each expressing a prevailing basic orientation of "disposition.". From the multitude of limited meanings in which such a disposition manifests itself and is obscured beyond recognition by the peculiarities of external circumstances, the meaningful and essentially transparent expression in the mythical-religious consciousness and moral will of a people emerges early on. Here, the meaning, which is present in every meaning, albeit still in a peculiar form, becomes clearly perceptible, perhaps most clearly through the claim to universality that such symbols—they are largely symbols—tend to inhabit. World myths emerge; the supreme deity is, if not the only universal god, the most powerful and excellent of all gods, both native and foreign; the customs of one's own people are considered to be the right ones among all possible ones, and that they want to be so is, of course, an inadequate expression of the striving towards a better meaning that lives in all morality: the anticipation of a "goal" is characteristic of primitive and also of less primitive experiences of meaning in both theoretical and practical areas. The search for an expression of an all-encompassing and all-justifying meaning is then taken up by philosophy with conscious theoretical intent; it only makes sense as an undertaking of fundamental reflection, a task set not for every human being, but for humanity as a whole.

THE MEANING OF EXPERIENCES

Experiences are multifaceted and indeterminate. Within each person lives the human being, with all his or her aspirations and inclinations, opinions, convictions and perceptions; he or she always experiences something that manifests itself as the fulfilment of manifold meanings or, as it were, leaves an inner impression. In the average life, clear meanings of will and knowledge do not occur too often as directions in lived and experienced reality — whether actively set in motion by us in the true sense of the word or merely perceived by us. Their "pure" meaning, as it were, pure direction in the practical and theoretical, is inherent in valuable, vivid content as its mode of fulfilment. It is set as "pure" when we deal with demands and statements in a very general way; in such an attitudinal setting, the pure meaning of the demand or statement is conscious, which is not an object to be regarded as a deceiver of determinations and, as it were, from the outside, as their bearer. The original unity of the mode of fulfilment is still clearly experienced,
e.g. in artistic perception; it is the content of expressions such as *light, dark, warm, cold, soft, hard, flexible, rigid, sparse, wide* and *narrow, heavy* and *light,* which, in terms of historical experience, do not first have a purely sensory and then a "figurative" meaning, but rather a uniform, value-laden, illustrative meaning — without which it would be impossible to understand how those "figurative meanings" could have come about. Gradually, values and sensory perceptions begin to diverge, though never completely separating, just as the meaning of a demand differs from that of a statement — of comprehension and knowledge — as "purely" practical from "purely" theoretical meaning.

This results in a simple, broad connection between the main types of meaning: pure demand meaning and pure factual meaning, values — the whole diversity of content experienced in feelings and moods — as ways of fulfilling of the meaning of demand and contents of perception as modes of fulfilment of factual meaning, factual meaning. When events occur as the fulfilment of demands, they do so not in terms of their factuality, but through values, which represent the actual mode of fulfilment of the demand, as they determine its content.

Language plays a significant role in the process of separating meanings, since the diversity of colourful, unexplained content that occurs as random, momentary meanings in the individual applications of a form of expression gives way, in human linguistic communication, at least in certain areas, types of traffic situations, a specific, consistent meaning emerges as the minimum meaning of the word or phrase, as "the meaning of this form of expression".

At this point, we shall not discuss the intellectualist prejudice that ideas about target objects and perceptions of facts are "psychological" and that their contents are meaningful prerequisites for evaluations and demands, nor shall we discuss the sources of this prejudice, some of which are still effective today.°4* However, this prejudice should not be replaced by a contradictory one, as there is a tendency to do so at present. Instead of a one-sided relationship of presupposition and sequence, there are manifold connections between theoretical and practical meanings and, in a different way, between their experiences. Investigating these connections is a task that will be more successfully tackled once the distinct meanings of the individual types have been considered separately. It was probably always closest to theoretical endeavour to reflect on theoretical meaning, especially since what emerged from the consideration of non-theoretical meanings naturally had to be understood in terms of theoretical statements, whereby their unique nature was completely lost sight of more than once. Certainly, the situation is clearest in the theoretical field, even if there is still much to be clarified.

11

THEORETICAL MEANINGS IN **GENERAL**

We distinguish between two types of theoretical meaning: statement content and determination content. These are also referred to as statements and determinations — or judgements and concepts. When we speak of judgement and concept, we sometimes mean an act of judgement, of conceptual thinking, or an ability to perform such acts, and sometimes the meaning of such an act. A statement or assertion, a propositional sentence, is the linguistic expression of an act of judgement, which in this context means: the ascertainment, whether it be a recognition or an activation (actualisation), an execution, of already existing knowledge or even of false opinions.°° Of all this, only the meaning of the statement or observation is considered here: the content of the statement. Where there is no fear of misunderstanding, a brief statement should be made. Likewise, the meaning that may appear as the content of an act of determinative positing should always be designated as the definition.

In order for these meanings, of both types, to be "considered", i.e. developed and clarified, their linguistic expression is of course necessary; meanings that lack such expression, for which we may first have to search, should by no means be denied, but must remain outside the actual discussion. Even when consideration originates from language, language always remains an expression of meaning and a means of communication; the means of expression, the sign, does not enter into the meaning it expresses and is therefore irrelevant to meaning, just like the psychological experience and the mental act of positing. Even if, in communication with one another, we are dependent on the use of signs in acts, it is the meanings alone that are decisive here, which would remain the same if they were experienced in other experiences and expressed in other signs.

The fact that meanings should not be attributed to the existence or timeless permanence of "ideal objects" is a fundamental point of discussion.

This has already been clarified. Questions of being are not questions of meaning and must not be confused with them. This seems to be contradicted by the fact that the question of whether a proposed sentence or definition has a specific meaning is permissible and meaningful, and yet seems to imply that a statement or definition has a specific meaning. However, such questions can be asked and answered without asserting, denying or even considering the existence of meanings.

If *p* is a sentence that has the linguistic form of a proposition (propositio), then the phrase "There is a (specific) propositional meaning of *p*" should mean: *p* is a meaningful, decidable sentence, which can be summarised as follows: the sentence *p* is a proposition. If it is not, then such a sentence (despite its form) will not be called a proposition. If p is a linguistic expression that has the form of a definition, then the phrase "there is a definitional meaning of p" should mean: the expression p is an expression of a definition, or, in short, a definition. If p is not this, then it will not be called a definition. More on definition in Chapter 16.*6* These stipulations protect us from inadmissible assertions of being in questions of meaning. They take on their living significance in contrast to expressions that are apparent statements or apparent definitions. Such expressions form the core of those sophisms with which the ancient logicians occasionally concerned themselves, without, however, arriving at a fundamental clarification. As "paradoxes", they have been seriously addressed by modern logic with the intention of revealing the reason for the logical inconsistencies and indicating a sure means of avoiding them. But even outside these sophisms and paradoxes, there are enough sentences and concepts for which the question of meaning — whether they have a specific one — is justified and necessary.

12

THE PARADOXES. CIRCULAR REASONING AND APPARENT DETERMINATIONS*

A much-discussed example of logical paradox, a refinement of the ancient "liar" or "Cretan" paradox, is the sentence "This sentence — which I am just saying or writing — is false". According to its wording, the sentence would have to be false in order to be true, and vice versa; it is meaningless. The essence of the error becomes clearer if we change the example so that there is no contradiction. It now reads: "This sentence (or: what I have just said) is true." If we stick to the wording again, we would have to say that the sentence is true if it is true, and not true if it is not true.

One cannot infer any possibility of a decision from it, because a decision would presuppose that the decision has been made: the proposition is undecidable. The reason for this undecidability becomes apparent when one asks what is claimed to be true in the proposition "This proposition (q) is true", i.e. what is the q that must be true if the proposition is to be true. This q would have to be the sentence "This sentence (q) is true" itself. The meaning of the sign "q" should therefore be explained by the meaning of the sign "q is true", which would only have a specific meaning, a statement, if "q" had a statement meaning. The purported explanation of the meaning of the sentence is circular; it is not an explanation. In other words, in the sense of the sentence that has been formulated, and based on its wording, it is not possible to specify what the meaning of the sentence is. The sentence is not a statement, but — this must be countered to the usual understanding or expression of more recent logicians.
- It is not meaningless (devoid of meaning), like something that does not appear as a sign at all, but has a meaning — not a propositional meaning, but a sentence meaning — that makes it impossible to decide whether it is true or false.

The meaning of a statement is predetermined by the decision of true or false; it is a decidable sentence because its meaning is determined in such a way that, on the basis of this meaning, the decision is possible. The sentence 'This

The sentence is true, but its wording implies that the decision itself is the stated content, about which the decision has yet to be made. Therefore, this wording has no meaning. *'p* is true'. The predicate expression 'is true' can only be applied in a meaningful way if *'p'* stands for a decidable sentence content, i.e. for a statement. The expression that *'p'* then represents cannot be *'p* is true'. This is because: 1. the phrase *'p* represents the expression *'p* is true' is a false statement; by saying *'p'* means the same as *p* is true', it has not explained what each of these expressions means; 2. because *'p* is true' contains the predicate expression in the sentence that is to be decided as true (or false), which would only be appropriate as an expression of the decision about this sentence. — Therefore, *'p* is true', where *'p'* stands for *'p* is true', is not a decidable sentence. The decision about *p* is "made the content of *p*", according to the wording. Thus, *p* has no propositional content, and a decision is impossible.

The peculiarity and meaning of such a paradox lies in the fact that a linguistic expression appears with the claim to convey a meaning, but the fulfilment of this claim depends on the fulfilment itself, which is assumed to have already taken place (and is therefore impossible).

If one replaces the word "meaning" with "definition," this characterisation also applies to the pseudo-definitions known as circular definitions, one of which is found in the paradox of the liar; The reason why the expression of a meaning does not come about here is that a determination of the meaning of the expression "this sentence" or "q" does not come about due to a faulty self-assumption. The expression "'q' means 'q is true (false)'" is a pseudo-explanation (pseudo-definition). If *"p"* means a certain statement, then "*p* is true" is also an expression of a certain statement content, and both statements, *p* and *p is true,* are equivalent: if one of them is true, so is the other.° But the explanation that the sign *"q"* expresses something that is true if and only if what the sign "q is true" expresses is true, this explanation only establishes that "q" should be an expression of a statement, and such an establishment does not make "q" an expression of a statement. In general: the statement "The expression 'R' means the same as the expression 'B'" is only a determination of the meaning of 'R' if 'B' already has a meaning. However, this rather obvious observation shows that the efforts of some modern logicians to answer the question of the meaning (or sense) of an expression (or sign, as they prefer to say) by referring to the "class of signs that are synonymous with it," when at the same time it is meant that synonymous signs are those that can stand in for one another within a given (fixed) "language." If 'R' and 'B' can stand in for one another according to the definition, each of them can still be meaningless. Mere definitions or, after definition, considerations of the relationships between signs do not yield any meaning of signs: it is not possible to dismiss meaning as a "sham relation" — even if it cannot be grasped as a "rel", i.e. as a relationship or assignment between a sign and the object it designates.°® A sentence such as "I always lie", "Every sentence I utter is false" — in the sense of the sophism of the "Cretan" — or "No sentence (in the form of a statement) is true" or "is decidable (a statement)", is absurd, and if one omits the negation in it — "I always tell the truth" etc. —, it is meaningless without self-contradiction. The first then contains the sentence "What I am saying now is true", as does the second; the third contains the sentence "This sentence (which has the form of a statement) is true" or "is decidable (a statement)"; i.e. such a general sentence consequently includes a sentence that has no meaning as a statement and is therefore not a statement. The meaning of the general sentence, which is not a statement, nevertheless justifies the conclusion of the self-referential individual sentence: the consequential relationship is not bound to the meaning of the statement (and determination); otherwise, those conclusions from a sentence that prove it to be meaningless would not be possible. Nothing can be extracted from something that is utterly meaningless.

Expressions that are apparent determinations can be formed in any quantity. When someone makes provisions "for unforeseen circumstances," this implies the real determination of "circumstances that I do not foresee in the specific manner that I have envisaged." This becomes a pseudo-determination as soon as the "foreseeing" is taken so far that "what I do not foresee now" means "what I do not meet in this determination (which I am setting)". The result is a turn of phrase in the form of a definition that is self-contradictory; its non-contradictory counterpart, "what I do meet with this very definition", "what satisfies this definition", proves itself and the definition to be self-referential and a mere pseudo-definition. What the expression "this determination" or "‹p" means is indicated by the expression "satisfy this determination (‹p)": this does not set any determination.

An expression such as "that which cannot be defined by any provision" or "that which does not fall under any provision", "does not satisfy any provision", which has the form of a concept name, is not a concept name. The pseudo-definition it contains, namely "does not fall under any definition", includes the pseudo-definition "does not fall under this definition" and is absurd and meaningless. The meaninglessness of circular pseudo-statements and pseudo-definitions has become sufficiently clear from these examples. None of them are affected by the "circularity principle"°1 of *Principia Mathematica,* which states: "If a certain set, assuming it forms a whole, contains elements that can only be defined in terms of this whole, then this set does not form a whole" (Mokre, p. 56)°* — to which the following comment applies: "By saying that such a multiplicity does not form a 'totality', we mean first of all that no meaningful statement can be made about 'all its elements' *(ibid.,* p. 55)." The examples just presented are all constructed in such a way that totalities are not presupposed in them; the word "all" does not appear. In some examples, the word "every" plays an essential role; but when we speak of "every sentence" of a certain kind or of "every determination," this does not necessarily imply that there is such a thing as "all sentences" of that kind or "all determinations" — even if this may be the case in individual examples. Furthermore, it has been shown that the "every" examples owe their lack of meaning to the fact that they contain a pseudo-statement or a pseudo-provision that does not contain the expression "every" either, but has the form of an individual statement or an "individual provision", if, for the sake of brevity, this unusual but but understandable name is permitted for the sake of brevity: these expressions are self-referential in a flawed way and each represent what can be called the "pure circularity error" of the example.

13

E SENTENCES AND TERMS°3*

A second remark that should be added here concerns a matter of fundamental logical importance. Logical propositions, understood to mean propositions of logic, are general, and many of them are applicable to themselves, as are logical concepts; nevertheless, they are not circular. One example is the "law of excluded middle" in the form *p or not-p (applies generally).* However, if we rephrase it as "For all statements *p*, *p* or not-p applies", we introduce an erroneous generality: there is no such thing as "all statements". But this version is unnecessary; instead, one can say: *For every statement p, p or not-p applies,* or: *Every statement is either true or false.* This establishes a formula that applies generally, i.e., as soon as any statement occurs, it can be applied to it and applies to it. The "generality" that we attribute to such a proposition means nothing more than this. In particular, we can apply it *as follows*: *"'p or not-p' applies to 'p or not-p'",* therefore *"'p* or not-p' or *'p* or not-p' not". This is a correct application and free of circularity. The proposition *p or not-p* has been asserted in its generality and subsequently, in a second proposition, applied to the proposition itself. The procedure does not lead to a circular proposition of the type "This proposition (which is just being uttered) is true or false" — where the expression "this proposition" should mean nothing other than the whole phrase that contains it as its "subject" —; the application can be expressed as follows: "The sentence that has just been uttered — namely *'p* or not-p' — is either true or false." What the application deals with is not a pseudo-statement, but a specific statement with independent content. In short, one can say that what the sentence *'p* or not-p' extends to are *statements*.

p. But the expression "What I have just said" in "What
I have just said is true or false" expresses is not a statement

is neither true nor false, but rather meaningless. This is where the remarkable "retroactivity" of applying the logical proposition *"p* or not-p" to the proposition itself is rooted. In the proposition *'p* or not-p', the proposition "*'p* or not-p' or *p* or not-p' is not" is "potentially" contained in terms of attitude; however, the execution of this second proposition is not necessary for the meaningful execution of the first.

An expression such as *'x* is red' can be described as 'x is red' is a determination; 'to be a determination' is a meaningful predicate: *'x is a determination' is a determination.* This sentence makes perfect sense. The determination to be a determination is applicable to itself without any faulty self-presupposition. One can make statements about 'every determination'; they also apply to the determination to be a determination. Thus, for every determination ‹p: ‹p is either fulfilled in any case (within its scope of application) or non-‹p is fulfilled; or:

In every case of application, either p or not-p is fulfilled; the applicability of a determination can be decided in every case. Such sentences about every determination would not be possible as statements if 'x is a determination' were not a determination.

14

SCH DETERMINATIONS. "PREDICATE"°**4***

The sentence 'To be a predicate is an expression that applies to itself (and is accurate)' is a statement and differs significantly from a sentence with similar wording, which leads to the 'explanation' of the expression 'autological'. It reads: "An expression (in the form of a predicate) is called 'autological' if it applies to itself, otherwise 'heterological'." The expression 'short' is short, the expression 'word' is a word, the expression 'long' is not long. If one now asks whether the expression 'heterological' is autological, the result is: if the expression is autological, then it is of the type of heterological expressions, i.e. heterological; if it is heterological, i.e. does not apply to itself, then it must be autological. If one asks whether the expression 'autological' is autological, the result is that it is autological if it is autological; that is not a decision. The reason for the lack of meaning is easy to recognise and eliminate. It lies in the fact that the determination of the use of the word 'autological', in application to the word itself, presupposes that a decision to be made has already been made. If, on the other hand, one explains that "an expression of a given definition (in the sense of the explanation in chapter 13) is called 'autological' if it applies to itself", then the application to the word 'autological' is excluded and thus the circle is broken; because the word 'autological', whose meaning is to be explained, cannot be used in the explanation as an expression of a given definition, i.e. one that is independent of it and its explanation. In the version given, it is free of circularity, and there is no objection to 'autological' (and 'heterological') being explained in this way. Examples of a similar nature are partly known and partly easy to invent, and they do not have to contain expressions such as 'autological' that seem tricky. If the 'village barber' is described as the man who shaves every man in the village who does not shave himself, then applying the formula to the barber himself results in that if he does not shave, he should shave, and vice versa. If someone declares, "I respect everyone who respects themselves (and no one else)," it follows for the man himself that, in accordance with his declaration, he respects himself when he respects himself, i.e., no decision is required. To avoid this lack of meaning, one need only formulate the statement in such a way that it becomes a determination that does not presuppose itself. The village barber shaves his *fellow villagers,* who — regardless of the definition of his activity — have already decided whether to shave or not to shave; the statement is then inapplicable to him. The declaration of respect must read: "I respect everyone else who respects themselves (and no one else)"; and if it is to include the speaker, it must be supplemented with something like "and myself, as long as I have this and that reason for doing so (which is of course not my self-respect)".

15

DECISION-MAKING. OUTPUT AND OUTPUT FORM

The logic that is nowadays referred to as Aristotelian understands a statement to be a proposition that is either true or false; such propositions are called decidable. This term suggests that it is always possible to decide whether a given statement is true or false. "Possibility" is a somewhat unclear concept, and so the simple fact that statements are decidable gives rise to a number of questions and interpretations. The opinion arises that if a proposition is decidable, then it must be possible for subjects of a certain type and ability, or for any subjects, to recognise whether it is true. One thinks of decidability for humans, finds oneself faced with a great diversity of cognitive abilities among humans, perhaps retreats to decidability for "humans," for what the human species is capable of, and comes to no clear conclusion. This leads to the question: "Is the proposition that there are mountains on the far side of the moon that are higher than any mountain on Earth decidable?" and similar questions and considerations about what would be possible if this or that were different in our world than it is; whereby it is again questionable whether such a difference is possible, and in what sense. One might agree that it is possible if it is compatible with the laws of nature.°° This is a very dubious determination, since no one knows "the laws of nature," and the old question arises again as to whether they are knowable "to humans." To avoid all these difficulties, one ultimately introduces the "ideal subject of knowledge" and declares that what this subject could decide is "fundamentally decidable". This "ideal subject" is, of course, defined by the characteristic that it "knows everything" **or**, more cautiously, that it can decide whether every statement presented to it is true or not**, thus** proving to be a completely superfluous invention. "*p* is decidable for an ideal subject" means "if x is a subject that can decide on every statement".

can, then *x* can decide on *p*. "The proposition *p* is fundamentally decidable" or, in short, "decidable" means nothing other than *"p* is either true or false". A proposition is fundamentally decidable if its content is such that either it or its negation is true; to be fundamentally decidable means to be decidable in terms of meaning.

In order to be reasonably certain, the question of practical decidability must always presuppose certain subjects and other conditions. Thus, the question of the mountains on the moon is currently practically undecidable for the people of Earth, even though it is decidable in a meaningful sense. Such questions, however important they may be to us, are not purely questions of meaning; they in no way affect the logical distinction between decidable and undecidable propositions. The fact that there are decidable propositions, i.e. statements, and in particular that there are true statements, cannot be proven by appealing to the absurdity of the proposition "There are no decidable (or no true) propositions". It only shows that we have no reasonable reason to concern ourselves with such statements — except to reveal their lack of meaning. Once we have recognised that someone who argues against the existence of decidable (or true) propositions is in fact saying nothing, it is clear that there is no point in asking whether such **propositions** exist at all; but for *this* reason, it cannot be asserted either. Through every statement we make, i.e. assert, we express the opinion that it is true — even if we do not explicitly think so.

—, and to avoid this position, we would only have to refrain from making any statements. However, it is a fact of experience that we do make statements. It is also a fact of experience that we have already decided in favour of the existence of decidable and, in particular, true propositions, in such a way that there is no revocation (even if we were to overturn any number of the assertions we have made). But which given propositions and which sufficiently specified types of propositions are decidable and which are true are meaningful and important questions. One must simply not demand a general "criterion" of decidability and, in particular, of truth. An explanation of the form "A proposition *p* is true if *‹p(p)",* i.e. "if it has the property ‹p", is meaningless; it contains the circular proposition "This proposition (which I am just setting up as a criterion) is true if it has the property ‹p". Thus, the truth statement of pragmatism, "A proposition is true if it is useful" cannot be taken in an unrestricted sense; it would have to be restricted to propositions of a certain kind, e.g. those that do not deal with the truth of propositions, and the truth of propositions of another kind would be recognised as independent of usefulness. If, for example, the sentence "There is a God" is to be true in the pragmatic sense, then it must simply be true that it or the belief in it is useful. This does not achieve what was intended: the elimination of the concept and assertion of "absolute" truth.

Recent and latest attempts to relate the truth of a statement to a subject, a type or a community of subjects° * and to make it dependent on them are no different from the theory of truth in pragmatism. In their unrestricted form, they are *meaningless;* correspondingly restricted, they do not make the concept of absolute truth dispensable. Just as the pragmatic "usefulness" of a statement is a benefit for certain people, so, for example, the "fruitfulness," "recognisability," and "accessibility of a piece of knowledge" are related to people of a certain kind, and the truth that it establishes or creates is supposed to be a "truth for someone." In particular, the realisation of how great the diversity of human types is and how manifold the barriers that seem to make general understanding among people impossible, especially on the most important issues, has favoured the new relativism.However, the relativities that exist here are not relativities of the truth of certain propositions, but belong entirely to their content: the establishment of a relationship, the relation of a determination to certain circumstances and the dependence of its validity on these, is either true or false, absolutely and not "for someone".[40] Those questions that concern "essentials" and in which understanding is so difficult are questions of value or are linked to questions of value. Now, to mention only the simplest example, something can be conducive to a certain purpose and valuable in that respect, but hindering or irrelevant to another. However, the natures of human beings are determined precisely by the directions of their aspirations, i.e. by the goals that are "appropriate" to them. To establish this means to establish a relationship of "values" — which, incidentally, gives no reason to assert a "general relativity of values"; but it does not mean asserting a relativity of truth: the

A theoretical statement is an assertion that claims to be absolutely valid. In people's value-related speech, it is quite natural for one person to say that something is valuable (beneficial, desirable, etc.) and another to claim the opposite, but each means "valuable (beneficial) for purposes that are in line with my way of thinking". Both can be right in a non-subjective sense, because values are based on demands, i.e. on their meanings, and not essentially on the people who represent them. There are, of course, differences in rank between the demands and the systems of demands to which they belong, and it is natural for people to attribute greater validity to those they champion; but the conflict that arises from this does not take place in the theoretical realm. Herein lies a practical root of the new theoretical relativism. The theoretical root, however, which was probably decisive for the older one, is the fear of "absolute truth," which one believed — wrongly — to have to think of as an "ideal object" or as the content of a perfect consciousness. In addition to the diversity of prevailing attitudes in which people find themselves, the diversity of the locations they occupy in their spatio-temporal environment is a cause of relativistic error. The aspiration in which I live and the place where I stand are the location from which I perceive; in relation to it, I determine and easily overlook that it itself is a member of the relationship. This gives rise to statements such as those that appear to be unconditionally evaluative, such as "Today is Wednesday," "It's cold here," "It's raining (here now)", "The right way is to the right": sentences that are only understood in relation to and application to specific points in time, statements that are true or false, but without such specific application — only as a grammatical example of a German sentence and not asserted — are not statements, but only forms for statements. The sentence "Today is Wednesday", which I assert in application to 1 October 1941, is true; an identical sentence referring to the following day is false and is not the same statement (it does not refer to the same day). Any sentence that contains a referential "this" can only be a statement in a specific application, referring to something pointed out, and without it is only a form of statement and not even true, sometimes false, but always neither true nor false. As obvious as this distinction is, it is nevertheless missing.

Those who say that any opinion — e.g. that there is no transformation of chemical elements, which prevailed among natural scientists in 1890 but no longer prevails in 1940 — was once true and is no longer true. Provided that the quoted sentence has the same meaning in both cases — only the meaning of a sentence matters — it was as false in 1890 as it was in 1940; more precisely, it is false regardless of time and independent of it. However, if it means "We know of no elemental transformation in 1890" in one instance and "We know of no elemental transformation in 1940" in the other, then these are two different statements. Such statements should not be necessary, but unfortunately they are, as evidenced not only by the daily refrain that "there are no eternal truths," but also by the resurgence of relativism among scientists, which is mostly based on a failure to consider such obvious facts. Truth and falsehood apply only to statements, and then irrevocably; mere forms of expression, however, do not have a changeable truth value, but none at all.

DECISIONS

The meaning of a proposition is most clearly understood in a free, accepting setting: *'x* is a natural number', *'y* is a positive proper fraction' or '0 < y <1', 'r is less than u' or *'z <u'*, 'u-F *o* ——*w'*. These mathematical examples have the advantage of a high degree of certainty and clarity in the content of the proposition, compared to examples that contain 'empirical' or 'descriptive' determinations, such as 'x is red', 'y is the father of r' or 'you get three apples and then two more' (which begs the question: 'How many apples do you have then?'). In the examples of the second type, modes of fulfilment of determinations are assumed in which pure determinations of the first type are set.

'*y* <1' cannot, according to its meaning, be asserted, either rightly or wrongly. This expression is not an expression of a statement, neither true nor false, as long as the meaning of the symbol *'y'* is not defined in a sufficiently specific manner. Rather, the free assumption *that* j'< 1 provides a definition for *y*, albeit one that is ambiguous and, in this sense, incomplete.

With it, some other determinations, such as j' }, k 1, are compatible; others are incompatible, such as y= 1; still others are not only compatible, but included in it, such as *y*<2. This relationship of one determination being included in another, the "general implication" between determinations, is particularly important. *y*<1 includes, implies, *y*<2; *y*<1 *does not* include *y* ——1, *y* 1, which is the incompatibility of *y*< l and *y* ——1; *y*<1 does not include y *not <§* and does not include *y §*, which is the compatibility of *y*<1 with *y<(* and with *y* —- .

"'*y* <1' *implies* '*y* <2"', or *"If y*<1, *then y*<2" can be asserted without 'y < 1' or *'y <2'* being asserted on their own. *If y*<1, then *y*<2' applies regardless of whether the conditions are met, as a conditional implication, a pure relationship of meaning. In asserting of valid determination implications with *'y* < 1' as the antecedent, the meaning of this determination is unfolded; its implications are its meaning.4'

A scientific investigation, initially one for which a precise logic can be established, always moves within a specific domain; for example, number theory in the domain of positive integers, ordinary arithmetic in the domain of real numbers. In relation to such an "entire domain", the negation of the validity of a determination *fx* is equivalent to the assertion of the validity of the

negation non-fx or *fx* and defines a specific negation class or complement class to the class defined by *fx*. The determination *fx vfx — fx or fx* - is empty in relation to the universal domain of

permissible x-values, because it is satisfied by every x in this domain, but

not empty in an absolute sense. A determination *fx* presupposes *fx* vfx as the meaning that is its common partial meaning with *fx*; it is the basis of determination on which, as it were, every

particular, i.e. non-empty, determination in the assumed universal

. Without such a basic determination, there can be no strict and feasible determination: expressions that presuppose an absolutely unrestricted universal domain, such as "the class of all classes", are meaningless; no statements can be made about "everything conceivable" (a fundamental and fundamentally general reflection, such as the one undertaken here, must be wary of these deficiencies in meaning. The question of "how it is possible" takes on a deeper meaning).

D EFINITIONS AND CLASSES
(CONTENT AND SCOPE)

In terms of concrete thinking, we assume that there is a multiplicity of elements — things or cases — as the universal domain of our determinations. Each determination *fx* defines a class *x:(fx)* within this universal domain; there are as many different determinations in terms of meaning above this domain as there are different classes within it. The classes are obtained by forming combinations that can be formed from the given elements. If n is the total number of elements, then there are n single-member classes,

$$\binom{n}{2} = \frac{n(n -}{2} \quad \text{two-membered classes,}$$

$$\binom{}{3} \quad \frac{2(u - 1)(n \quad 2}{2\cdot 3} \quad \text{three-part, etc. ,}$$

$$\binom{n}{n-1} \quad \binom{n}{1} \quad - n(n \quad - \quad 1) \text{ member;}$$

If we add the class consisting of no elements, the null class, and the universal class as marginal cases, this gives us 2' classes and just as many different definitions in terms of meaning. This overview of the classes based on the elements also provides the answer to an old question in logic: Are there simple, ultimate definitions, i.e. definitions that cannot be broken down into sub-definitions? There are: every definition that excludes one and only one element of the universal class is, in this domain, a simplest definition. It is not empty because it does not allow or encompass all elements of the total domain; it does not do nothing, but what it does is minimal, the exclusion of a single element.4° Such definitions, each of which is a class of n —1 elements specifies, there are n: as many as there are (excluded) elements in our universal domain. Each of them is to be called a determining element, and from such elements one can conceive of any non-empty determination composed by conjunction, if n is a finite number. It is justified to say that a determination has the greater or stronger content the more it excludes elements of the universal class, i.e. the more determining elements it contains. If one wants a measure of the strength of the content, the number of determining elements, which is the number of elements of the negate class, is suitable for this purpose. The old rule of the inverse relationship between scope and content takes on a definite form: scope (as the number of things in the class) and content (as the number of excluded elements of the universal class) add up to a constant (namely, the number of elements of the universal class). It is clear that a useful measure of content can only be established in a certain relationship to the measure of extension. If the universal class is not finite, the simple procedure described above will of course fail; but even in the case of a continuous universal set, for example, a measure of content will always be given for determinations if there is a negate class for each class and one has a measure of extension — in mathematical language, this is called a 'measure of content', e.g. for areas or volumes — for it.

The question of "content strength," to which this brief digression was devoted, is not a question of modern logic. However, the consideration shows or suggests what assumptions must be made in order to give the question a specific meaning and the possibility of an answer. Above all, it is noteworthy that a determination that includes only equivalents of itself and empty determination is only 'simple' in a relative sense, namely in relation to the presupposed empty determination fulfilled by every element of the universal *domain* and therefore exceeding it by a minimum of content. There is no 'absolute zero point' among determinations and therefore no absolute ultimate unit of content strength.

Two determinations are called equivalent — *fx gx* — if and only if they mutually include (imply) each other. In this case, they have the same implicates, i.e. — if the implicates are the meaning — the same meaning. Nevertheless, each can be thought of in a thought with different content, such as 'x is an equilateral plane'.

Euclidean triangle', and *'x* is an equilateral Euclidean plane triangle', we then call them different in content — which, compared to the explanation that they have the same meaning, represents a linguistic harshness. Without doubt, in the proposition of one of the above definitions, something different is immediately thought or understood than in the proposition of the other; proof that the equivalence of both must be proven in elementary geometry. But in each of the two propositions, the same thing is posited in terms of meaning; proof that the equivalence of both can be proven. Whoever posits one of them implicitly posits every implication of the other, including itself, which, like every definition, is its own implication; one can know as much or as little about it as one wishes, or posit it 'explicitly'. The meaning of a determination goes far beyond any explicitly posited content; it can never be exhausted by positing partial determinations (implicates).

Where there is equality, one looks for identical elements in the terms being compared. Things of the same colour have "the same colouring", colours of the same brightness have "the same brightness", tones of the same pitch have "the same pitch", quantities of the same number have "the same number of elements". Equivalent determinations "achieve the same thing". One can formulate and express it differently, as long as each of these formulations "does the same thing". If fx and *gx* are two equivalent determinations, they do the same thing in terms of implications, since each partial determination of *fx* is a partial determination of *gx* and vice versa: they have "the same meaning". But they also achieve the same in terms of which cases — "values of *x"* — they allow and which they exclude: they "define the same class" of x values that satisfy them, and this class includes "the same negation class" of x values that do not satisfy them, in such a way that one can say that they satisfy the negation of one and then always also of the other determination. What matters is that determinations that "achieve the same thing" are considered equivalent or identical in meaning, but this identity can be established by different explanations that again "achieve the same thing" because they are all equivalent to each other. Such explanations are: *fx* and *gx* are equivalent if they define the same class — if they *do not* contain the same determining elements in relation to the universal class; if they have the same implications — if the classes defined by them have the same superclasses; if they have the same implicants – if the classes defined by them have the same subclasses; if they contain the same determining elements – if they exclude the same elements (things), they correspond to the same negation class. The dash "—" connects explanations that correspond to each other in terms of scope and content. It can be seen that sometimes one is more natural to our thinking, sometimes the other. But they are all equivalent, and there is no reason to prefer the scope-related ones or one of them in particular.

If *F(f)* is a determination, set function, of the determination /, and *F(f)= F{g)* applies as soon as *fu— g*, then *F(f)* is called an extensional function of *f*. The designation is one-sided, taken from the identical scope of equivalent determinations, but must not and should not be understood one-sidedly. Thus*, fx v fx, fx nd fx* are extensional functions of *fx;* if *fx = gx,* then *gx* v qm will always apply precisely to those x for which *fx* vfx applies, etc. These are, of course, trivial cases. *F(f, h)* will be an extensional function of *f* and of *h* if *F(f, h) = F(f', h')* as soon as *f = f*.

h h'. Thus*, fx* v *hx, fx n hx, fx* v *hx and fx m hx* are extensional functions of fx and *hx*.

In contrast, *'y* sets (assumes, presupposes) /' is not an extensional function of *f: whatever f* sets, it does not necessarily have to set every equivalent of /. It cannot do so explicitly, because there are arbitrarily many such equivalents for a given determination /; but it will hardly do so in the sense that, after setting *f*, when asked whether it would also accept a q —which is equivalent to /— as set (assumed or presupposed), it would always immediately say yes, because the existing equivalence does not necessarily have to be obvious to it. 'I set' is an 'intension' sentence function of *f*. It is clear that this non-extensionality of the comprehension function represents a weakness in our comprehension. We do not comprehend the full meaning of our settings. We do not comprehend and do not mentally set what is *meaningfully* co-set by our setting and is therefore set. However, it must be said that there is a condition in relation to which, as soon as a determination *f* is posited by a subject *y*, it is more probable that *y* posits a given implication — i.e. also an equivalent — of f than that y posits the negation of f (as posited with *f*). The condition thus characterised is what we call the 'understanding' or 'logical ability' of *y*. If

Even though our understanding is not 'extensional', it nevertheless has, in the sense indicated, a 'tendency towards extensionality'. In setting the determination *f*, I experience the attitude — the intention — towards the full meaning of /, which is at the same time the meaning of every equivalent of /; but it does not stand opposite my intention as something to be attained; it is the meaning — the 'direction' — of the intention itself. It depends on my abilities and my attention how clearly the meaning becomes clear to me, how far I, maintaining the attitude, retain it in subsequent positing. Since the determination *x is an equilateral triangle* can only apply if the determination *x is an equilateral triangle* applies, and vice versa, the question arises as to how the two differ. Or do they differ only in the same way as *the author of Faust* and *the author of Werther*, as *the North Pole of the Earth* and *the northern end of the Earth's axis*, namely not at all; are there just two expressions for the same thing?

D ET ER M I NAT 45*

It is relatively rare for the objects or cases of a class of scientific investigation to be given individually and completely, such as the pupils in a school class or persons who were present in a locality at a certain time. Such classification by summarising existing things is far more a matter of practical statistical or historical thinking (the members of a ruling dynasty, Napoleon's wars) than, for example, natural science, which, when it finally observes and examines many cases of a kind, never focuses on the cases for their own sake: they are merely examples (specimens) of a more comprehensive natural species, which is only 'given' in terms of content and for which the respective individual stock is not essential. The types of descriptive natural sciences, as well as those of the exact sciences, will be discussed later; it is immediately apparent that these are not classes in the strict sense of logic: for each of them, cases are conceivable whose membership of the class cannot be decided without arbitrariness. Since the 'given' things and cases of reality are always vaguely defined, so too are the 'classes' to be formed from them.

Classes in the strict sense can only be found in the formal realm. Here, however, there are no given or existing objects that could appear as elements of a class. Of course, one can summarise 2, 3, 5 as 'the first three prime numbers' and form the concept 'prime number smaller than 10', to which the 'things' 2, 3, 5, 7 are pre-assigned. But such and any other 'ideal objects' are not predetermined by *every* definition that applies to them; rather, to put it briefly, they are always either definitions themselves or 'results of definitions', 'determinates', 'definition or conceptual objects'.

Let us first explain this with some examples. We say: "The (Euclidean plane) triangle has an angle sum of two right angles" and, linguistically, treat 'the triangle' as an object to which every property applies that applies to every (Euclidean plane) triangle, and no other triangular properties, i.e. no special properties such as equilateral or non-equilateral, right-angled or oblique. Since every triangle, every *x*, for which 'x *is a triangle'*, *'x z* triangle', is either equilateral or non-equilateral, right-angled or oblique-angled, it is clear that one has no right to say that 'the triangle' is a triangle — just as one cannot say that 'the triangle' is any non-triangular geometric figure and in this sense a non-triangle. It is unjustified and absurd to say that 'the triangle' fulfils any triangular definition. If one nevertheless attributes the general triangular definitions to it, then, if this is to make sense, it must mean a kind of 'accident' that is not fulfilment. In fact, it must be said that 'the triangle' ('in abstracto') is only a conceptual content; being a triangle is *constitutive of* it, as its defining property, not actually coming to it as a property that it fulfils; it is not the bearer (i.e. fulfiller) of its defining **property,** but rather the result, *the determinant* of this property. The determination that fulfils 'the triangle' is that of being the determinant of the determination *'x* is a triangle'. In order to deal with triangles in general, I need the concept of 'triangle'; to apply this concept, I think *of* its content, but not of its content itself, rather *of* something that satisfies the determinations that constitute it (regardless of whether such a thing exists or not); thinking of the content, the determinant, I "aim in the direction of fulfilments of its constitutive determination", in the direction, as it were, in which such fulfilment would be found, if it existed.

One could say: "In comprehensively grasping the determinant of *fx, x(fx),* I aim at a carrier of *fx* and grasp it accurately, if it exists." When I speak "of the triangle (in abstracto),"
i.e. of the determinant of the triangle definitions, I comprehensively grasp — as a "form" — the determinant of the definition, the determinant of the triangle definition, and accurately grasp (if it exists) the carrier of this definition, namely the determinant of the triangle definitions, and so on. The accurate grasping of a carrier (fulfiller)

of a determination comprehensively grasps its determinant (as a "form").4°*

But it is unnecessary and not appropriate to regard them as objects. In the thought "triangle", I orient myself "towards" fulfilment, an object of a certain kind; in the thought "a triangle", I orient myself towards or, rather, to the attitude of that first kind (by meaning the determinate of the determination triangle); I can adjust myself again to this second-level attitude in a third-level attitude (by meaning the determinate of the second level), and so on. Objective fulfilment could only take the form of the first level if one. A determination is *"fulfilled"*, if at all, only by "cases" of something completely determined: that is, in the proper sense, an object or a case, a concrete event. A "form object", such as the form "triangle", which "satisfies" the determination of being the determinant of the general triangle determinations, *does* not actually *fulfil* any determination: what only "satisfies" the determination of being the determinant of the triangle determinations ... The concept of a triangle first and naturally appears in the definition *'x* is a triangle', in the predicate position. The expression 'triangle' *means* 'a closed plane figure formed by three straight lines with three different points of intersection'; this *meaning* is such that the expression can be applied imprecisely in a true statement 'That is approximately a triangle' to a corresponding real figure; i.e. there are real objects to which such an application is possible. In such a statement with the predicate noun 'triangle', one can say, in precise usage, that the object in question *is* (not entirely correctly) *designated* as a triangle*:* as a thing of *the kind* that the expression 'triangle' *means,* independently of its application to an individual object. If, in view of a given real object, one speaks of it as 'this triangle', one has again designated it as a triangle; but the expression 'triangle' now seems to appear in the subject position: 'This triangle is (approximately) equilateral'. In truth, it is of the same predicate function as in its original use, but the words 'this triangle', as *a designation* of a present object, are an expression of the apprehension of this very individual subject as the 'bearer' of the predicate 'triangle'. 'This one here, and it is a (thing of the kind) triangle'. The statement 'This

'Triangle is equilateral' means 'This *(x)* is a triangle, and this *(x)* is equilateral'. The subject object is the given individual, and the name 'triangle' used to designate it in 'This triangle is equilateral' is a generic name and has a predicative meaning. It is always the same in different uses of the name, whether it occurs in the predicate or subject position. In the phrase 'This is a thing of the kind triangle', the *meaning* of the word 'triangle' is objectified, treated as an object and *designated* by the compound name 'the kind triangle'. If I say 'triangle is a kind', I simply use the word 'triangle' itself to designate what it means, its meaning.4'

The importance of this objectification of types in our theoretical thinking could easily be underestimated if one were to think only of examples such as 'the triangle'. It becomes apparent as soon as one tries to clarify what kinds of objects occur as things of such a type. First, we encounter the well-known fact that there is no real thing that could be called a triangle in the strict sense, nor is there a real sphere, cube, etc. Geometry does not concern itself with this and deals with triangles, spheres and cubes as 'geometric structures'. In the spirit of an old logical distinction, one notes that the empirical scope of such a concept is zero, but its logical scope is infinite; however, 'individuals' that fall within the logical scope of the concept of a triangle are, for example, the triangle with side lengths 1, 1, 1; the triangle 3, 4, 5; the triangle 1, 1, 2; the triangle n, 1, z, etc. It is immediately apparent that each of these 'structures' is a determinant of a completely 'geometrically unambiguous' definition within the geometric (not physical) triangle definitions, but does not fulfil them. And 'the Ltinge /' that appears in these definitions is the determinant of the definition of being a unit of length, and not a given unit of distance.

The class of prime numbers smaller than 10 comprises the 'elements' 2, 3, 5, 7. Again, these are not given objects. The number 2 is easily recognised as the determinant of the definition of being a set consisting of one and only one element, and the remaining 'positive integers' are determined accordingly. A 'prime number' is determined by the definition of being a (finite) set to which there is no majority of mutually equal, at least two-member subsets that are foreign to the elements.

Like any number, it is not a quantity, but rather, by virtue of its defining characteristic, a 'representative' of every quantity that has as many elements as units are constitutive of the number. It was evident that 'geometric structures', even those that are 'geometrically unambiguously defined', as well as numbers, are not fulfilments or realisations of their constitutive determinations, but rather their determinants; they are consistently types or categories — modern logic uses the term 'classes' for this. The result can easily be generalised. In the field of pure formal science, we are moving in a universal domain to which no things or cases belong. If they did belong to it, i.e. if there were cases that fulfilled the set determinations, they would be of no immediate relevance to formal science and would not fall within its scope of investigation. We know or believe that there are no real objects that exactly satisfy the conditions of 'geometric structures'; that there are approximations to their 'ideal forms' in reality is very important for the applicability of geometry, but falls outside the scope of purely geometric consideration. But there seem to be enough sets of ones, sets of twos, and so on in reality to fulfil those definitions that are constitutive of the concepts of natural numbers. Nevertheless, and although the possibility of counting real things and events is extremely important, no one would call the arithmetic of natural numbers a theory of countable sets of real objects. The existence of such objects is not a mathematical matter.

It is irrelevant to the validity of arithmetic.

In order to construct the theory of natural numbers, we assume that for every finite set there is a set that contains one more element, or an equivalent assumption. That is, we formulate propositions such as this: If for every finite set there is a set that contains one more element, then there is no largest finite set. Such a proposition is true even if the assumption is not satisfied.There are numbers that are so large that our reality may not have a set of objects of such cardinality. This in no way prevents us from calculating with these numbers according to the same rules and as meaningfully as with 2, 3 or 5. If *a and b* are numbers of such supermundane size, then the following theorem certainly applies: If *a* is a set of cardinality a and f› is a set of cardinality b that is foreign to it (or would be), then there is a number i= o + b that is the cardinality of the union set c of a and b.

, then there is a number i= o + b that is the cardinality of the union of *a* and *b*.
- which indicates how many elements this set would contain, if it existed. If this were not the case, a statement or assumption such as that a set of d real objects does not exist would be meaningless. Incidentally, the assumption that there are objects in a certain number, e.g. five, never finds exact fulfilment in reality — because objects, individual things or cases do not exist in the strict sense — will be explained later.[4®] Here, it is only important to note that its fulfilment is not relevant to the meaning and validity of arithmetic.5°

Since a science of form does not consider real objects, but only their 'ideal objects' or 'forms', the determinants of their determinations, which are only apparent objects, it is clear that a science of form does not consider any objects at all. It establishes determinations — whether in completely free assumption or prompted by observation of cases of approximate fulfilment, which is practical and relevant for an investigation of scientific thinking, but without any relevance for the meaning of the determinations — and it develops the meaning of the determinations in pure determination implications: a pure explication of the implied. The classes that define the assumptions of a formal science are thing-free (individual-free) classes; their "things" are at most determinants. The statement that "two determinations are equivalent if they specify the same class" does not have the meaning — which was probably originally attributed to it, and which may still be "in the minds" of some particularly nominalistic, thing-free thinkers — that it freed the determination from meaning, so to speak, by specifying an objectively given thing. The assumptions 'z is the intersection of the medians *a* and *b* of triangle *ABC'* and 'x is the intersection of the medians h and r of triangle *ABC'* determine 'the same point', but this is nothing more than the determinate of these determinations; what they actually determine is one and the same singular class, and if one disregards this apparent object, nothing remains but the identical meaning — whereby these words objectify what is only directly expressed in the determining expression. So now, when talking about things or cases of a certain kind, initially in the formal realm, it should always be noted that these are phrases that

Using determinate or class names as if objects — things or cases — fell under them, whereas it is irrelevant to the expressed meaning, the meaning of determinations, whether any fall under them, and it is clear from the outset that, in the precise sense of the word, none fall under them. One speaks of objects, but what one has 'in one's hand', namely in one's thinking, are always only meanings: one carries out meaningful thoughts and expresses them in sentences, assumptions and statements based on them. It is completely irrelevant to the meanings that they are thought and spoken, irrelevant whether they are fulfilled, irrelevant and out of place any question of being and mode of being: a fact that, as self-evident as it is, some people's thinking will first have to get used to.

19

FULFILMENT

The assumption that for every (finite) set there is another that contains one more item may or may not be fulfilled in reality, but its mere meaning guarantees that for every (natural) number there is one that is greater by one. For every determination, there 'is' its determinant in the same sense that the determination 'exists'. One has a sufficient explanation of this meaning for every case in which it is actually used if one stipulates — as already suggested 51* — that we say there is a determinative meaning of a given expression if the expression is a determinative expression (if it 'determines something'). This means that the phrase 'there is a determinative meaning of this expression' should be understood as actually characterising the expression — and the thought expressed — in a certain way; it should not introduce a peculiar 'being' of a meaning.

Now, there are meanings that can be fulfilled, and there are meanings that cannot be fulfilled.

bare determinations. This distinction is purely semantic, a matter of pure meaning, and completely independent of fulfilment or non-fulfilment in any reality. A determination is semantically fulfilable, in short: 'fulfilable', if it is consistent, i.e. does not imply its own negation; otherwise it is semantically unfulfillable, in short 'unfulfillable'. A tautological determination such as x= x, of which one might say that it is more than just fulfillable, can only be called 'always fulfilled' in an improper sense: there are no objects that do not satisfy it; but whether objects exist at all is irrelevant to it and its validity. 'x = x' as an 'always fulfilled', i.e. empty, definition means: if x is an individual — of the area under consideration — then x is identical to x. Such a definition is unconditionally fulfilled, i.e. 'actually fulfilled', if there is at least *one* individual, one individual object, one individual case, an individual 'entity', and 'improperly fulfilled' if there is nothing of the sort. Thus, the negation of an unconditionally valid determination is also semantically unfulfillable, not because there are no fulfillments for it, but because, according to its meaning, there cannot be any.

By now stating: "There is no x that satisfies x -1- x, in short: no x(zJ z)", we set the determinant z-1-z and consider its determinant:
- not as a content (meaning) of the form of an object, thereby proving that, in the same sense as the determination, it also 'exists' as its determinate: in the realm of the 'really possible', there are corresponding meaningful propositions and expressions. In this sense, the 'forms' of 'impossible objects' exist as determinants of unfulfillable determinations, but not impossible objects as their external fulfilments.5°' It is usually said that there are concepts of impossible objects — the statement that a round square cannot exist would not be possible if the concept of a 'round square' did not exist. When we say that there is the concept of 'an integer greater than 1 and less than 2', but no such number, whereas there is an integer between 1 and 3, we obviously mean an 'existence' that is more than the mere presence of a concept, i.e. a determinate. The difference lies in the fact that **'integer** between 1 and 2' is the determinant of an unfulfillable condition, while 'integer between 1 and 3' is the determinant of a (meaningful) fulfillable condition, namely the condition *that* z *is a set containing one thing and one more thing and no more things.* In the realm of the meaningfully possible, there are sets of two; that is the meaning of the statement. There is an integer two. 'There is *x(fx)'* can therefore mean at least two things: either *'x(fx)* is the determinant of a determination fx', or *'x(fx)* is the determinant of a (meaningful) fulfilable determination *fx'.*°° The second 'there exists' is, to all appearances, an expression of what is called mathematical existence, while the first only means the existence of a meaningful concept — with fulfilable or non-fulfilable content. In mathematical considerations, this further meaning of 'there exists' does not seem to play a role at first. The question may arise as to whether an existing expression that has the linguistic form of a mathematical problem actually has the meaning of such a problem — that is the question of 'existence', if not of determinants, then of certain determinations.

— but it will have to be regarded as 'metamathematical'.

However, the given explanation of the term 'mathematical existence' — which, of course, corresponds to the widely accepted definition based on the 'freedom from contradiction' of the determination of the 'mathematically existent' — does not do justice to the case of zero. The number zero exists; there are systems in which it appears as an element. There is the determinant of the definition '*x* is an empty set (class)', because empty classes 'exist', but in the sense that there are *unfulfilled* definitions — in the formal realm, they are unfulfillable. The 'existence' of zero is only the 'existence of a conceptual content' and does not — like that of one or two — go back to the existence of the determinant of a fulfilable determination. It is, one might say, a 'purely conceptual existence' — which, of course, in no way means reintroducing 'existence in thought'.5*

20

DETERMINATION AND CLASS 5°*

The old universal questions are not resolved by conceptual definitions such as those of determinate and class, but rather stimulated anew. The oldest attempt at a solution was probably there before the questions were explicitly asked, just as ways of understanding exist before opinions, and opinions before they become questionable. An original conception is closer to "entities" than to strictly isolated and sharply defined things and cases; a distinction between such objects and the supra-individual may exist, but it is uncertain and fluctuating. The common name, which appears as soon as language exists, is more an expression of such a fluctuating conception of reality, long before it can express a logical class concept. The statement "The raven is black" is probably more original than "All **ravens** are black"; it is also more correct and better expresses the opinion of today's biologists; This becomes very clear in the example "Humans have 32 teeth". But such a sentence cannot be taken as an expression of a general implication ("If *x* is a human, then ...") — since it appears as a statement of experience — and certainly not as a mere dissection of a concept of humans or ravens. Words such as "the raven" and "the human being", which appear as subjects in declarative sentences, do not denote determinates or classes in the logical sense.°®* However, they are expressions for universals as realities, primordial precursors of concepts that only develop purely in formal terms, in concepts such as "the Euclidean plane triangle" or "(the number) five". Just as "the raven" is considered a reality, even if it does not have to be thought of as a supra-individual individual, so too "the triangle," "the **sphere,"** and "the five" are later conceived as beings that somehow exist in the realm of the "ideal," or they are to be found "in the objects of the world of things" or "on" exist or "exist" for them. The determinate – "the triangle", "the five" – is essentially a single entity: "the ideal representative" of all things or cases of a kind. Nominalistic thinking, which is essentially thing-thinking, since it only allows individual things (and cases) to exist, turns it into a "word", a "sign", and since "the sign Y" is again a universal, a type, it ultimately refers to the individual cases of its occurrence and the habit of its use. It has already been explained that this approach is an inadequate attempt to eliminate meaning; "usage" is also a manner of occurrence and behaviour and is not given in any individual case, nor in a totality of cases, which, incidentally, is never complete and can never be presupposed. But time and again, attempts have been made to replace the determinate of a determination with the "elements" it "represents"; hence the tendency of modern logic to give precedence to the class over the determinate and the determination itself, the tendency towards an "extensional" conception. It is recognised and emphasised that a class is not the same as its elements. However, the idea whose content is the concept of the triangle class should serve to refer to "all triangles", and the expression whose meaning is the class ("all triangles") should serve to designate all elements of the class. The determinate idea and the determinate name — "(the) triangle" — serve to refer to and designate any (individual) element of the kind ("this triangle"). The determinate, as a "singular form", is conceptually closer to the original, singular "essence" of the "species"; the class, as a generally "plural form", is closer to the "totality of elements that satisfy a determination" presupposed by belief in things. But talking about these "forms" does not refer to objects or entities of their own kind; it is only a meaningful expression, justified — and even made possible — by the fact that there is a meaningful reality, especially in thinking and speaking, in punctuation; their meanings are neither "ideal entities" nor "forms" whose existence and mode of existence it makes sense to question, and yet one does not do justice to this fact of the meaningfulness of realities if one only wants to recognise individual things and cases, which can only be done by establishing or presupposing the fulfilment of certain meanings ("forms"). It has been shown that they are not prerequisites for the meaningfulness of experiences and expressions.[5®]

THEORY OF TYPES

In order to reliably avoid circular errors, Principia Mathematica introduces the theory of types. It requires the following explanation: If
If px is a predicate for x, then “px” — and therefore also #(px) — should be a meaningless expression; if « is a class, then **“«t«”—“z** is an element of «” — hence also “«t«” and “«eä” — be a meaningless expression; if *p* is a statement, then *“p* is applicable to *p”,* or *“p* applies *to p”* — hence also “@ is applicable to *p”*, or “@ applies *to p”* — should be a meaningless expression. The case in which a definition ‹p or the class defined by it becomes the "argument" of ‹p, or in which a statement says something about itself, cannot therefore be considered meaningful at all; not because such a case does not exist, but because even talking "about such a case" is meaningless. This can hardly mean that such speech cannot be given any meaning under any circumstances without changing the usual usage of language; but it can be regarded as a definition of a usage of language or signs, namely the use of the signs '‹p', 'xt' and *'p*'. The Principia Mathematica excludes any use of these symbols that might give rise to the appearance of certain self-references.

The provisions for provisions, classes of classes, and statements about statements that do exist are interpreted accordingly. A determination for determinations is always of the next higher "level" than those that are its argument values; a class of classes is of the next higher level than the classes that can be its elements or elements of the negate class; a statement about statements is of the next higher level than those. Accordingly, one cannot form a determination "for every determination", a class "of all classes", or a statement about "every statement"; but one can form a determination for every determination of the nth level, given n, which is itself of the (ii + 1)th level, and correspondingly a class of classes of the nth level, a statement about "every statement of the nth level", given a value of a. Since such a construction always produces an expression of the next higher level for the meanings under consideration, it or its meaning always eludes self-application: it evades it, so to speak, by moving upwards.

Natural thinking will sense something correct in the theory of levels, insofar as it goes beyond a mere definition of the use of symbols, but will have obvious reservations about its unrestricted acceptance, apart from reservations of a technical nature that arise in view of the complications it entails in logic and mathematics (and complications give rise to justified suspicions of factual inaccuracies). Thus, the prevailing opinion seems to be that the theory of levels cannot be maintained in all its rigour, but that an improved substitute for it is indispensable.

An approach to the necessary relaxation of the theory of levels is already contained within it: in the introduction of the concept of "systematic ambiguity". Expressions such as "determination", "class", "statement", "negation", "true" are ambiguous in this sense; for sentences about "determinations", "classes", "statements" ... are not considered meaningful at all, because there is no such thing as "all determinations" etc. Such sentences are therefore not definite statements, but incomplete forms or formulas that only become statements when each expression of systematic ambiguity is converted into a definite one by adding an unambiguous level determination. For example, the formula *p* vp, or more clearly, *(p).p v p*, only becomes a definite statement when one specifies that *'p'* should stand for any statement of a certain level, for example the first, and the other symbols in the formula should correspondingly stand for disjunction or negation of the same level. Now the formula is valid for statements of every level. So if *p* stands for any statement of the nth level, then
(n) *(p).p* vp, be a valid formula; let it be denoted by '#'. Then *P* is a valid statement, but not of a specific level: it falls outside the level order, and with it others that we consider to be propositions of logic.5° One will have to recognise propositions that do not belong to any level as meaningful and valid.

A proposition such as *P* is not merely an empty form for valid statements, but is itself apparently a valid statement, even though it is a proposition "about "any statements". A sentence "about any statements" or "about every statement" (of course, according to the explanation of the term "statement", "about every statement of a certain meaning") is not a sentence about "all statements", let alone a sentence "about all sentences"; it does not presuppose a totality (or class) of all statements (or all sentences). It is applicable to itself without leading to the application of a meaningless expression such as "This sentence (what I have just said) is either true or false" and without necessitating a conclusion of the content "Every statement is either true or false (sentence *P)*, therefore this sentence *(P)* itself is either true or false." "This sentence" here refers to sentence *P*, which has just been stated and has a completely specific content, and not to a sentence that has just been uttered and says nothing other than "What I am saying is true or false". What matters here is not the difference between a previously uttered and thus temporally "finished" proposition on the one hand and an "unfinished" proposition that is only in the process of being spoken or thought on the other, but only the difference between a proposition with an independently given meaning – such as *P* – and a proposition that has no given meaning and thus no meaning at all. If one pays attention to this difference, the circular error can be avoided without the unfulfillable requirement of a generally applicable distinction between stages.

Assuming, of course, that every expression that means something designates at least in the case of non-contradiction — in order to eliminate the unacceptable case of impossible objects — a specific object, or that every non-contradictory thought captures or hits an object, then one easily arrives at the following representation.

An expression expresses a meaning *that* someone *thinks* when they use the expression in accordance with its meaning — at least when they use it meaningfully —; this meaning is the *content* of the thought that thinks it and *the meaning* of the sign that expresses it. If there is something that can be regarded as *fulfilling* this meaning (by satisfying the conditions set by it), then this something is *the object* of the thought (of thinking), and it is *designated* by the sign that expresses and signifies the meaning. Of a thought (as an act of thinking), one can say that it *grasps* both the meaning that is its content and its object; to be precise, one must then say that distinguish between two types of comprehension. This can be done by speaking of *"thorough"* comprehension of the content and *"accurate" comprehension* of the object; when thinking conceives a meaning, it "aims through it at an object", i.e. "in a direction in which an object must lie" if thinking is to find its "intention" fulfilled, and if that is the case, then thinking has "accurately grasped" an object. If the meaning of a thought, since it can always be a thought of a higher level in the position of the object (to be hit), is taken as an object (of a higher level) in the absolute sense, one arrives at an upwardly open sequence of objects: the ideal object, which is the content (I) of a thought, can be accurately grasped by a thought whose content (II) is of the next higher level. For this to happen, however, it seems necessary that content I be present, so to speak, in the "realm of the ideal" before content II, which is to grasp it; and it is easy to interpret this, in the sense of a shift into the real, as meaning that the thinking – and, in linguistic terms, the expression – of I must be completed before the thinking (or expression) of II can occur and accurately capture (or designate) I. In fact, I can only affect content I through II currently and immediately if I have thought I explicitly (explicite). Thus, I apply the sentence *p* v@ to itself retrospectively, after I have posited it, but without applying it to it. The retrospective application, however, takes place in the statement *p* v also applies to *p* v itself: a validity without which the application would be inadmissible, and apparently one that is independent of the explicit reference of the sentence to itself, which is expressed by substituting *p* v@ for *p* in *p* vp. What matters is this "potential" relationship of the sentence to its own content, to its applicability, not to its application. However, it is contained in the simple positing of *p* vp, in terms of meaning: the proposition "meets itself" by applying to itself, and this relationship of meaning is implied in the positing of *p* v@ in that the act of positing this meaning accomplishes the attitude of (explicit) positing of every implication, every application – in this it has "its full meaning". This attitudinal co-setting of the application of the proposition to its own content is what actually takes place when one "potentially encounters" it and therefore thinks one should attribute to it the character of an (ideal) object.

The test of the self-applicability of a piece of content is, of course, its application. It succeeds in the case of those pieces of content which, because they do not fit into the hierarchical order, are replaced by mere signs of "systematic ambiguity" in the Principia Mathematica, i.e. as long as the hierarchical determination is missing, signs without meaning. However, this content is of the utmost logical importance, for it consists precisely of the concepts and propositions of logic (such as "statement", "implication", "conjunction", "disjunction", "negation", "determination", "class", the "**law** of contradiction", the "law of excluded middle", etc.). In the end, instead of a general prohibition against violating the hierarchical order, the applicability of one meaning to another meaning or of one expression claiming meaning to another will be decided only by the attempt at application: if there are deficiencies in meaning If it is, then it is prohibited; otherwise, there is no reason to prohibit it.

THE GENERAL AND THE SPECIFIC

To declare a logical formal proposition such as *p* v or (n) *(p).p* v to be a mere empty form from which a statement emerges only when a specific value is substituted for *n* seems to me no more justified than the corresponding procedure with regard to a formula such as "For every natural number *n* there is a next one". This leads to the "discovery" of a number, and that is a *specific* number, *n+ I,* only when a specific value is set for ii; but what it states is precisely that such a result occurs with every permissible substitution. Similarly, our logical formula says that for every specific assumption of the value of ii, a (step-bound) true logical proposition results: but that is precisely the meaning of the formula and is a genuine meaning of the proposition, and indeed a valid one.

For Principia Mathematica, a kind of atomism seems to apply, which associated with the extensional conception and gave it a tendency to make the individual, as it were, the building block of the general. Thus, here the class presupposes its elements and non-elements as given, the determination or sentence function, the true and false sentences that represent its applications. In order to judge that "Socrates is a human being," one does not have to have set the propositional function (determination) "x is a human being"; it stands to reason to say: certainly not in this form, but one must still have the concept of a human being in order to make such a statement. It is also conceded that, in order to grasp a class, at least an infinite one, it is not necessary to enumerate its members, but only to grasp them intensionally; it is not clear what the claimed semantic priority of the members is supposed to consist of in this case. Instead of primacy here or there, it will have to be established that, on the one hand, the defining determination (or the class) and, on the other hand, every "object" whose membership in the class is to be decided must be "given";

That is to say, at least in the formal realm, a general definition that is not meaningless *and* a content that defines the "individual object" or "case" can only make it possible to decide whether the object belongs to a class. It depends on whether the individual content (e.g. that defining the number z) implies the general definition (e.g. that of the algebraic number) or implies its negation; one of the two must be the case if the question is to be decided, and in this respect there is a dependence of the individual content and its determinant, the individual "object", on the generality of the class; in this sense, it presupposes it as given. Of course, the class definition does not have to be explicitly established before the definition of the "individual object", but no meaningful priority of the individual can be asserted.

At the same time, it becomes clear what a "single object" is in the formal realm. Five is a single object in the class of natural numbers and is clearly a single object only in relation to this class or a class subordinate or superordinate to it. What an "individual object" is in relation to a formal domain is determined by a conjunction of definitions that are permitted within the domain, namely a connection that either implies or excludes each of the permitted definitions by implying their negation. In this limited sense, five or the number a or the triangle with side lengths 1, 1, 1 is an "individual object" that is "in every respect" or "unambiguously" determined within its domain.

In another sense, however, one could say that every meaning presupposes the individual case, insofar as it is true that everyone "aims" at fulfilment, and fulfilment is the individual case. But this is not a presupposition of the kind that an expression which means an unfulfilled meaning must therefore be meaningless or lacking in meaning, and in formal terms, fulfilment remains completely out of consideration·

"*The cube z lF'urfef*— Wü(Wü)" – "Cube" is a cube – does not work – because "cube", the subject word, does have meaning, but one that requires *predicate position*. The sentence "(The) cube is a regular solid" stands for "xsCube . m . xt regular solid". How would one translate "'(The) cube' is a cube" (or: "is a regular solid")? Something like this: "(xs cube) s cube" or: "s regular solid" — which obviously makes no sense — one sees no possibility of deciding whether it is true or false. — If "the cube" is not hypostasised — which would be inadmissible — then a sentence like the second one cannot be formulated at all; it is not even *linguistically* possible. —

Wü(x) is a determination whose *"fulfilments"* are bodies (of reality). "Wü(x)" is an expression (of a striving "towards a certain body form") of a certain *striving for body formation. !**

'That is a W'iirfel' – expression of the *fulfilment* of such an aspiration, namely in *linguistic punctuation*, which at the same time (in addition to the intention) expresses an act *of 'setting'* such fulfilment (mental act) –

Fulfilment. Your judgement

The reality that is reliably judged is that *which is experienced* (and *lived) in the striving for individual cases* of concrete form, in approaching fulfilment — whether the concrete fulfilment is precise enough to be accepted as "fact" is determined by the extent and accuracy *of the associated spatio-temporal separation.* Where it *breaks off,* the *deception* or *error* is revealed — it consisted in the fact that a (partial) striving was experienced in a one-sided form which, on the whole, *does not prevail.* A completely logical "fantasy" that fits into all contexts would be a completely correct grasp of the world of things; taken seriously, it would provide perfect knowledge.

P '(p). p v p', setting this, I denote the attitude towards setting. *“*Any *p;”* and explanation of *t.p; v p;.* In this setting, I am also set to the proposition of *P* and explanation of *t. P v P; which would be correct: P* is *applicable* to #. The proposition “f*” *takes* place *without* “reference” to *P*, *does not require* an impossible self-relation, self-presupposition. I say on 14 February 1942 at 11:29 a.m., so that I am finished with the sentence before 11:30 a.m.: *S.* 'Every sentence I have uttered up to 11:30 a.m. today is true'. This is a decidable sentence: if all my previous statements are true, then *S* is also true, otherwise it is not. No harmful self-presupposition – *definite statement content. 'This sentence is true',* however, remains undecidable because it has no statement content. The preceding sentences and their truth values give *S* a specific content, so that *S* is also applicable to *S* as an individual case and proves to be either true or false. If, instead of *S*, I say *S'*, which can be derived from *S* only by substituting 'false' for 'true', then S' is undecidable.

If all my previous statements are false, then *S'* must also be false in order to be true: *S'* is *contradictory,* but not meaningless. If at least one of the previous statements is true, then *S'* is definitely false. Therefore, a statement such *as S'* is a *false statement* under all circumstances. *Not meaningless.* - The same applies to 'Everything I say is false', whereas 'Everything I say is true' and 'I always tell the truth' are true, as with *S.*®**- This does *not* include a meaningless sentence of the form 'This sentence is true' or 'This sentence is false', where 'This sentence' = 'This sentence is true (false)', i.e. it would be *meaningless*; for 'this sentence' now has *a specific content and can be decided.* It does not have to be true of itself (without relation to Yorgegebenes), so to speak, in order to be true, but its truthfulness results from its relation to the sentences *given to* it (even if they are later in time). 'I always tell the truth' is true if I tell (told or will tell) the truth in every *other* case of utterance, i.e. in cases *other than* the present one: then *S* will *'automatically'* turn out to be true, otherwise it will be false. —

MATHEMATICS: “AN SCHA ULI CHE BEGRÜNDUNG" **THE** NUMBER THEORY (HILBERT)

The importance of intuition and thinking for mathematics can be characterised by the statement that mathematical knowledge cannot be achieved without intuition, but that intuition does not enter into any mathematical theorem. This is a negative, imprecise and only provisional characterisation. The attempt to make it more precise and satisfactory will initially be limited to the field of arithmetic. We know how, in primitive societies, a series of notches or lines represented a number, how addition and subtraction were performed on such line figures, and how children of civilised peoples still learn to count and calculate today. *D. Hilbert* builds on such methods and shows how they can be developed to form the basis of elementary arithmetic or number theory "from a finite point of view".°4* Everything is very clear. One works with figures such as '1', '11', '1111', called 'digits' by Hilbert. If one sets '2' for the digit '11', '3' for '111', etc., then only abbreviated symbols are introduced; what we mean by them are always only the figures or 'digits'. It is easy to see, for example, that '1111 l' is a longer series than 11', and that the first is a larger digit than the second; it is also clear that '11' plus '111' gives the digit '11111', and we call this the sum of the two, writing '11 + 111 = 11111' or, abbreviated, '2 + 3 = 5', etc.

Even if "the considerations are made in the form of *thought experiments* on objects that are assumed to *be concretely present*" ®5*, it can still be seen that they differ significantly from experiences, inductive generalisations of individual observations; but it is not the manner in which the insights are gained that matters here, but their content. And this is such that, for example, as soon as one has understood in a case of "digits" or arbitrarily "given" or assumed to be given quantities of objects that two things and three things make up five things, the possibility of a counter-case does not come into consideration at all. The nature of the "things" (or cases) is irrelevant; if a child is still at the stage of "object-bound" understanding of quantity relationships, it does not pursue arithmetic from a finite point of view, but rather no arithmetic at all. If, in Hilbert's sense, we agree on certain stick figures as "digits" and do not consider any accumulation of distinguishable shapes on paper or anywhere in space to be digits, this only has the meaning of a restriction of the means to a certain type of representation. Here we are already disregarding differences between the figures within a fairly wide range, but not those differences and similarities that matter, namely those relating to the number of lines in a figure. We take a figure with the same number of lines to be "the same digit": we do not work with the individual given figures, but with types of figures. A figure has its representational meaning by virtue of the type – the "digit" – to which it belongs; and what it means *is* the type of quantities of things of any kind that can be "paired" with it, i.e. quantities of the same number. The belonging of a given quantity to the type of quantities that a certain digit represents by virtue of its figure type is determined by some unique assignment of the things in the quantity to the lines of the digit, which we usually do by counting those things and those lines, i.e. by using a system of linguistic signs of a certain type, thus applying the name to the set that signifies that type (and therefore designates that set as belonging to it).

It turns out that (1) the finite method of elementary number theory uses intuitive means of representation—like any method that relies on communication—but (2) in such a way that the propositions obtained, insofar as they are arithmetic, contain only arithmetic determinations (of sets) and no intuitive ones. One is dealing with figures in one's perception, but in such a way that what ultimately "is meant" is not determined by the figures, but by certain types, determinants of set definitions. Such a procedure, not very clearly characterised by the expression "thought experiment", differs from genuine experimentation and observation essentially in its result, which is not a statement about reality, but one about effect...

An independent if-then relationship, a pure determination implication. If x is a set consisting of one thing and one more thing and no other things, and *y* is a set consisting of one thing and one more thing and one more thing and no other things, and no thing belonging to the sets *x or y* belongs to the other set, then there exists a set *z* for x and *y* – the "unification set" or "sum set" of x and *y* — that contains every thing from x and every thing from y and no other things, and r is a set consisting of one and another and another and another and another and no further things. Every set that can be paired with *x* (including x itself) is called a set of two, every set that can be paired with z is called a set of three, and every set that can be paired with *z* is called a set of five. The statement that a set of two and a set of three that is foreign to it give a set of five as a sum set is expressed in arithmetic language, which works with "numbers", namely "natural numbers", as specifically defined types of sets — determinants of set definitions — is expressed in the form "The sum of two and three is five", "2 + 3 = 5".

Such a sentence only develops implications of the provisions that are considered

Constitutive elements of the types that are understood are set, and perception is only insofar as those types are "meant" , mediator of an "evidence," the insight into meanings which, as arithmetic, are independent in their validity of everything perceptible — even if they may be "interwoven" with it: this validity is of a purely logical nature. "The examination of the initial principles of number theory and algebra served to demonstrate to us the direct content, which takes place in thought experiments on vividly imagined objects and is free of axiomatic assumptions, in its application and handling. For the sake of brevity, we will refer to this type of reasoning as *'finite'* reasoning and, similarly, to the methodological approach underlying this reasoning as the 'finite' approach or the 'finite' standpoint. In the same sense, we will speak of finite concept formation and assertions, using the word finite to express that the consideration, assertion or definition in question adheres to the limits of the fundamental conceivability of objects and the fundamental executability of processes, and thus remains within the Concrete consideration takes place within a framework."^'

Even if one is initially dealing with "visually imagined objects", it cannot be assumed that the essence of arithmetic thinking adheres to and is bound by such "objects"; and even if axiomatic assumptions are not explicitly introduced, one nevertheless proceeds in accordance with such presuppositions, which would be expressed as axiomatic assumptions. In such "finite" arithmetic, the vivid structures only become the occasion for an "abstract" consideration, in which, however, it is not the "abstraction" that is essential, but what remains, what enters into the conception of the "digit". This means an "axiomatisation of the intuitive given", assumptions of the existence of sets or things in and the number and again in and the number ... Number theory conducted in this way is not axiomatically structured in that the existence of "numbers" as objects that satisfy the and the (such as Peano's) definitions, but uses axiomatically set definitions of *sets;* these definitions themselves, which it sets, *are* the numbers, and their "existence" does not have to be assumed first, *because* they are handled as *the content of the posits* of our theory.

The premise "Let there be one thing and another thing" is
experienced as an "intention" that leads to a logical, otherwise undefined or "negligible" fulfilment in a clear way, and cannot be separated from this connection with perception; nevertheless, it is purely logical in content, or there is no purely logical content. Even in the non-finite assumption that there is a sequence of three digits Z in the decimal fraction development of a ("somewhere"), it is experienced as an "intention towards vivid fulfilment", but as strangely leading into a void, without that "evident" certainty that it can be fulfilled, without the knowledge of the "producibility" of what is meant, because a way of "production", and whether there is one, is not known.

In the case of natural numbers, the "manufacturability" that ensures fulfilment is already given in the thinking of the definition. "Let there be one thing and another thing"—this thought is carried out in a proposition that itself includes a set of two individual propositions. If one refrains from any closer determination of the nature of the "things" whose one and another are assumed to be given, then in these propositions...

tongues already "set" a set of two, as it is meant, in fact: a set of two files. Although the files certainly do not initially refer to themselves, it is clear that they are applicable to themselves in terms of content, even if this may happen in a second thought. It can always be established that in the assumption "Let there be one thing and another thing", in its execution, the meaning has already been fulfilled. (The concept of number is independent of levels). Incidentally, real files are always "things" of the lowest level.®

Here, it is worth mentioning a comment that is often raised as an objection to attempting to explain the concept of numbers using purely logical means: that in its formulation — if, for example, the number two is defined as a class (or type) of sets *m*, each of which contains an element x and an element y that is not identical to it, and no element that is identical to either x or *y* — the concept of number is already being used, and is therefore necessary for the definition. In our case, one must make exactly two assumptions, *xtm*, *yarn*, in addition to the appropriate assumptions of identities and non-identities; anyone who could not "count to two", even without using the word for number, would not be able to grasp the concept of two. This is only natural; by making the assumptions that constitute the content of the concept of number, one grasps the concept. One needs certain intellectual abilities to do this, including memory, but none of these belong to the content of the concept; none of them *are* part of the meaning that one "assumes" or "grasps". Nor is the act of grasping a particular number an "original positing" of indissoluble peculiarity as far as its meaning is concerned; the positing are assumptions of the belonging of elements to a set or class, of the identity and non-identity of elements: all meanings of a purely logical nature. It is certainly a very remarkable and important achievement that takes place in the grasping of numbers, in its measured "distinctions" and "equations," but it is a *logical* achievement in terms of its meaning and nothing more. The logical definition of the "natural number," as first given by G. Frege, however, achieves what all previous efforts to clearly define the concept have failed to achieve: it states with complete clarity the property of a set by virtue of which it is a set of a certain number of elements, thereby also expressing that sets, and only sets, are numbers as elements.

number, as a "property". It has already been sufficiently explained how this last statement is to be understood: that neither a quantity nor anything else can be regarded as an object and carrier of determinations, which, bearing this in mind, it will not seem strange that the number, even as a "property of a quantity", cannot be expressed quite freely. The fact is that a determination, by "defining a class" that has a number, also establishes the identities and non-identities of its members as its implications. To understand such a determination as a property of something corresponds more to the habits of our objectifying thinking and speaking than to its pure meaning. The fact that one only has the concept of a number by applying it — in the original form of counting (still independent of numerals) — is something it has in common with all logical concepts: assertion, negation, determination, conjunction, disjunction—they all have or grasp the meaning that is their content by virtue of a peculiar "active character." But this is nothing other than the fact that they are pure acts of meaning, independent, in terms of their meaning, of the concrete fulfilments in which they are experienced, and, of course, of being experienced.

Even a "basic" number-theoretical theorem such as "three is greater than two" is, despite the specificity of given numbers, a universal theorem, even if it appears to be couched in a judgement about "digits" that do not enter into it as things, but only in terms of a typical determinacy. A theorem such as "For every x and every *y:* if *x* is a set of three, j' is a set of two, then x contains more things than *y"* presupposes no less and no more than an "infinite set of sets of three" etc., as does the (not "elementary") "For every x and every *y:* if x is a (natural) number and *y* is a (natural) number, then either x > y or *x - y* or *x < y"* presupposes "the infinite set of (natural) numbers". The meaning of the expression "infinite sets" will be defined later. In any case, the fact that every digit of the type three is larger or "longer" than every digit of the type two follows only from the fact that a digit of the first type contains one thing and one more thing and nothing else, and each of the second type contains one thing and one more thing and nothing else, with "mathematical certainty" and from nothing else that may lie in the perception of the digits. Only at the level of its typical "objects" ("three", "two") does an "elementary" number-theoretical theorem appear as a special theorem.

The "general" theorems of this level, which are universal statements — theorems about every number and every digit — and existential statements — of the form "There is (at least) one number it with the property *H(n)"* or "There is a digit *n* with the property R(n)" — pose certain difficulties for the finite conception.

“A *general* judgement about figures can only be interpreted in a hypothetical sense

sense, i.e. as a statement about any number presented" 71 How can one make a statement "about any number presented", i.e. **about any** number that

might or could be presented? 71 Evidently, and with good reason, only from a general concept of the "digit" that is not bound to the occurrence of any single vivid circumstance. Whether it may have been gained from such a circumstance or on the occasion of it is irrelevant to its meaning; it does not enter into it. This meaning is that of a general implication between the defining (albeit unarticulated) determination of "digit" (in which sensory vividness does not play a decisive role) and what is then stated as true of every digit. It is no different with "every set of three" in the "elementary" judgement. A judgement of the form 'Every digit has the property fl(ii)' therefore means 'If n is any arbitrarily chosen specific digit, then R(n) always applies'. Such a judgement can only be justified if the determination fl(x) is included in that determination concept which is explicitly or merely intuitively (from which it can be read, as it were) assumed to be constitutive of "digit (in general)"

; then it is also clear that a digit n for which R(n) — non-fl(n) would apply *cannot* be 'presented'.

"A *statement* about numbers, i.e. a statement of the form 'There is a number n with the property fl(n)', can be understood as a 'partial judgement', i.e. as an incomplete communication of a more precisely defined statement, which consists either in the direct specification of a digit with the property fl(ii) or in the specification of a procedure for obtaining such a digit, — whereby the specification of a procedure requires that a certain limit be set for the series of actions to be performed."[3] Instead of this explanation, three more precise versions are presented for consideration.

(1) The existence proposition, as a 'partial judgement' or 'incomplete communication', is not a judgement in itself, but a reference to that 'more precisely defined statement' which is a true statement of the form 9J(n) — a reference which does not itself have the character of a statement.'[4] An objection to this statement is that the reference can be understood even without knowledge of the statement to which it refers, and that understanding takes place as a recognition in a judgement. The content of this judgement is: 'The speaker knows a true statement of the form R(n)'.

(2) The 'partial judgement' interpreted in this way is equivalent to: 'There is a true statement of the form 91(ii), and the speaker knows it'.

In the usual understanding, one would interpret further:

(3) 'There is a certain number n for which R(n) is a true statement

, and the speaker knows this true statement (i.e. that specific number n)'.

If (3) is accepted, then the reference, in order to be understood, i.e. in accordance with its meaning, presupposes the meaning of 'There is an *n* with property 91(n)' as given and cannot be considered a definition of this expression. If (2) is accepted without the addition made by (3), an existence clause is still required for the understanding of the reference, i.e. for its meaning, except that 'there exists' refers to a specific statement rather than to a digit or number, which in turn does not contain 'there exists'. 'There is a true statement of the form W(n)' is the partial statement of the existence clause in version (2), which alone has a purely objective or mathematical meaning; only it can, at most, belong to the set of statements of a number theory. A theorem of this form, 'There is an n with the property *R(n)'*, which is to be valid, must of course be meant in a certain way; this follows from the systemic context of the theory in which it occurs. In an axiomatic theory, this means: "Assuming the axioms of the theory — which are consistent — there exists an *n* with the property R(n)", i.e. "the assumption of its non-existence contradicts the axioms". If there is no explicit axiomatic system, what is nevertheless assumed to be decisive is what is "implicitly" contained in an undefined concept of "number" and in the other undefined basic concepts that are assumed; with regard to these assumptions, always "within the theory", "there is" a number of one kind and "there is not" a number of another kind. What 'there is' ultimately means remains undefined even in this context. It does not mean 'such a number is constructed' or 'is constructed in such and such a way', 'is known to me'. Each of these statements *confirms* 'there is', but none of them gives it meaning. The same is true in the empirical realm: 'There is a tower called the Eiffel Tower' is confirmed by certain perceptions, but this does not mean that such a tower is or was perceived, nor that it *can* be perceived, and a phrase such as 'there are possible perceptions of such and such a kind' obviously adds nothing. Possible here and there: definition by negation of a universal proposition that presupposes 'there are values of x'. Judgement of an object i m p l i e s 'there is', perception has *judgemental value,* otherwise it could They do not prove the existence of judgements. Analogue: "primordial intuition".

The other part of the statement, 'The speaker knows this particular true statement', is merely a communication of a random piece of knowledge and does not belong to the set of propositions of the theory. Nevertheless, this second part of the statement is not without significance for the theory: assuming its truth, it ensures the *justification* of the first part of the statement within the theory. It is now essential to separate the content of a statement from its justification. The justification, the reasoning behind a statement, is not part of its content; attempting to grasp it in this way leads to a circular argument. The meaning of a sentence must be established if one undertakes to show that asserting the sentence, in a given context, has the good meaning of justified assertion — which is a meaning of action — for itself.

If one makes this distinction, one will only allow the first part of the statement — 'There is a true statement of the form *A(n)'* — of the sentence in (2) as an interpretation of the existence statement 'There is a digit *n* with the property W(ii)', and regard the second part of the statement as a justification that at least satisfies those who know the specific true statement. If such justification is required by direct or indirect demonstration of a true statement fl(n) and, within it, a specific *n* for which it is true, in order to justify the existence statement — the first part of the statement — and any other proof is rejected, then one still finds oneself on a finite standpoint; it could be distinguished from a *"content-finite"* as "justification-finite". Everyone must be free to choose the manner of justification they require for the propositions they wish to consider as belonging to their theory. Ultimately, what matters is what the theory achieves.

A content-finite theory according to interpretation (1) cannot include existential propositions, as non-statements, in its content. However, if one accepts the "reference" as a communication, which can hardly be rejected, it is indeed a statement, but not a proposition of number theory: content-finite number theory does not contain any existential propositions. A justification-finite conception will only consider the objective partial statement of (2) as a theorem of number theory. The situation becomes even clearer if one attempts to determine the meaning of *the negations* of general theorems of a finite number theory.

In conception (1), which does not recognise any number-theoretical existence theorem, the question of the meaning of the negation of such a

The negation of (2) yields: 'There is no true statement of the form *9J(n)*, or the speaker does not know it'. It is clear that this disjunction provides no objective information: even if it is true, it does not belong to the set of propositions of number theory. In the finite conception of (2), only the first part of the statement or its negation is a theorem of number theory. The negation has the meaning of *the universal statement:* 'Every statement of the form R(n) is false'. This expression could be taken as a definition for the meaning of 'For every digit *n*, non-H(n) applies'. The *justification* for such statements lies in the knowledge that the determination *A(x)* contained in R(n) is incompatible with the definition of "digit *n"*, i.e.
that the definition of "digit n" implies the definition *X(n)*. The statement that

it is "impossible" for a statement of the form *A(n)* to be true, the assertion that a digit *n cannot have* the property fl(n), expresses not only the purely objective meaning of the sentence *"*There is no true statement of the form *X(n)"* also expresses an understanding of that implication — or at least its obviousness — and the resulting claim to justification for the assertion. The same applies to the assertion of the "necessity" of the validity of a statement. The (objective) content of the proposition must always be distinguished from the justification for asserting it. The current justification or reasoning in the given case of assertion, based on insight, corresponds to the more "objective" justifiability based on the "insight" of the implication. The requirement of insight into the determinative implication, in order to justify the assertion of the universal proposition, can be called finite if it demands that the insight be gained "from intuition", from the imaginative visualisation and embodiment of the type "digit" — a clearly justification-finite "point of view". Now, **"**neither is the alternative logically apparent that a general judgement about digits must either be true or lead to a contradiction in its consequences, i.e. be refutable, nor is it self-evident that such a judgement, if it is refutable, **is** also refutable by a counterexample**"**. These are two statements of a technical nature. That the second is so is obvious without further explanation. The first can be understood by attaching to it the meaning that it is not logically obvious that a false universal statement about numbers in a foreseeable series of shots must result in a contradiction – the very definition of "number", with which one works in the illustrative finite process, must not be approached with such conceptual clarity that the chain of reasoning can be linked to it. If one now concludes that, from the finite point of view, "it is not possible to find a negation of finite content that satisfies the principle of excluded middle for the existential as well as for the universal judgement" °6, then one regards the requirement of modes of justification — for the negation of an universal proposition — whose availability is not taken for granted here, as part of the content of that negation. But the inclusion of the justification or a required mode of justification in the content of the statement to be justified will be impossible in the case of the universal proposition, as it is in the case of the existential proposition. It seems more likely that the finite standpoint cannot be maintained than that the principle of excluded middle must be abandoned. The application of this proposition to a general judgement about digits shows that the defining concept of "digit" either implies a given definition of number-theoretical content or does not imply it, and that there is no third possibility. If the implication does not apply, then the assumption that it does apply is in contradiction with it and, as can easily be seen, in contradiction with the definition of "digit". The principle of excluded middle does not claim that this contradiction must be provable in a series of foreseeable steps of inference and, in particular, by demonstrating a counterexample (against the universal statement). It is not affected by the possibility of the absence of such proof procedures. According to a theorem by Gödel, in every "formalised language" that contains arithmetic, there are undecidable propositions, i.e. propositions that can be formed using the means of expression of the "language" according to its rules, but cannot be proven or disproven according to its "transformation rules". However, by enriching the "language" accordingly, it is possible to achieve the decidability of such a theorem. Ordinary language — language in the true sense of the word, which is essentially incapable of expanding its stock of means of expression beyond any given limit — does not allow for undecidability, at least in the formal domain. Classical logic presupposes such a language — which also contains the "syntax language" (which establishes its rules) for every "language" formulated in it — and thus the undecidability of such a proposition.

The separability of every sentence that can be designated as a statement. It transcends the boundaries of any specific formalised language, just as it transcends the boundaries of specific finite levels (in the sense of type theory) in its sentences. It can do both without falling into contradiction.

WIDTH OF FINITISM

Finitists claim that a mathematical theorem only acquires meaning through proof; without proof, it remains meaningless. In order to take a clear position on these claims, one must first define their meaning. To do so, it is first necessary to know what is meant by a mathematical theorem. A theorem such as that the sum of the angles in a flat triangle is two right angles is a linguistic expression to which, however, one c a n n o t attribute the meaning of a specific statement. However, if it appears in the context of a representation that is to be understood as part of Euclidean geometry, then it is true in a certain sense that it only acquires its meaning through proof; it states 'In a Euclidean plane triangle, the sum of the angles is two right angles'. However, "the Euclidean plane triangle", "its interior angles", "their sum" and "the right angle" are names for objects whose existence cannot be asserted and for which the validity of the theorem, insofar as it is correct, i.e. understood in the meaning relevant to mathematical theory, is not required.[7] If one says that these objects are given in intuition, it suffices to point out the inaccuracy of intuition, which cannot be eliminated by any amount of careful measurement; because of this property, intuition cannot "contemplate" the content of our general theorem on the sum of angles by means of example and "ideation". Unless, that is, "ideation" effected the transition from vague intuition to a sharp conceptual determination, which, although suggested by it and recommended as the "simplest" assumption before others, is neither prescribed to judgement by the facts at hand, nor imposed on it by some peculiarity of our "cognitive faculty". If our sentence is to become a true statement, it must be given the meaning of a general implication: under the conditions of Euclidean

In Euclidean geometry, the sum of the interior angles of a plane triangle is equal to two right angles. These assumptions are definitions that exist either in an undeveloped form, in undefined basic concepts, or explicitly, in a system of axioms. The fact that only the determination of the implication between the basic assumptions and the content of the "theorem" ('The sum of the angles of a flat triangle is two right angles') rather than this 'theorem' itself is a statement — and a valid one at that — is only vaguely expressed by the assertion that the 'theorem' only acquires its meaning through proof, i.e. through derivation from the premises. It already has meaning before the derivation, the same meaning it retains after it, namely that of a determination (theorem function, statement form) that cannot be asserted on its own because it is neither true nor false. What is to be asserted in a particular statement is only the relationship that the premises of Euclidean geometry imply this determination. If the conjunctive connection of those premises is H and the content of the "proposition" is B, then neither R nor B, but only R—+B is a statement, and indeed a true one. This is independent of whether someone can accept the present implication relationship as plausibly proven, or whether they only recognise the derivation as accomplished after demonstrating some intermediate links between the concept W of the basic assumptions and the "theorem" B. Therefore, the statement "In a Euclidean plane triangle, the sum of the angles is two right angles," which merely expresses R—+B in a different way, is a genuine statement and true even without the derivation described above.

What has been shown here using this example applies in general: a linguistic expression — including sign language — in the form of a sentence has the meaning of a statement and is either true or false, regardless of whether proof or refutation has been provided, or it does not have this meaning, in which case it cannot acquire it by any means without changing linguistic conventions.

Every (purely) mathematical statement has the meaning of a "general implication", i.e. an implication between (pure) determinations. For the sake of preliminary clarification, it should be noted that "pure" determinations, i.e. determinations in the true sense of the word, are "sharp": in every case to which a determination with meaning is applicable, either it or its negation applies, but not both at the same time. It can be seen that

At least the first of the "empirical" or "descriptive determinations", such as the illustrative ones, does not apply; they are vague. - To prove an implication 9J—+i8 between determinations fl, i8, it is generally necessary to demonstrate intermediate links that make the implication relationship between the basic assumptions fl and the final link B plausible. The degree of detail required depends on subjective conditions; an experienced mathematician may be satisfied with a hint of the "proof process", whereas a beginner may demand much more; nevertheless, even the beginner can understand what the statement 'R—+B' means, even if he does not yet have the proof. It is not appropriate to attribute meaning to the implications W—+ß , Bl —• 2› ., ,—•B, but to deny the implication R—+i8, which results from the transitivity of the implication relationship. Similar differences to those between beginners and accomplished mathematicians exist in some respects between earlier and later states of science; much of what was once a great achievement to prove has now become "self-evident", and such self-evident truths shorten a line of reasoning. None of these circumstances can determine whether a mathematical theorem is a statement or not. An objective limit to the detail of a derivation exists in the case of a precisely defined calculus, a "formalised language" in which the theory operates. Here, a limited number of very specific transformations are introduced as forms of immediate derivation, and a proof is considered to have been provided if a chain of such immediate derivations can be established between the basic assumption and the final link.

One does not usually go through them in detail; it is sufficient to refer to them briefly: the essential thing is that such a chain *exists* — a non-finite statement —; that this can also be understood, and how it is made understandable, is again dependent on subjective conditions and therefore does not belong to the mathematical facts: not to the meaning of the mathematical statement. It is clear that any statement that includes recognition, its existence, nature or conditions in its content presupposes the meaning of this recognition, and that is the purely mathematical meaning here. It is completely independent of the cognitive achievement that is present in the proof provided, i.e. it contains no relation to it and cannot contain it. This is decisive in contrast to the doctrine that the meaning of a statement consists "in the

Method of verification. Certainly, in order to know what a statement asserts, one must know "what would have to be the case" if, and what one does not usually mention, only if it is to be true. But that *is* precisely what the statement *says*, and every sentence that has the same meaning. He does not need to specify how this is to be proven; it merely states what must be proven if its validity is to be assured. Behind the stated explanation of meaning lies the opinion, at least once held by *Wittgenstein*, that only "factual" (non-analytical) sentences are statements °*, and that a statement can be "verified" by demonstrating the "fact" it asserts; which corresponds to the view that "the world" consists of a set of elementary or "atomic" facts, each of which must be thought of as being uniquely designated by a proposition that states it, as if by a signature (the analogy is mine, but it seems to me to characterise that view). Assuming the correctness of such a view, it would still have to be maintained that the statement only says *what* "fact" would have to be demonstrated in order to "verify" it, if one so wishes, not how this should or can be done. The former is semantically independent of the latter; it is semantically presupposed by it. '*p* is to be proven in such and such a way' cannot be stated without circularity as the content of *p*'. A sentence that is a statement has the **meaning** it has, regardless of proof and being proven. However, anyone who has proven it or understood a proof for it will, at the same time, have a richer content of consequences present or readily available with the "next" meaning — an implication R-+ i8 — Meaningful connections will "resonate" that, without knowledge of the proof, are not so conscious or close to consciousness, but, and this is essential, are equally included in the ("next") meaning of the sentence — only for this reason is it possible to bring them to mind in detail in a proof, to explain the implied — always to a certain extent.

ON THE APPROACH **OF THE "**PRINCIPIA MATHEMATICA"

Propositions that can be derived from *Hilbert's* "finite standpoint" are characterised and justified as arithmetic propositions insofar as they are conclusions from conceptual definitions that one posits in contemplation of the figures or on the occasion of this contemplation; the visual does not enter into the content of the propositions and therefore does not belong to the content of the definitions: they are purely logical in nature. It was G. *Frege's* great idea to construct arithmetic by purely logical means; independently of him, *Russell* and *Whitehead's Principia Mathematica* undertakes a logical foundation of mathematics in general. These undertakings have also been of the utmost importance for the development of logic, which had to and still has to create the means to master the new tasks, and it can be said that in the endeavour to logically constitute mathematical concepts, logic first constitutes itself in an exact manner.

Here we shall concern ourselves only with the constitution of the concept of number in more detail — 'number' stands for 'natural number including zero'. Jrege's structure contains what *Russell* calls a violation of the requirement of type (level) distinction and is thus not secure against logical paradoxes. In their endeavour to do justice to the order of levels, *Russell* and *Whitehead* went one step further and included the "axiom of infinity" in their logic, which states that there are infinitely many things of the lowest level, thus making logic and mathematics dependent on a property of the real "world". This linking of formal theory to reality begins with the statement that a class exists if and only if at least one thing in the class exists. If the number one is the class of single-member classes, then for this number to exist, there must be at least one class with exactly one element, i.e., according to the same

Explanation: there must be at least one real thing. For only if a class of the first level exists that has one element, does the class with the single element z also exist as a single-member class of the second level, and the class with the single element z as a single-member class of the third level, and so on. If there is to be a number that is one greater than any given number, i.e. an infinite sequence of numbers, then there must be a first-level class for every class that contains one more thing, i.e. an infinite number of things in reality.

This structure has been criticised on several occasions. Any impartial thinker will initially be struck by an objection that one cannot
e.g. can take the following form. If the Fates Klotho, Lachesis and Atropos exist, then at least three Fates exist. This sentence contains a numerical statement that is correct regardless of the existence of the "counted objects". One might say that the number three exists, regardless of whether there are "actually" three things. Yes, the sentence 'There are not three things' seems to have meaning, whether true or false, only if the concept 'three things' exists, and that means, surely, if the number three exists. The following consideration is of the same nature: if reality contains only a finite number of things or — in order to render the indeterminacy of the concept of a "real thing" harmless — if there is no infinite set of somehow determined objects (things or cases) in reality, then there is a number N such that a set of N + l things does not exist. Nevertheless, 'N -F l' is a meaningful expression; one will say that the number N+ l exists, even if there is no set of N+ l real objects. The "feasibility" of the operation 'N + l' is linked in the theory of *Principia Mathematica* to the applicability of the term "N + I"; the meaning of the expression 'N + l', the decision as to whether it is meaningful or not, is linked to the decision as to whether it can be applied to reality. **

Principia Mathematica does not seem to draw this conclusion, at least in the first introduction. There, the view is taken that in a world of only N things, the number N+ l coincides with zero; the reason is that in such a world, the class of classes of N+ l things would be empty, a null set. However, an exception is made for the empty set, which is allowed to be a "set" or "class" even though, by definition, i.e. according to its concept, it is not a

Thing contains: here "there is" the concept, e.g. of *an x*, for which x = x does not apply, regardless of the fact that it has no application; therefore, the class of empty classes is not empty, and "there is" the number zero. Incidentally, even in an empty "world", or if there is no "real thing", the concept "single-member class", i.e. "the class of single-member classes", is well distinguished from the *concept* "empty class", the "class of empty classes"; the former does not apply at the first level (there is no single-member class of real things here), the latter applies to every class at the first level (because the class of things in this world is empty). If the largest set of things in the world is of finite number *N*, then the following generally applies: "There is no class of *N* + l things in this world" and "There is a class of zero things in this world"; the "class of *(N*+ l-membered classes) of things in this world" is not identical to the "class of zero-membered classes of things in this world"; even in an extensional conception, the term *"N*+ 1" does not coincide with the term "zero".

It would be tempting to try to free the *Principia Mathematica* system from these difficulties by abandoning the 'existence' of zero. In the context of this system, one could say that 'The number of things of the type *'perpetuum mobile'* is zero' means 'For every x, if *x* is a real thing, then x is not of the type *'perpetuum mobile''*. This is a statement about things. The formal structure of the system should be achievable without the "zero", albeit with greater complexity, just as, according to the "Introduction" (to the 1st edition of the work), it would be achievable without the prerequisite of the existence of classes. But no logical or mathematical system can be achieved without the concept of determination. Only this concept, and not the concept of 'class' – which would be something other than determinacy – is invoked in the assertion that the concept of a number *N*, for example three, is independent of the existence of *N* things.

CONSTITUTION OF THE CONCEPT OF "NUMBER"

The following constitution of the term 'number' differs from the so-called logistical one in *Principia Mathematica* essentially only in that it does not use an existence axiom and thus avoids the connection to the existence of fulfilments (lowest level) of the introduced or considered determinations. Wherever the following definitions contain a statement of existence, the fulfilment of a definition, this is not done in the sense of a statement, but in the sense of a free assumption that belongs to the meaning of the expression to be defined; only the concept of fulfilment is used, nowhere is a fulfilment asserted.

A constituent meaning (conceptual content) that is constituted is thereby demonstrated. Its "existence" does not need to be axiomatically established or postulated. Constitution occurs through definition. A defining content is independent of fulfilment; it makes no difference to its meaning, i.e. to itself, whether there are fulfilments of it; the meaning is predetermined by the question of whether this is the case; it is only made possible by it. If there are fulfilments of a first-level defining content, then it is applicable in the true sense. Questions of meaning must be fundamentally distinguished from questions of application. Only questions of meaning are questions of logic, of mathematics. These include the question of the (meaningful) fulfilment of a determination or a system of determinations, i.e. the absence of contradictions. A procedure for proving this cannot be specified in general terms; it does not have to consist in demonstrating fulfilment.

If etx and • *x* are provisions and are equivalent to each other - *(x) (a x= azx)* -, then they have the same meaning, denoted by '«' (One says, "equivalent definitions define the same class", "have the same scope". These expressions often lead to misinterpretations, e.g. as if the fulfilments were the scope, while the

Salary identity is purely a matter of definitions, i.e. of meaning; these phrases are not used here on the definitions page). This makes no significant difference to *Principia Mathematica*, because for them the term 'class' is an 'incomplete symbol', and every sentence that contains it can be replaced by a sentence that does not contain it, and such a transformation can be achieved by means of the concept of determination. ‘‘’, ‘§’, ‘y’ are symbols for the determination contents of determinations ex, §x, yx; when we speak of ‘determinations’ in the following, we mean only the determination contents; the particular forms in which they occur are irrelevant. The definitions in which the concept of "number" is constituted are given here in verbal language and are not entirely exact; a more rigorous logical version can be found in the notes. °•

11 Definition of "equivalence", the expressions 'is equivalent to' with #’, ‘« §’. ‘« is equal to #’ means ‘There is a one-to-one relationship *Rxy* between the values of x that satisfy «x and the values of *y* that satisfy §y’; the relationship Rxy is one-to-one if and only if it is “one-to-one forward” and “one-to-one backward”; fixy means "uniquely forward" if every value of x that satisfies ex is assigned a value of *y* that satisfies *[1y* by *Rx y* and no other; *Rxy* means "uniquely backward" if every value of *y* that satisfies *§y* is assigned a value of y that satisfies *§y* by *Rxy* and no other.®°* Explanation 11 applies both to the case where "x defines a finite class (finite set)" and to the case where "x defines an infinite class". Since "number" is used here as synonymous with the expression "non-negative integer", the second case can be ruled out. This is done in the following definition of "equivalence", which makes use of the fact that "there is always a proper subclass of an infinite class that is equivalent to it". (The terms 'class' and 'subclass' do not appear in 12.)

= means the same as the expressions ‘z and § are equal in number’, ‘the number of « is identical to the number of §’.

12 =$ means the same as the expression ‘« is equal in number to $ (in the sense of 11); and if any determination y is an implicans of e and e is not an implicans of y, then « is not equal in number to y’.

On the constitution of individual determinations of the type "number" - the "specific numbers" — in II, the term "zero-dimensionality" is first defined, as are the expressions "e is zero-dimensional", "e0" or "the number of e is zero", "/,=0".

II ‘es0’, as well as ‘/,=0’, means the same as the expression ‘There is no fulfilment of e' and the same as '(Ex) ex'. Any further definition of the type "number" — any "specific number other than zero" — can be constituted in a chain of definitions as soon as the term "number greater than one (for a given number)" is defined. This is accomplished in III.

‘/ means the same as the expression, “the number that is one greater is ”. What this means is specified in III.

III ‘/,=/ means the same as ‘There is an implicant y of e that is equal in number to (/,=), and there is one and only one fulfilment of a that is not identical to it that is not a fulfilment of y’. (The fact that the number, the number of satisfactions, of z is one greater than that of § is expressed by means of the class concept using the phrase that Q is equal in number to a subclass of « that excludes one and no other thing from «). The term "number" is explained by specifying what the expression 'fle is a unique number determination' or 'R«t$' means. "/" means 'number', 'natural number (including zero)'. Each of these expressions is explained by

IYa (Jz3)'DF(3,=0) v(E 2) (,= }).

‘$, is a number’ therefore means the same as is (either) identical to zero, or there is such that the number greater than by one is greater than . Taking into account the preceding definitions, it is clear that the explanation is not circular, despite the wording. The scope of the term 'number' is defined in such a way that every single term, the term of a specific, 'given' number, an 'individual of the type 'number'' can be constituted by a chain of definitions. The appearance of the use of the term 'one' in the definition of the 'number' that is one greater than it disappears in the symbolic representation. The names of 'numbers' appear first and, so to speak, in their natural position as part expressions of sentences such as 'There are eight — "exactly eight" — large planets (of our sun)', 'The large planets are eight', equivalent to the unusual but clearer expression 'The set of large planets is a set of eight (things)', even clearer '(The definition) of being a large planet has eight fulfilments, is eightfold', equivalent to: 'The number of e is 8'. Such a sentence, when applied, gives a clear determination of number. This corresponds to the fact that, instead of the term "number", the term "unambiguous number determination" is explained, because in an "unambiguous number determination" a "number" term has its "natural position" as a dependent component. Let it be a "predicate predicate", i.e. a determination for determinations (e); '/b' means as much as the expression 'unique number determination'.

IVb '9I' is a unique number determination' means '9I' is (either) equivalent to = 0 or equivalent to the determination: ' is one greater than ';

$$(\mathfrak{A}\alpha\varepsilon\mathfrak{Z}\mathrm{b}) =_{\mathrm{Df}} [\mathfrak{A}\alpha \equiv (\mathfrak{Z}_\alpha = 0)] \vee (\mathrm{E}\mathfrak{Z}_\beta)\, [\mathfrak{A}\alpha \equiv (\mathfrak{Z}_\alpha = \mathfrak{Z}'_\beta)].$$

It should be noted that '$ = 0' and '/ =/3' have been explained without using the term 'number' and are only used here as shorter designations.

"EXISTENCE" OF NUMBERS

Although number determination is for determinations, it also appears as an "object" and "carrier of determinations". This means, first of all, that there are sentences in which numerical names appear in the subject position or, in the case of statements of relationships, in subject positions, and there are settings of determinations with "object variables" whose "values" are to be thought of as numbers. The expressions '/ =/', '/ = in the definitions just presented are examples of this, as are the statement '3 + 2 = 5' and the statement forms *'x* + y ——5', *'x* + *y* ——*z'*. In addition, there are the posits of the "existence" of numbers with the property that play such an important role as assertions or assumptions in mathematics, at least in its linguistic usage. In all these cases, there are 'objectifications'. They are irrelevant to formal theory, provided that rules are specified according to which propositions of this theory in which objectifications occur can be translated into propositions in which they do not occur.

An attempt to define "the idea" that arises when "number is conceived as an object" — be it "a number" or "the number" — leads to definitions of a non-extensional nature, i.e. it concerns certain forms of definitions and not only the meanings that are important for formal theory alone. Nevertheless, it seems to have more than just psychological significance, because after all, forms of meaning are being considered, not just modes of experience.

Here, only the question of the "existence of numbers" needs to be addressed. It leads to the general question of the necessary and sufficient conditions under which a "class", i.e. ultimately a determination, exists. This is not purely a matter of language usage, at least not one that has yet to be established; one will have to pay attention to the conditions under which mathematical practice considers a determination to be

“Existent”. Now, this practice is by no means uniform in this respect; it is therefore advisable to start with the strictest interpretation and then see how it can be expanded in a useful way.

Sentences that affirm or deny the specified determinations of "existence" also occur in non-mathematical language. For example, one says "There is tensile strength greater than that of steel" or "There is no such thing as perfect rigidity," clearly meaning that there are instances that fulfil one determination but not the other. No one would think of denying the meaning of the term 'perfect rigidity (x is perfectly rigid)'; it is, after all, a necessary prerequisite for the question of the 'existence' of rigidity and, moreover, has been well proven in the 'mechanics of rigid systems'. It turns out that more is required for the "existence of a determination" than the mere "presence of the meaning of the determination". The examples considered were determinations of the lowest level. Here, the usage corresponds to FiuSseff's explanation that a class exists if at least one element of the class exists. Incidentally, it is clear from our example that 'there is' is by no means unambiguous. In comparison to the much-cited 'round square', for example, the 'perfectly rigid body' does 'exist'; the difference, of course, is that in one case the defining determination can be fulfilled in terms of meaning, while in the other it cannot.

"There is", in its strongest sense, refers first and foremost to reality, and from there the expression is applied to a determination that has fulfilments in reality; "there is", in a weaker sense, refers to a determination insofar as it is fulfilable, and its determinant ("the rigid body"); "There is," in the weakest sense, every determination, as the meaning of a meaningful determinative expression, regardless of fulfilment (such as the determination 'x is round and *x* is not round'). The task here is to determine the meaning of "there is" that is decisive for the application of the expression to numbers. But numbers are determinations for determinations. If, for example, one explains that the number 1 exists if there is at least one determination (lowest level) with a fulfilment, one accepts that connection between mathematics and “random” conditions of reality which proves to be foreign to formal science and not very acceptable (chap. 26). The Ent-

The decision as to whether a specified number "exists" within mathematical theory must be made within mathematical theory without regard to the facts of the "world" that are external to it. This is indeed the case; in no instance is the existence of a number made dependent on the existence of any things in reality. [4] It is a question of meaning. We say 'There is a number between zero and *two'*, 'There is no number between *zero* and *one'*. One remains as close as possible to the meaning that the expression 'existence' had in determinations of the lowest level in the formal realm when one translates the example sentences using the phrases 'The determination (second level), e is a determination of which there is not no fulfilment and of which there is not at least one fulfilment that is not identical to this one, is fulfilable', or 'The determination, e be a determination of which there is not no fulfilment and of which there is not at least one fulfilment, is unfulfillable'.

These examples would correspond to the following statement: A higher-level (at least second-level) definition "exists" if it can be fulfilled (in terms of meaning) (D1). Applied to a number, this results in: The number exists if a definition with the same meaning can be fulfilled with $. This does not require that a determination n with the property — and therefore with as many fulfilments as prescribed — exist, but rather that the assumption of its existence be consistent. It should be noted here how the view that starts from the assumption of a multiplicity of things and cases as a given material for logical comprehension and theory formation already leads to difficulties in view of the concept of number. /“The number of e”, it seems, for such a view, is “abstracted” from a given class e, with given members, by means of the concept of “classes with the same number”; and the constitution of the concept of number just presented begins with the explanation of the “equality” of determinations «, Q. However, it is essential to note that at no point is the existence of a number to be specified /" such as zero or one bound to the existence of a given determination (or "class") e of never-lowest order. In the relationship /,=/3 or « Q, e and Q are not, so to speak, the first given; they are not given at all — 'e', '§' are variables, not proper names — but the relationship is set as one through determinations that would appear as "values" of e, Q, to fulfilling and, in terms of meaning, also fulfilable – regardless of whether it is actually fulfilled. The assumption is consistent that for every specific /" that can be constituted on the basis of definitions I-IV, there is a determination "of some level to be specified, and in particular of the lowest level. Nothing more is required for the "existence" of the numbers, i.e. nothing more is stated by the phrase that every number to be constituted in this way "exists" ( ).

The question has been complicated by whether a determination can always be found for any given number, i.e. a determination that defines a "class" of things, such as the determination "x is a finger on my right hand" for the number 5.

B. Russell resolves the question by stating that for any given number n, in the sense of an infinity axiom, there are some things a,. a and can be grouped together into a "class" defined by the definition (x=o,) v (x= o,) v ... v (x= o,),

each of them and thus no thing suffices. Such formation of a determination z, given /" "in terms of scope" is in fact always possible, i.e. the setting of a determination "x, which is equivalent to

(x= o,) v ... v (x= o,), for n =/ , assuming that et. •2. are any non-identical things (lowest level), is in itself

consistent; it can be fulfilled in terms of meaning, even if the "axiom of infinity" does not apply, and independently of all reality. The constitution of the concept of a specific, "given" number by means of the concepts of "zero" and "the number greater than one" (II, **III)** works with such assumptions: it assumes a zero number " to be present and, to it and to each subsequent one, one that has one more fulfilment. None of these "values of «" are actually demonstrated, nor does any of them need to be demonstrable; the concept of each is formed by the mere assumption of a "scope" property: there is no *x(ax)* or there is an x(«x) and no other that is not identical to it, and so on.

It all depends on whether these assumptions can be fulfilled. Proof of this would be provided if it could be shown that the assumption that they lead to a contradiction in their consequences is itself contradictory. 5* A kind of positive existence of fulfilment arises from the fact that the individual specific number can be "constructed". The assumption "There is an x(«x)" can always be countered by the assumption "There is an further x(«x) can be added in the subjunctive; this conjunction again assumes 'There is another x(«x)'. The process leads sequentially to the concepts '(There is) at least one thing of type z', 'at least two things of type «', 'at least three things of type «'. The consistency of this procedure is evident in a peculiar way. If 'at least *n* things' are assumed, there are in fact n positions of 'non-identical things'; one can say: apart from the necessary positions of non-identities, there are n thing-positions, which we may call constitutive in the positive sense for the concept of at least ii things. These positions are accomplished in the positing of the concept; in retrospect, one might see the real acts as proof of the consistency of the concept; it is consistent because in the positing of 'at least n things', t constitutive acts of positing are accomplished. This proof of fulfilment based on fulfilment is not circular; but it is not mathematical, and one could accuse it of drawing on empirical facts when only matters of meaning are at stake. However, one does not first have to refer to the reality of the acts of positing, nor does one have to pay attention to them at all, but only to the fact that the positing of "n non-identical things" has no other meaning than the conjunctive linking of ii positing of one thing each, none of which is equivalent to another. This is purely a matter of meaning: if one wants to develop the meaning of assumption (1) "There are n things x(«x)", one must use the conjunction (2) of two assumptions with different meanings, "There are", in which lies the proof, namely the demonstration, of a meaningful "executability" of that assumption, which is not to be understood psychologically. Of course, the demonstration of (2) does not prove that there are n things of the lowest level. However, since absolute level numbers are not of decisive importance in the consideration, they show that the assumption that there are n elements of level m is only possible (i.e. meaningful) if there are at least n elements of level m + 1; and since such an assumption exists as a meaningful concept, there are certainly n elements of every level other than the lowest. Whether there are it elements of the lowest level for any assumed ii is a question of existence, not of meaning, and is not a matter for formal science. The consistency of the assumption of the existence of *n* elements of any level is ensured by the reasoning presented above, together with the fact that this assumption always has the same form, regardless of the level; and only this consistency is necessary for the "existence" of the number n at any level. It is not the fulfilment of the assumption by elements of a certain type that matters, but only its fulfilment. No property of the real "world" can change this level-independent fulfilment of the assumption "There is a determination with n fulfilments", or more precisely, "There is an n-ary determination for every specifiable n". Here, "every specifiable n" means nothing more than "every uniquely determined number". The fact that a level-independent concept of number is used here does not, I hope, constitute a violation of the justified ®* prohibition of level confusion. The concept of number can be constituted without taking into account things of a certain level, and in particular "real" things. The expression 'number of elements of any level' makes **sense;** it is not just an empty form that would only acquire meaning through the insertion of the designation of a specific level. Nor is the concept of number strictly bound to elements of the same level; 'A determination and an object that satisfies it are *two* elements of different levels' is a meaningful sentence. The genetic paradox has shown that one must be careful with such constructions; however, it is certain that the conclusions and demands that were once attached to it go too far. The view presented here, which will be developed further shortly, does not wish to be accused of introducing "ideal objects" — as numbers or as counted items. Even if it claims meanings as counted or to be counted, it essentially does no more than any logical system that uses, for example, "there exists" statements with "predicate variables". The precautions required by the consideration of the order of stages must be observed in the further development of arithmetic, especially in the extensions of the concept of number. Perhaps this indicates the meaningful basis for the cognitive advantage of the "primordial intuition of number" claimed by the intuitionists, and the advantage of the "constructibility" of number concepts on the basis of that "intuition". First, it is shown in what specific sense the individual number is "constructible," in what sense the formation of the concept here is already an "erection," namely a demonstration of "fulfillability." This most original construction is found in a

For the nearest extension, we must take those "abbreviated constructions" that allow a clear numerical specification using a specific "number system", such as the decimal system, without going through the original construction by executing the individual settings; it is sufficient that we know that they are feasible.

ON "EXISTENTIAL" LOGIC

A "finite standpoint" is only possible without contradiction in the sense of foundational finitism. It essentially consists in rejecting unlimited propositions of being, whether as non-judgements or, in mathematics, as inadmissible judgements. This has the particular consequence that the refutation of a universal proposition about numbers, a proposition of the form "For every n, *R(n)* holds*"*, does not give us the right to say

“There is (at least) one n with the property *R(n).”*

In particular, the rejection is directed against an "existential" logic of general propositions, which assumes an infinite set of values of an unrestricted variable as given, a universal statement *"*For every *x, R(x)* holds*"* as a conjunction and an existential statement "There exists an x with the property fl(x)" as a disjunction of infinitely many individual statements: as a proposition of the form "R(o) a fl(b) ...", or of the form "R(o) v R(b) v ...", where o, b, ... are the values of x. ° First, we should think of "the set of (natural) numbers" as the variable range.

This conception of general statements has been repeatedly criticised and rejected. Perhaps those who adhere to it do not pay enough attention — which seems essential to me — to the fact that a universal statement and likewise a statement of existence cannot be replaced by an equivalent conjunction or disjunction of individual statements, even in the finite variable range, insofar as one can work with such a range.

Assuming that the variable range comprises exactly n items o„ ..., o„ and that 9J(•)• • R(o,). applies, if the universal proposition ‘(x) R (x)’ is to be replaceable by this conjunction, then the conjunction must imply the universal proposition

and vice versa. However, if we write

'S(°)• ---•*(•• •(*(•)'.

then the formula lacks a prerequisite that has been made and must not be missing, namely that o„ ..., o, are *all* values that x can assume. If this addition is made, the valid relation is obtained

$$(1) \qquad \mathfrak{A}(\mathfrak{a}_1) \wedge \ldots \wedge \mathfrak{A}(\mathfrak{a}_n) \wedge (x)\,(x=\mathfrak{a}_1 \vee \ldots \vee x=\mathfrak{a}_n). \rightarrow .\,(x)\,\mathfrak{A}\,(x)$$

(The dots are only intended to emphasise the implication sign as the main sign, which separates more strongly than the other connectives). The conjunction of the individual sentences 'R(li;)' has been supplemented by a *universal sentence* in order to obtain an implicans of the universal sentence '(x) G *(x)'*. The implication in the opposite direction, '(x) fl (x) . —+. R(o,) ... a fl(o)', is considered valid in the logic of *'Principia Mathematica'* and *Hilbert's* according to the principle '(x) R (x)—+H(o)', in which 'tt', as an indeterminate constant, stands for any specified 'value of x'. Because this is not expressed in the formulation of the principle, but rather a constant appears on the right-hand side that does not appear at all on the left-hand side, the sentence seems like a magic formula rather than a "tautological" implication, which it is supposed to be. To remedy this deficiency, the indeterminate constant must be properly introduced on the left-hand side of the formula by stating that fi is a value of x, i.e., that d refers to a thing (an object, an individual, an element) that occurs in the range of the variable x. A partial expression must therefore be introduced that stands for the designation of any such thing (no matter which). I write !, which can be read as 'a occurs as a value of x' and described as 'expression of *the* (assumed) *judgement* or designation of the object a in the value range of x'. If one now writes

$$\text{I} \qquad (x)\,\mathfrak{A}(x) \wedge !^{x}_{\mathfrak{a}}. \rightarrow .\,\mathfrak{A}(\mathfrak{a}),^{90}$$

this is the expression of a valid, "tautological" implication, i.e. an analytical proposition. If !a . li expresses the judgement of things a„ ..., a in the value range of x, then we have

(2) (a)W(x)a!Ai„j,A,.-+.W(A,)a.:afl(A,).

The universal proposition together with the demonstration of the things • , o, implies the conjunction of the individual statements relating to them. The implication, like that in (1), is not reversible.

Formula (1) can be supplemented to form a more meaningful implication, which is still not reversible:

1') *(°i)• °¥(°.) (x) (x=ay v ... v *x——a)m(x) Z (x)* !
$\mathfrak{a}_1, \overset{x}{\ldots}, \mathfrak{a}_n.$

Here, use is made of the relationship

II ‘(o)-+! .

The predication 9J(o) includes the designation !o. The designation !' can be understood as the borderline case in which predication degenerates when the predicate is the empty determination of the form W(x) v R(x). Only because of
this relation is

III G(a)—•(Ex) fl *(x).*

Because in ‘R(a)’ the designation of the thing o from the range of *x* is included, it is included in R(o) that there *is* a thing with the property fl(x). Thus, as a “reinforcement” of III, we have

IV *A(a)m(Ex) A(x) n ! p.*

This means "If d has the property 9J(x), then there is a thing in the range of x that has the property fl(x), and o is a thing in the range". It does not follow from the expression on the right-hand side that a has the property *R(x)*: implication IV is also not reversible.

Based on these findings, it is already clear that a propositional statement ‘(Ex) R(x)’ cannot be replaced by a disjunction of individual statements. First of all, it is clear that it does not imply such a disjunction: it does not apply

(Ex) *R(x).—+.'II(at)* v ... v fl(a)

but rather

(3) (Ex)‘Jf(x) a(x)(x=a/ v ... v x=a.) -•€( )v ... v II(o,) and in order to supplement the existential clause with an implicant of the disjunction, an *universal clause* is needed to ensure its completeness.

Instead of the completeness condition in the antecedent of (3), one can introduce a weaker one and then obtain a weaker consequent of the implication:

$$\text{(4)} \quad (\mathrm{E}x)\,\mathfrak{A}(x) \wedge !\mathfrak{a}_1, \overset{x}{\ldots}, \mathfrak{a}_n. \rightarrow . \mathfrak{A}(\mathfrak{a}_1) \vee \ldots \vee \mathfrak{A}(\mathfrak{a}_n) \vee (\mathrm{E}x)\,(x \neq \mathfrak{a}_1 \wedge \ldots \wedge x \neq \mathfrak{a}_n).$$

A disjunction of individual cases, without the completeness addition, irreversibly implies the existence theorem and the demonstration of the individual values that occur in the disjunction:

(5) \!t(0y) v ... v fi(a,).-+.(Ex) ‘St(x) !a„/, a .

The following is essential for assessing the interpretation of the universal proposition as a conjunction of individual cases: a conjunction of the form ‘fl(o,) a ... a iiI(o,)’ always contains something that the universal proposition ‘(x) G(x)’ does not contain, namely the demonstration of cases of the form ‘9J(q)’. and in it individual "objects" of the range of the variable *x*, the universal proposition again contains something that no conjunction of individual statements achieves, namely the statement that it is the *entire* range of values of the variable to which the determination extends.

The relationship between the conjunction of individual propositions and the universal proposition, and the relationship between the disjunction of individual propositions and the existential proposition, can finally be expressed in the following equivalences

(6) *(x)* '(x) a(x)(x=0y v ... v x=ü). -.‘¿f(ay) a ... a'(a) a(x) (x=a, v ... v *x* —-a)

(7) $(\mathrm{E}x)\,\mathfrak{A}(x) \wedge (x)\,(x=\mathfrak{a}_1 \vee \ldots \vee x=\mathfrak{a}_n)\,.\!\equiv\!.\,\mathfrak{A}(\mathfrak{a}_1) \vee \ldots \vee \mathfrak{A}(\mathfrak{a}_n) \wedge (x)\,(x=\mathfrak{a}_1 \vee \ldots \vee x=\mathfrak{a}_n).$

The *universal proposition* 'For every value of *x*, fl(x) applies' and likewise the *conjunction* (of individual propositions) '(It applies) R(a,) and ... and R(a)' is supplemented by the designation of the values of x, which also has the form of a *universal proposition*, 'Every *x* is either a, or ... or o,' to an expression that may be called an *'exhaustive universal proposition'* and also a *'conjunction* (of individual propositions) *marked as* exhaustive' (6). Similarly, the (indefinite) *existential statement* ‘There is a value of *x* to which the determination 9J(x) applies’ and likewise the disjunction of individual statements, 'For o, or ... or a, the determination R(x) applies', supplemented by the *same listing of all values* that x can assume, to an expression that can be called an *'exhaustive existential proposition'* and also a '*disjunction* (of individual statements) *marked as* complete' (7).

These relationships make it clear that the meaning of the universal proposition is determined by nothing other than the universal proposition itself, and the meaning of the propositional sentence by nothing other than can be expressed by the existential clause. In particular, it becomes apparent that the complete conjunction — as well as the disjunction — of individual statements, where it is to be performed, namely in the finite domain, requires supplementation by an *unrestricted universal clause* in order to be characterised as complete (or "certain"). This means, however, that we are in fact always working in an unrestricted domain, i.e., one to which a determination 9J(x) — and likewise a multi-place determination, 9J(x, *y,...)* — refers. Based on the above, one could say that even the finite domain cannot be "logically mastered" without unrestricted general propositions, which makes it clear how much *Brouwer's* opinion *that* logic is a language usage derived from and adapted to the consideration of finite sets misjudges the essence of logic.

The statement ‘Every prime number between 4 and 8 is of the form 2"+ 1’ is proven by the conjunction ‘5 is of this form and 7 is of this form, and every prime number between 4 and 8 is either identical to 5 or to 7’. However, this addition is usually expressed in the form 'For every x, (x is a prime number and $4 < x < 8$) implies ($x = 5$ or $x = 7$)', which is an unrestricted universal statement. If we say that the range of values of x is the class of natural numbers, or that it is the class of numbers in general, then we would have to add what we mean by the terms 'natural number' and 'number'. Then a statement of the form 'x is a (natural) number if and only if x satisfies the definition i8(x)' would occur, and so our universal statement in fact means: that 'for every x (in general)', the specified definition applies, i.e. for every object to which it can be meaningfully applied, so that a true or false statement results. If *R(x)* is a given definition, then the universal domain to which the statement (x) 9J(x) extends is the widest it can be: absolutely unlimited; for the fact that it encompasses all things to which *the determination A(x)* "can be meaningfully applied" is no restriction if, as assumed, *'A(x)'* is precisely the expression of a determination: it can only refer to things for which it is meaningful, a determination.

A provision "refers to objects that it fulfilled (would fulfil)", regardless of whether such objects exist in any discernible way, whether they are fulfilable or — like x-l• *x* — unfulfillable; it "demands" and "designs" such objects, as it were, which is precisely why it *can be fulfilled* or remain *unfulfilled,* and is independent of actual fulfilment in terms of its meaning.

If R(x) is a given definition, then 'There is an x (a value *of* x) such that fl(x) applies', ('Ex) 9J(x)', means that there is such a 'value' in the *unrestricted* range of the variable *x*. A statement such as 'There is a prime number between 4 and 8' can, however, be proven in a finite manner by 'demonstrating an object', but it has no 'finite' meaning and also no meaning as a finite or infinite disjunction of individual statements.°' This statement of meaning also takes the unrestricted variable range into account.

SCH R ANK E N "EXISTENTIAL" LOGIC

According to the concept of "existential" logic, a (infinite) totality of things is given for logical consideration, and an universal proposition is to be understood as a conjunction of individual judgements and, as it were, as the result of a complete "survey" of this stock of things. The fact that such a procedure would be not only "impractical" but fundamentally impossible to carry out on an infinite totality lies in the concept of the "procedure", which is supposed to be endless and yet have an overall result, and points to a fundamental difficulty in the concept of "infinite totality". The difficulty becomes all the more significant when one considers that this conjunctive conception of universal judgements already fails in the case of a finite multiplicity of individual cases, and that the recognition of an unlimited range of variables for every **universal proposition** proves to be necessary.®°

Thus, one must exclude the opinion that the universal domain consists of any given things; the universal class, like any class, is not identical with its elements and is independent of their existence.

Universal determination, range of meaning. We replace the dispensable concept of "class" with that of determination and find that if 91(x) is a determination of a given content, then the "universal determination"

9J(x) v $\overline{\text{fl(x)}}$ does not define a range of things – which may not even exist – but, despite the "emptiness" of this determination, it does define a range of meaning "within which the observation moves". This means that

91(x) v $\overline{\text{fl(x)}}$, and therefore also by G(x), which kinds of determinations, i8(x), *E(y), D(x, y)* and so on, can appear meaningfully connected in logical connections, conjunction, disjunction, implication, with *A(x)*. This "primary domain of meaning" is not the domain of meaning of logical theory.®° It is not predetermined by this theory, nor does it have to be; the theory merely assumes that there is one, through some specific, but equally valid, meaning of the "undetermined constants" R(x), or *B(x, y),...,* given or "chosen". The "primary" domain of meaning may itself be the domain of meaning of a theory, be it a real scientific or a purely formal one — an "absolute" level is not presupposed for it in logical theory. The domain of meaning of logic itself is "secondary" to it; the determinations it sets and translates into statements by means of the universal operator *(x)* or the existential operator (Es), for example, are of a higher level than those determinations that belong to the primary domain of meaning, such as the determination that the statement

print 'R(x) v $\overline{R(x)}$' with the (primary) meaning of 'fl(x)' left undefined, and the statement (91) (z) (G(x) v R(x)) in the same case, which is a statement of logical theory. The determinations and statements
of logical theory, referred to briefly as 'logical determinations and statements', refer to determinations and statements of any, unspecified primary domain of meaning, of which it is only assumed that it is maintained throughout the entire logical consideration, unless the contrary is expressly assumed.

The existential logic of Principia Mathematica makes no distinction between the fulfilment and fulfilability of a determination; the concept of fulfilability and its opposite, meaningful unfulfillability, which lies in the inconsistency of the determination, is absent from it. This has been noted on numerous occasions, and some have argued that this deficiency should be remedied by a "logic of modalities"; however, the proposals for its construction, like the assertion of its indispensability, have been subject to numerous objections. A brief discussion of the matter is therefore necessary.

A statement such as "All residential buildings standing in Adorf in 1940 are covered with tiles" or "Every featherless biped is a human being" is treated in the same way in existential logic as, for example, the statement "If *x* is a cube, then *x* is a regular polyhedron" or "Every multiple of 9 is a multiple of 3". Each of these sentences is represented in the form '(x) (M» i8x)'. However, there is a significant difference between the sentences of the first type and those of the second type: not only in the "modes of cognition", which are characterised by the terms "empirical" and "non-empirical" or "a priori" — this would not be significant for logical consideration — but also in their meanings, and in such a way that logic must not overlook it. The first example sentence refers explicitly to a specific spatial temporal realm, the second, less clear but no less certain in its existential conception, likewise: it refers, for example, to the featherless bipeds that existed on our earth at some point until 1943, or for as long as it has existed and will continue to exist. Each of these statements asserts something "about all real things". The first is that each of these things is either not a residential building in Adorf in 1940 or is covered with tiles. This becomes particularly clear when one considers the case that someone does not know whether there is such a thing as "residential buildings in Adorf" at all, i.e. at any time and in any place in the world; they would then be compelled to search the "whole world" to decide on the sentence, if, for example, misfortune prevented them from discovering Adorf or sufficient evidence of it. We probably find ourselves in this situation with regard to the **sentence** that arises from our first example when the place name is replaced by "Adorf". It is of no use to say that the existence of one — and only one — place of this name outside the German settlement area and outside our earth is "virtually impossible from the outset". Sentences of this kind claim probabilities, i.e. even in the best case scenario, they cannot be strictly decided. However, one could object that they are either true or false in themselves, i.e. in terms of their meaning and in relation to reality "as it is". That is not the issue at the moment; it was only necessary to make it clear that these are sentences "about everything real".

The second example is again a sentence of the same kind. "Every thing in reality" is, as he states, either not a featherless biped, or, to put it more cautiously, "not a featherless biped of our Earth," or a human being (of our Earth). However, there is something offensive about this sentence. This stems from the fact that, if we omit the reference to "our Earth", which does not actually appear explicitly in its formulation in *Principia Mathematica*, we can give the sentence another interpretation that suggests itself and would result in a false assertion. Namely, if the sentence is intended to assert that "the concept of a featherless biped includes the characteristic of 'being human'," then this would be considered, at the very least, an unjustified statement. The expressions "x is a featherless biped" and *"x* is a human being" are both of little definite meaning, their meanings being rather vague; but it is fairly certain that the

The former does not include the latter, although the reverse may be true. The *meanings* of 'featherless biped' and 'human being' are not equivalent, they do not have the same meaning°4*, they are not different names for the same species; they both serve to *designate* the same individuals on earth, as far as known times are concerned, they have the same scope of application (within this area). One is reminded of the old distinction between the 'logical' and 'empirical scope' of concepts.Here there are no clearly defined "logical scopes", but they are nevertheless defined to such an extent that one can say that the first is broader than the second, "superordinate" to it. The fact that "the class of featherless bipeds on Earth" coincides with the "class of humans" is not a matter of "original definitions".°® A "class" such as "the humans of our Earth" or "the books on my desk at such and such a time", "the inhabitants of Vienna on 10 October 1941", whose defining characteristic contains a reference to a given spatiotemporal domain, may be called a "collocation". It is determined "by its scope", even if not (always) given by an enumeration of its members. This is the essence of the "empirical" character of those concepts that have "collocations" as their scope. The "extensionality" of the lowest-level definitions is achieved in extensional logic by taking them consistently as defining definitions of collocations. The vagueness that is essential to them is thus limited, but not eliminated.

The third and fourth examples are sentences that contain only precise determinations

‘(x) (Xxx i8x)’ now means “for every *x* (the following applies): x is not a cube or x is a regular polyhedron”. Here, the characteristic “being a regular polyhedron” is undoubtedly “contained” in the “concept of a cube”: the determination lx *includes* the determination *lx.* It does not mean a "collocation" of cube-shaped things in reality and does not state that they fall into the collocation of regular polyhedra. The *determination implication* or *inclusion relationship* is independent of any configuration of real things; the sentence does not assert or imply any reality. It does not mean that in all reality there is no cube that is not a regular solid, but it means that the determination "x is a cube and x is not a regular polyhedron" is unfulfillable in terms of its meaning, i.e. that it is contradictory.

The independence of a containment relationship from reality can be illustrated particularly clearly using the example given. It consists not only in the fact that our sentence of the form '(x) (lx v *iEx)'* remains true even if lx is never fulfilled — which also applies to the interpretation of this sentence form in the sense of our first or second example — but also in the fact that the sentence is valid in the current interpretation, even though the precise definitions that appear in it are, in their exact sense, not applicable to reality at all. There is no thing that fulfils the definition "to be a cube" or its negation, no Fulfilment of *lx v lx* in our interpretation. Fulfilment of M would have to be fulfilment of some determination incompatible with M; the case of a "non-cube" would be the case of a thing that satisfied a geometric determination that was incompatible with "being a cube" but otherwise arbitrarily "complicated". Only for determinations of the same semantic range can the proposition be established that a determination or its negation applies in every case. Despite the, at least highly probable, inapplicability and also in the case of the inapplicability of the determination "to be a cube or a non-cube" to reality, it and the other geometric determinations have their meaning. "For every x, x is a cube or a non-cube," "for every *x*, x is not a cube or is a regular polyhedron" are "absolutely valid" statements, i.e. they apply regardless of fulfilment (in reality). Of course, if one replaces "being a cube" with "being approximately cube-shaped" and the other geometric predicates accordingly, one has achieved applicability, but one is working with fuzzy definitions.

If the universal domain is "the domain of real things", whereby it remains rather unclear what this is supposed to be, then only vague definitions can be directly applied. A definition M, however, is characterised as vague in that its meaning cannot be determined in every case, i.e. it is not always clear whether it or its negation applies; (x) (lx v M) does not apply. It is always possible to find a thing for which it cannot be decided without arbitrariness whether it is cuboid with sufficient approximation or not, an edge for which it cannot be decided whether it is 1 cm or 1.1 cm long with better approximation, a body surface for which it cannot be decided whether

whether it should be called yellowish or not yellowish, but rather, for example, "pure grey" or white or greenish; and this not because of an "imperfection of our senses," but because of the nature of the determinations and the lack of certainty of "things." Countless coarser indeterminacies are disregarded here; only the finer ones, which have gained fundamental recognition in modern physics, are recalled. Unclear definitions do not satisfy the higher-level definitions that we consider to be valid formulas of "predicate logic"; they are not definitions in the sense of this theory — the name "modes of fulfilment" is intended to take this into account.°°* Because only fuzzy statements, propositions of an imprecisely defined content and, accordingly, of an not necessarily fixed truth value can be formed with fuzzy concepts, the "propositions about reality" that arise in this way, propositions about "things and cases of reality," do not satisfy the formulas of propositional theory either; they are not statements in the sense of this logical theory. A theory that has a "domain of real objects" as its "domain of things" is not a system of logical sentences; it is not a component of logic.

Now one might object: it has long been known that provisions, as accepted by logic, and statements in the logical sense **do** not **apply** to reality in the strict sense; however, this has not prevented us from developing strict logic on the one hand and applying its propositions on the other, with the necessary tolerance of inevitable inaccuracies. That is correct. We also need such application; it is indispensable for our actions and knowledge. Nevertheless, it is not mere pedantry, but rather a necessity to take these inaccuracies into account in theory as well. Then one must conclude that an axiom of existence, in the sense of a connection to the reality of things or cases, does not belong in a logical theory. Logic is independent of the fulfilment of any of the determinations with which it works in reality. Such fulfilments may or may not exist, as is to be believed, but they do not concern logical theory and do not enter into any of its propositions. How, nevertheless, an application of logic to reality is possible, in what way it can be achieved without violating the character of reality and without violating the meaning of logical concepts and propositions, is a very important question—a fundamental question of knowledge, but not a question of logic. It will be dealt with in the second main part of this book.

“MATERIAL IMPLICATION”, “FORMAL IMPLICATION”, CONCLUSION AND CONSEQUENCE° *

The "material implication" between two statements is defined as a relationship between their "truth values" as follows: If *p* is a statement and q is a statement, then *p* implies q, or written *as "p ∃ q"*, if and only if p is false or *q* is true: p» i Dr *p* v *q.*

This represents the "truth table" for *p,* q and *p» q:*

I *p* *q* @ v *q p* m q or p v q is only false if *p* W W W is true and at the same time *q* is false, i.e. if W F F *p* a applies.

F W W

F F W

The relationship marked in this way is one of the 16 truth relationships that can exist between a statement *p* and a statement q. The complete table of these relationships will not be written down here. Instead of two statement variables *p,* q, we now consider two arbitrary determinations Wx, *Bx* and the links between them that are not synonymous, which can be formed from them by means of conjunction and negation.

, namely lx a Bx, M a fix, Mix a i8x, M a Be. *A.*

?fo@er1°0* has noted that one can determine "the relationships between two conceptual ranges" – i.e. "classes" — *a, b* — by asking the questions "Are there (things of) *a* that are (things of) *b*?", "Are there *b* that are *a*?", "Are there *a* that are not f›?", "Are there *b* that are not *a*?", and considering all possible answers. Here, the second question, alongside the first, is obviously superfluous; on the other hand, one has been overlooked, namely "Are there things that belong neither to *a* nor to *b*?". The questions then present themselves as follows:

$$(\mathsf{E}x)\,(\mathfrak{A}x \wedge \mathfrak{B}x)? \quad (\mathsf{E}x)\,(\mathfrak{A}x \wedge \overline{\mathfrak{B}x})? \quad (\mathsf{E}x)\,(\overline{\mathfrak{A}x} \wedge \mathfrak{B}x)?$$

$$(\mathsf{E}x)\,(\overline{\mathfrak{A}x} \wedge \overline{\mathfrak{B}x})?$$

II	1	2	3	4	5	6	7	8	9	10		12	13	14	15	16	
(Ex) (9tx fix)	W	W	W	W	W	W	W	W	F			F	F	F	F	F	Ex] *(9Jx > Bx)*
(Ex)(9Ix iE)	W	W	W	W	F	F		F	W	W	W	W	F	F	F	F	[Ex)(91 Bx)
(Ex)(9Lr iEx)	W	W	F	F	W	W			W	W	F	F	W	W	F	F	(Ex)(M Bx)
(Ex)(9J_raiØx)	W	F	W	F	W	F	W	F	W	F	W	F	W	F	W	F	ĘEx)(Unfix)

All possible answers to these questions result in a table of relationships between the "classes" defined by two determinations M, i8x and, therefore, between the determinations themselves (p. 163). In the sense of existential logic, the "class relationships" can be regarded as relationships of "empirical extents" that are themselves empirical. In (1) and (2) we have the "intersection", but in one case "with exhaustion of the universal class" and in the other without it; in (3) and (4) we have the "subordination of ö under *a"*, once "without exhaustion of the universal class" and then "with exhaustion" of it, namely by the superordinate class; in (5) and (6) we have the reverse relationships; in (7) and (8) "range coverage". In all cases with 'F, F' in the upper lines, a is the empty class; in the cases with 'F' in the first and 'F' in the third line, f› is the empty class; in (15) both are empty, in (16) the universal class is also empty. This case excludes "existential logic", but without logical compulsion.

For the provisions M, i8x, defined as "in terms of scope", relationships arise for which no appropriate names are available, even where the scope relationships have common names. In case (1), for example, compatibility between the provisions and between their negatives seems to be a suitable designation, or 'complete independence of the provisions among themselves' in (9) 'mutual exclusion' or 'incompatibility'. But one senses that the designations do not quite fit in the case of 'empirical concepts'. Two determinations that have common fulfilments are certainly "compatible", but two determinations that have no common fulfilment need not be "incompatible" or "mutually exclusive". Even before the discovery of the platypus, a reasonably cautious zoologist would not have claimed that laying eggs and suckling young were two mutually incompatible determinations. In the cases that have 'F' in the second line, — with the exception of (16) - there is a relationship which in existential logic is called "formal implication" and is expressed by '(x) (lx» i8x)'; the cases with 'F' in the third line result in the reverse relationship. In existential logic, it is supposed to have the meaning of a conjunctive link between material implications in individual statements; the difficulties involved in such an interpretation have already been discussed.

If one also sees how inevitable it is to depart from this interpretation and leaves

If the peculiarity of universal statements applies, then a formal implication is still a statement of an inductive generalisation. This is initially plausible insofar as it refers to statements at the lowest level, statements about reality. No one would deny that a statement such as "Every cube is a regular polyhedron" is of a fundamentally different nature. However, a proponent of existential logic would say that the difference lies in the mode of cognition, and that it is irrelevant how one arrives at a claim; only its meaning is decisive for logical consideration. But the difference lies in the meaning; this could be seen in the present example. : What the sentence "If x is a cube, then x is a regular polyhedron" means is completely independent of the fulfilment of the determinations occurring in the sentence, such as their negations. It applies "for every *x"*, i.e. "for every value of x", regardless of whether such values exist or not. It does not concern given objects, nor does it presuppose any for its validity. It expresses a relationship between the meanings of determinations. Their meaning is to "go towards fulfilments"; they use the concept of fulfilment as every determination does, but no fulfilment is asserted in our sentence. The relationship that it posits as "unconditionally valid" — namely, valid independently of any **fulfilment** — is a determination implication or "inclusion". "x is a cube" implies "x is a regular polyhedron", and this is purely in terms of meaning, not "existentially".

Es1°°* expresses the peculiarity of inclusion as opposed to formal implication

implication "(x) *(lx» B'x)"* is expressed by using the notation x) (9Ix»i8x)'. '(x) (M»i8x)' or '(x) (v i8x)' means "Of every thing in reality, as a value

of x, it is true that it either satisfies Mix or *lx*"; x) (lx» i8x)' or *[x}* (M vi8x)' means "For every value of x, whether one exists or not, it is true that it either satisfies M or Bx (would have to satisfy, if it occurs)." The fact that a formal implication applies is a peculiarity of the objects; the inclusion between determinations

is a matter of the determinations, a purely semantic matter.

heit. The formulas of formal implication

(•)(xxvax)=(Ex)(‹ •a),

(E.)(x •$)-(•›(x •a)

correspond to the formulas of the inclusion relation

l•J(üvs') EE'J(^•°8•).

fE•l (• •)-f•l (• +•).

where the symbol '[Ex]' does not mean, as '(Ex)' does, the existence of at least one fulfilment of the meaning of the following expression, from the opening to the closing bracket, but rather the fulfilment of the same meaning and nothing more. The rules for operating with formal implications also apply to working with inclusions, with the only difference being that the 'fulfilment operator' must be substituted for the 'existence operator' and an 'existence-independent' 'universal operator' for the 'existential' operator. If one replaces every (Ex) in Table II with an [Ext, one obtains a *table of inclusion relations* (and their negations) in the form of a table of *satisfiability* (and its negations). For the expressions in this table, there are now standard linguistic translations. "Ex) (M lx)" means "M and i8x are compatible", and case (2) is the case of "complete independence" between *lx* and i8x. “Ex) (lx i8x)”, indicated by an ‘F’ at Ex) (9Jx i8x)’,

therefore indicates "incompatibility" between lx and *lx*. "Ex) (lx i8x)" means that M is incompatible with Bx, i.e. "Mix includes Bx". "(x) (lx v i8x) or [x) (Mmi8x)". For the sake of brevity and with a view to later application, it should be explained that

$$(\mathfrak{A}x \rightarrow \mathfrak{B}x) =_{\mathrm{Df}} [x]\ (\mathfrak{A}x \supset \mathfrak{B}x).$$

In the sense of extending this definition to multi-digit determinations, the arrow symbol has already been used in earlier sections of this work; there, it always means inclusion

. If x] (lx v Bx) applies, then, particularly in mathematical usage, 9Jx is called a "sufficient condition for *lx"*, and Bx is called a "necessary condition for lx". *"Necessary"* in a clearly defined sense is that which is included in relation to that which includes, hence the relationship of inclusion can also be called "necessary", and this now in an unconditional sense. An inclusion that applies applies *"unconditionally"* or "necessarily". Only in this sense will "necessity" be discussed here. A "necessity of thought" as a property of thought or of acts of thought is not considered in this "logical necessity". If M and i8x are *compatible* in the specified sense, then the simultaneous fulfilment of both provisions is *"(logically) possible"*, their negation *is not necessary;* in any other sense, we are not talking about "possibility" here. The necessity

of a relationship such as *lx mlx, lx* v *Bx* is *the "non-possibility"* or "impossibility" of its negation. What is considered necessary is *contradictory* "in itself" when negated, and its fulfilment is "impossible", for which it would be more correct to say: "The negation of the necessary is unfulfillable."

The two 'F's in (4) indicate that both *lx* together with i8x are unfulfillable and lx together with *lx* ; here, M is assumed to be *unfulfillable*: (Ex] *(lx).* At the same time, the expression in the third line, in (4), can be interpreted as (x) (M v i8x), i.e. i8x—•Rx, i.e. M-+Bx; similarly, the fourth line yields the interpretations: [x) *(lx v Bx), M-• B'x, lx—•Ax.* The *unfulfillable W:* includes both *lx* and i8x, both in the case of the fulfilment of *Bx(4)* and in the case of its unfulfillability (12). We set

(lx -— lx) ——z .(lx—+ 9tx) (Xx—+iIJtx).

The following applies*:* [Ex) (9tx)= [x) *(lx),* for which one can say: The fact that Wx is unfulfillable is equivalent to Mix being *absolutely* valid (which does not mean that it would necessarily be fulfilled). Similarly, the unfulfillability of *lx* is synonymous with the unconditional validity of M. The sentences just stated now allow for the following interpretation: The unconditionally valid *lx* is included in both Bx and iBx, both in the case of satisfiability and in the case of unsatisfiability of i8z. This implies that the unconditionally valid is also included in the unsatisfiable.

In order to obtain a complete overview of the possible fulfilment and non-fulfilment relationships for *a* determinate variable, we insert M for *B'x* in our table [II) and obtain the following table **III**

III	1	2	3	4
[E) (M)	W	W	F	F
[Ex) (9Ix)	W	F	W	F

It is (1) the case of the fulfilable and not necessarily applicable provision, to be called "provision in the narrower sense" or "actual provision"; (2) the case of the fulfilable and necessarily applicable provision M; (3) the case of the unfulfillable and not necessarily applicable — "non-required" — provision M; (4) the case of the unfulfillable and — since its negation is unfulfillable — at the same time unconditionally applicable — "required" — provision *lx;* this is the case of the *antinomic* provision.

In order to understand these "cases" correctly, it must be noted that each of them represents an assumption that is a condition for conditions('. Whether each of these conditions(') can be fulfilled by conditions('), i.e. whether there are conditions([1]) that satisfy its conditions or "requirements" for each of the four "cases" must first be investigated and can only be decided positively by demonstrating such determinations. It can be said that for an expression to have the meaning of a determination, its use must be defined in such a way that it is possible to decide for what kinds of cases — not "for which given cases", because there need not be any given cases — it applies or would apply, and for what kinds of cases it does not apply. This is not a definition, but an explanation of the word 'determination'. Determinative expressions that satisfy this requirement, in such a way that the requirements of (l) in Table III are fulfilled for the expressed determinations, can be specified without difficulty, as can cases (2) and (3). Such determinations can be demonstrated. In (2), a determination is "required" that "must" apply to all cases, whether they actually occur or not, an "unconditionally valid" determination; in case (3), an unfulfillable determination, but one that is unfulfillable precisely because of its meaning. The determinations in these cases are for their part fulfilable and even fulfilled. An *lx* that is absolutely valid, like one that is unfulfillable, is already a borderline case of provision; the former can be called *"empty,"* the latter "contradictory" and therefore "absurd." But the fact that the "empty", namely "content-empty", applies unconditionally, that it must be asserted "for every possible case", and that the "absurd" "does not allow fulfilment", that it "excludes fulfilment" and must be unconditionally denied, shows that each of them is nevertheless a determination, that there is no expression devoid of meaning. In fact
is defined by the "empty" determination *lx v* 9Jx, which, in an "existential" interpretation, specifies a range of things, also independently of the

prerequisite of existence, namely, as one might say, a certain *range of meaning*. This is the range of determinations that stand in logical relations of conjunction, negation, disjunction, inclusion, etc. with a given determination Rx. This is a feat of meaning that shows that the "empty" in the meaning of the unconditionally valid cannot be called "empty" in every respect, that it is not meaningless, but still "determination". In case (4), however, a determination lx is "required",

such that *lx*, is unfulfillable, including Hx and Rz, again unfulfillable, including Wo. According to these requirements, the expression lx would have to be applied precisely to such cases — regardless of whether they exist — to which it does not and cannot apply. More precisely, it is required that lx applies if and only if Rx does not apply. It would therefore be irrelevant in a precise

sense, it would be irrelevant whether one assumes ("sets") "lx" or assumes "9Jx". This means that in "Gx", and likewise in "M", there is nothing at all

Assuming that the symbols 'lx' and '9Jx' are not expressions of a sense of determination, then the "antinomical determination" is in fact no determination at all. There is no reason to deal with it logically; indeed, it is impossible to deal with it because no such determination exists. What one can deal with, and what one does deal with in the question of antinomies of the kind under discussion, is the *concept* of "antinomical determination", that is, the epitome of the "demands", the determinations [2] with which our case (4) works. These are genuine but unfulfillable determinations. We have it, when we

'lx —lx' , not with a determination"', whose expression would be 'lx'

, but we do set, in 'Rx= Hx', a determination'[2] '. I believe that this determination [2] ', which cannot be fulfilled by any determination [1] ', is what is meant when one speaks here of an "antinomy". It has the meaning of making an unfulfillable demand for meaning, and thus differs significantly from the case of an object that has no meaning at all and is inaccurately called a "meaningless sign"

. Not only does 'lx= lx' make sense, even though it is an expression of an unfulfillable and therefore "absurd" determination*', but the sign 'lx' in this expression also has, not the value of a relationship for any determination'[1] ' that might exist, but the function of as a "variable" *to mean* as much as the word 'determination', thus has meaning.

When asking about determinations'(1) ' that are of this or that kind, i.e. that satisfy these or those determinations °, one cannot doubt without absurdity that there are determinations of any level at all, because one works with determinations; they are set and thus, for secondary consideration, demonstrated. This is a case of the self-evidence of logic that was already apparent in the constitution of the concept of number.104*

Case (4) in Table **III** and case (16) in Table II are to be excluded as cases that require "anti-nomical determinations" because these requirements cannot be fulfilled. If Table II — with the sign of being '(Es)' — is understood in the sense of "existential logic", then (16) must also be excluded here: the existence axiom prohibits the case of the empty domain. If it is dropped, i.e. if it is disregarded without introducing its negation as an axiom, Table II takes on a meaning such that case (16) also poses no difficulties. This is the case of two determinations *lx, B'x,* which have no fulfilment in reality, but may be fulfilable or unfulfillable; nothing is said about this; the fact that they cannot be unfulfillable together with their negations is already implied in the assumption that they are determinations. Case
(16) is present in the example already cited of the determinations from
the semantic domain of, for example, Euclidean geometry.

Logic therefore has good reason to retain the sign of being '(Es)' alongside the sign of fulfilment 'Ex)': it needs the concept of fulfilment alongside the concept of fulfilability, even if it does not assert any fulfilments of the lowest level, namely in reality.

A sentence of the form ‘(Ex) *(lx)’,* in which ‘lx’ expresses a non-first-order predicate, a predicate of reality, is not part of logical theory; it is never asserted in it, because a statement “!.”, as a judgement of reality, is not a matter for logical theory. Expressions of these two (and related) forms nevertheless occur in logic and other formal theory: (1) as unasserted components of asserted inclusion expressions (in connection with negation); (2) as independent logical assertions of the existence of "objects" of a higher level, i.e. ultimately of certain meanings, and as demonstrations of such "objects".

which justify those "existence" assertions — it has simply not been customary to express such proofs in formulaic terms, even though they have always been used. An example of (l) is the logical proposition (x) (Mix) !*—+.Wt, another is the proposition Alt.—•.(Ex) (lx); an example of (2) is the arithmetic, and likewise logical, proposition (E«) (/ = 1) — ef. Chap. 28 ... — and the self-reference of the proposition "(Es) (ex)", i.e. its meaning, not its actual reality, which is essential in the constitution of the concept "t" for the proof of its fulfilment.

The provisions with which a formal theory "works," which it "sets" in its terms, are always self-referential and reveal themselves in such work with them. Without them, the theory would not exist. They construct the theory, whether it may be fulfilable or unfulfillable, useful, "right" or "wrong", contradictory and absurd; it is only this through the meaning of its positing. This meaning "is present", exists in the sense that to claim it as non-existent in any sense would be the height of absurdity. We do not fall out of meaning. And when a madman utters "meaningless sentences" and thinks he has something to say in them, there is at least one **meaning** in this opinion**:** the meaning of belief that the sentences contain some kind of meaning, perhaps of a more specific kind; they "claim" to have such meaning. The self-evident propositions — as meanings, not as acts — that construct a formal theory prove the "existence" of these very meanings of the most general kind, such as determinations, statements — apart from levels of height — and at the same time prove their fulfilability by yielding cases of fulfilment of a determination, of a higher but otherwise indeterminate level: thus proving the validity, "correctness" of statements, and the invalidity of their negations. Every statement of being that is part of a formal theory is founded and justified by the demonstration of meanings, ultimately in the propositions that construct this theory.

A formal theory is one that only uses precise definitions in its structure. A definition Wx is precise if [x) (Rx v M) applies, and indeed applies in the strict sense that the assumption that there is a value of x that fulfils any definition of the domain of *lx* but does not satisfy either mix or lx, is contradictory and absurd. It is absurd to assume that something that satisfies any geometric metrically unambiguous definitions is neither a cube nor a non-cube. However, it is not absurd to assume that a thing in reality is neither "approximately a cube" nor "approximately any other geometrically definable shape". This indeterminacy lies in the vagueness implied by the word 'approximately'; it can be reduced by a 'sharpening' of linguistic usage, but not eliminated.

There are two types of formal theories: axiom-free and axiomatic. An axiom-free formal theory is a logical theory, a system or subsystem of logic. Examples of this type include purely logically based number theory and arithmetic. A number

/ is called "existent" if it is a satisfiable determination of the type — in the sense of Chapter 28 — if [E«) (/) applies. One can explain (Ez) (t=)= pt[E«) (/), where z is the "thing variable" belonging to the domain of the definition "/", which would be more precisely expressed by (z). In logical theory, propositions occur as assertions.

such as (EG) (EB) [Ex) (lx Bx), also (H) (EG) [Ex) (lx i8x); the first states that there are provisions *lx*, i8x of some higher level that are incompatible with each other. In the sentence itself, in which 'R' and 'B' are "predicate variables", the term "predicate R" appears; it is called "R'", and the corresponding term "B'", both of which are predicates"*' for predicates" , would have values of 'fl' and

of 'B'. In the determination "+" "[Ex) (Hx a i8x)", a determination "*" incompatible with "[Ex] (lx *fix)"* is set and thus demonstrated. Thus, the proposition that there are two incompatible determinations *of some* level proves itself. The other statement asserts that for every determination 91' there is a determination i8^ that is incompatible with it; it proves itself in the structure of the theory that uses the concept of determination by setting determinations for determinations. Determination involves setting what is valid, as opposed to what is not valid. It is inherent in the concept of "determination" – allowing a decision (on whether something is true or not) – to have a negation. Only in this sense can the universal proposition under consideration here be asserted. The "existence" of a determination i8x incompatible with *lx*, "for every Rx," cannot be proven other than by this inference "from the concept of determination." Our proposition would also be

write '(R) (EC) Ex) (9Ix *B'x)'* - but it should not be written

‘[[EB]Ex] (*@x)’*. It cannot be proven, like “(E‘?I) (EB) [Ex] *(lx n lx)”*, by a finite number of proofs; in other words, it cannot be proven by proofs at all. In the same way as our
first example, the number-theoretical theorem "There is a prime number number immediately followed by a prime number" is proven by demonstrating 2 and 3; however, the theorem that there is a larger number for every prime number is deduced "from the concepts of number, prime number, and greater than (as understood here)". What results is not a "construction" but the "constructability" of a number that satisfies given conditions "for every given prime number"; just as in our second example, *"*for every given determination *lx"* another one that satisfies the given determinations is "specifiable", i.e. demonstrable, according to the "concept of determination".

BASED ON LOGIC: IMPLICATION, ELECTRONIC COMMUNICATION, CONSEQUENCE

Material implication is a relationship between the truth values of statements. The rule of inference applies: if *p* is to be asserted and *p»q* is to be asserted, then q must also be asserted. *B. Russell:* "In order that it may be *valid* to infer q from *p,* it is only necessary that *p* should be true and that the proposition 'not-p or q' should be true... But inference will only in fact take place when the proposition 'not-p or q' is *known* otherwise than through knowledge of not-p or knowledge of q. Whenever *p* is false, 'not-p or q' is true, but is useless for inference, which requires that *p* should be true. Whenever q is already known to be true, 'not-p or q' is of course also known to be true, but is again useless for inference, ... In fact, ... inference only arises when 'not-p or q' can be known without our already knowing which of the two alternatives it is that makes the disjunction true. Now, the circumstances under which this occurs are those in which certain relations of form exist between *p* and q. For example, we know that if r implies the negation of s, then s implies the negation of r. Between '*r* implies not-s' and 's implies not-r' there is a formal relation which enables us to *know* that the first implies the second, without having first to know that the first is false or to know that the second is true. It is under such circumstances that the relation of implication is practically useful for drawing inferences." [106] In contrast, Ne/son'°7 states that inference always occurs on the basis of a relationship between the meanings of the statements and not on the basis of a truth relationship.

In fact, the conclusion is based on a relationship of inclusion between determinations, i.e. between elements that are neither true nor false. This relationship must be clearly distinguished from matters of true and false. The "paradoxical cases" of implication, namely that a false proposition implies any proposition (within the realm of meaning), whether true or false, that a proposition

implies its negation is false, have repeatedly caused offence.
.108

As the discussion in our Table II shows, the relationship of inclusion gives rise to cases that correspond exactly to the "paradoxical" ones of material implication. An unfulfillable determination *A(x)* includes any determination *B(x)* of its domain of meaning, in particular

in particular its own negation $\overline{A(x)}$, i.e. the unconditionally valid determination; an unconditionally valid determination is included in every determination and, for its part, includes nothing but the unconditionally valid determination, i.e. its identical meaning in any form. If K' means this unconditionally valid meaning of determination, 'd' its negation, i.e. the unfulfillable determination, then the following applies

(A) (A(x)—• K), *(A) (A*—• *A(x)).*

This seems strange; one might miss a contextual connection that would allow one to conclude from "x is a number" that *"x* —- *x"* is true, or that a statement of the form *"p* vp" includes the statement *"p"* and the like; at the same time, however, such reasoning seems unnecessary because what is to be concluded is "in any case" absolutely true. Similarly, there is no contextual connection in the reverse relationship, except that the unreasonable expectation of inferring, for example, from "x-1- x" on the one hand that "*x* is not a number" and on the other hand that "x is a number", and the conjunction of both propositions, seems even more strange. Now, it is important to note that in the true sense of the word, no conclusion is being drawn here — just as one concludes from "x is a multiple of 4" that "x is a multiple of 2". The only important thing is to recognise that in any given definition, for example *A(x)*, the unconditional truth is "included" and, with its proposition, "implicit in the meaning" in the sense that it would be meaningless to propose *A(x)* and at the same time propose the negation of the unconditional truth. The conjunctive proposition *"x* is a multiple of 4 and *x*- *x* does not apply*"* — which arises as soon as its first clause is combined with something that cannot be fulfilled — or *"x* is a multiple of 4 and is not a multiple of 4" and the like is absurd and, in terms of its meaning, impossible to fulfil. Whoever posits "x is a multiple of 4" "posits the unconditionally valid" in the sense that its negation together with that positing would result in something unfulfillable, whereas he "meant" something fulfillable. This fact, "The un-

"Conditionally valid is included in any proposition" is extremely important for our reasoning and conclusions, everywhere and also where we draw conclusions from the proposition of an actual determination that is neither absolutely valid nor absolutely invalid. This significance consists not only in the negative fact already established—that the combination of the invalid with an arbitrary proposition results in something unfulfillable—but is particularly evident in the fact that, in order to draw certain conclusions from a premise, we in many cases have to accept an unconditionally valid second premise, but the conclusion is still valid as a consequence of the first premise. Trivial examples of this procedure are provided by the "resolution" of a defining equation such as *x – a* ——*b*, "according to *x*" – this is done by adding a on both sides of the equal sign, on the basis of the "identical equation" *a* ——*a*, thus obtaining *x*- *a*+*b*. As in the equation *a* ——*a* here, in countless cases of mathematical reasoning, an unconditionally valid determination is conjunctively linked to the premise from which one starts, thus obtaining a conclusion that is "contained in the premise alone", i.e. included, but only clearly "emerges" through that auxiliary means;it thus proves that what is absolutely valid is included everywhere, i.e. in every proposition, in terms of meaning, and that its conjunctive addition "changes nothing" in terms of its meaning, but that it can serve to develop this very meaning, to make it clear in its unfolding. It is *A* ——*A n* P and *(A A n V)* -*(A*—• *V)*. With this relationship *A*–+ P, which is so important for inference, or more precisely *(A) (A*-+ P), its "contraposition" *(A) (A*—•*Afi* is also set, for which, since *'A'* stands for any determination, *"(A) (A*—• *A)"* may also be written. An absolutely invalid proposition, be it an unfulfillable determination or a statement that arises from such a determination through application or "through specialisation" or "through generalisation", must, according to its meaning, be considered incompatible with any proposition, including incompatible with itself.10®

"*x4 x*" or "2 +2-}-4", "2 + 2= 5" and similar expressions can always be supplemented or developed without changing the meaning of the result to "xJ x. a .x= x" or to "2 + 2-1-4. a .2 +2=4", to "2 +2= 5. n . 2 +2-l- 5" and so on. For example, "a value of x" is understood to mean something "individual" — in the broadest sense of the word — "that is identical with itself", "that is what it is", the same under "2 +2" as under "4", etc.

It thus becomes apparent that these assumptions are *contradictory,* absurd and therefore "necessarily" invalid. However, what is absurd in itself, when combined with any other assumption in a conjunctive manner, results in a contradictory assumption: it is "incompatible" with any assumption because it is incompatible with itself. It is therefore also "incompatible" with everything it includes.

These are *relationships of meaning,* not relationships of truth; it is not because the relationship of inclusion has been defined according to the formal pattern of material implication that "paradoxical cases" arise in it. As pure relationships of meaning, they must be "taken along" if one wants to grasp the relationship between theoretical propositions that is decisive for inference and conclusion.

The consequential relationship is independent of fulfilment and truth. It will be useful to explicitly define the use of language here. We say *"A*(x) *implies B(x)"* if and only if *A(x), B(x)*

Provisions are and “[x) $(\overline{A(x)} \vee B(x))$” applies, i.e. “$\overline{Ax} \vee Bx$” applies unconditionally, i.e. regardless of whether provisions *A(x), B(x)* or their negatives are fulfilled. The relationship of inclusion exists between provisions. This is also indicated by the abbreviated notation *'A(x)—+B(x)'* or *'A—•B'*. For example: "x is a multiple of 9" includes *"x* is a multiple of 3" or "being divisible by 9" includes "being divisible by 3".

If *A(x)* includes *B(x),* one also says *"B(x) follows from A(x)"*. If *"A (x)—•B(x)"* applies and t is a value of x, then *"A (c)—+ B(t)"* applies*;* e.g.: from "18 is a multiple of 9" follows "18 is a multiple of 3". Here, the expression "follows" seems to be even more appropriate

than in the cases of the first type mentioned. In *"{x] ($\overline{A(x)}$* v *B(x))"*, an inclusion between determinations is expressed, while *in "A (t}-+B(t)"*, as an "application" of this, there appears to be a relationship of following between a fulfilment of *A(x)* and a fulfilment of *B(x)* by the same object t. *"A(c)"* and *"B(t)"* are statement contents, no longer determinations, and one can *deduce* the second statement from the first. The *inference* from d(c) to *B(t)* appears as an application of the consequence relationship *A(x)-+B(x)*, which can be represented more precisely as follows:

“From (x) *($\overline{A(x)}$* v *B(x)) n A(t) follows B(t),* [x) *(A(x)* v *$\overline{B(x)}$) n A(t)* applies*;* therefore *B(t)* applies*”*. Here, “applies” is merely an expression of the assertion of what follows, not an assessment of its content. *‘A(c)’* expresses the fulfilment of *A* by t, in the example the fulfilment of "x is a multiple of 9" by the number 18. But in what sense does 18 fulfil the condition; what does "18 is a multiple of 9" mean? Expressed in the language of class theory, it means: "For every class «, if « is an 18-member class, then there exists a number n such that n are mutually disjoint subclasses Q" Q„ ..., § of e, each of which is a nine-member class and whose union class is «." If we remove the term 'class' from this statement, we get: "For every predicate «(x): if « has 18 satisfiabilities, then there exists a /,($,= n) such that n mutually incompatible implicants Q" Qt, ..., Q of n exist, each of which has exactly nine satisfiabilities and whose disjunction is «." The expressions '/,' and *'n'* could also be omitted on the basis of the definition of the concept of number; however, it can be seen that our simple statement "18 is a multiple of 9" would then become a rather unclear sentence;

In other words, one sees that the meaning of this statement is not entirely "simple" and recognises the advantage of simplified language. But this advantage comes at the price of using expressions that suggest the danger of erroneous interpretations, or at least interpretations that are subject to all kinds of justified doubts and, above all, non-logical interpretations. This is particularly true of the "ontological" interpretation: that the words '18', '9', 'multiple' or at least 'class', 'subclass' refer to some kind of "ideal objects" or "entities". Whatever one may argue for or against these views goes beyond the realm of logical and mathematical theory, which is the realm of pure conceptual development. It cannot be denied that a similar danger is also associated with those interpretations that are ultimately permitted here and use only the concepts of "determination," "fulfilment" and "fulfilability." But these concepts are ultimately self-evident meanings in the structure of logical theory; they are protected from fundamental doubt about their validity in theory by the fact that any attempt at such doubt must challenge its own meaning and prove to be self-contradictory. Herein lies the "self-evidence" of these terms and their "logical right". To attribute any onto-logical meaning to them is not a matter for logic or mathematics.

One can see how the statement "18 is a multiple of 9" represents a "fulfilment case" of the determination "x is a multiple of 9". The logical meaning of this statement is the assertion of an unconditionally valid determination, namely an inclusion relationship. "If e is a provision with 18 fulfilments, then there is ...", which means the same as "The provision content /" for "= 18, suffices as the value of x for the provision *'x* is a multiple of 9'". If we call the fulfilment of a determination in reality a "final fulfilment", then we can say that the statement '18 is a multiple of 9' expresses a fulfilment of the determination "x is a multiple of 9" that is (not a final fulfilment and) independent of final fulfilment. But the statement is not independent of ultimate fulfilment; it includes the demonstration or assertion of the 'existence' of numbers, and that is the ultimate fulfilment of certain determinations.

The "fulfilment" of "x is a multiple of 9" consists in the fact that (/ = 18) as the value of x transforms this determination into an unconditionally valid inclusion relationship (for "), but not into a statement that would have the kind of truth required for a true judgement of reality. That is why some people do not want to attribute truth to unconditionally valid determinations, but only "correctness". It is not important how one decides on this question of nomenclature, as long as one remains aware of the meaning. In any case, it is clear that the consequential relationship between "54 is a multiple of 9" and "54 is a multiple of 3" does not consist in the fact that both statements are true, but in the fact that "being a multiple of 9" includes "being a multiple of 3", which is a pure relationship of meaning in which truth or falsehood in the sense of fulfilment or non-fulfilment of the conditions and any "subjects" as fulfilment can be disregarded altogether.

This is expressed in the formula of the sequential relationship

$$“[x]\ (\overline{A(x)}\ \mathrm{v}\ B(x))\ n\ A(t).\!-\!+.B(c)”,$$

for which one can also write

$$"A(x)\text{—}+B(x).\ n\ .A(c):\!-\!+:B(t)"$$

. It is noteworthy that a correct formula also results if the expression is used instead of *{x]'* in the first

Form '(x)'. The sentence obtained in this way, namely

$$“(x)\ (\overline{A(x)} \vee B(x))\ n\ A(c).\text{-+}.But)”$$

or

$$“(x)(A(x)xB(x))\ n\ A(t).\text{—+}.B(c)”,$$

is more general than the previous one. It says that the truth relation of the "formal implication" from *A(x)* to B(x)!!0* is sufficient to give the consequence *B{c)* together with *A(t)*. This corresponds to *Russell's* statement [1]"* about the conditions under which a conclusion (according to this relationship) is *correct*, regardless of how it might be drawn. But the question is not logically irrelevant. A brief overview shows that knowledge of the first premise, *(x) (A(x) x R(x))*, without knowledge of the truth value of *"B(x)"*, and thus the actual conclusion, is possible (a) if, beyond the mere truth relation, the inclusion *A-+B* applies; (b) if the person drawing the conclusion accepts the truth relation as valid without having established it, either because they believe it on the basis of a communication or in the sense of an inductive generalisation of incomplete observation; both play an important role in science and practical life, but do not result in actual knowledge. In these cases, it can be asserted that
(b) only the *consequential relationship,* and it is purely logical in nature, independent of final fulfilments. It goes without saying that in case (a) the truth relationship is also purely logical; this means that the truth relationship that exists here is in fact a pure semantic relationship between the contents (not a relationship of actual fulfilments). A logic for which "formal implication" means the conjunctive linking of a set of individual statements cannot do justice to this fact. In it, the consequence relationship has no justified place. But inclusions and negations of inclusions are the contents of logical propositions; logical theory is built up by applying such relationships in inferences. Therefore, the theory of these relationships will be presented first.

NOTES

[1] It is the view of "behaviourist" or behavioural psychology and a **language** theory.

•* 'Judgement' is a term coined by Mally. In every perception, for example, something, an object, a process or a state, is first 'judged', i.e. opened up to consciousness, and can then be *assessed* in a second act.
°• A handwritten table of contents has the variant: The word 'meaning' and its uses (meanings).
•• The manuscript does not contain any chapter headings. In a letter to Dr. Laurin dated 1 June 1943, the heading ‘Philosophy as fundamental reflection’ is discussed but not finally approved.
5• At this point, a note by the author was intended but not executed.
•• The manuscript erroneously reads: 'ihrer'.
• Mally refers here to Rudolf Carnap's essay, 'Psychology in Physical Language', *Erkenntnis*, Volume 3, 1932–1933, 107–42, in particular pp. 123 and 126ff.
® The spiritual is not initially found and in this sense "given". It is experienced – in fact, it is experience. – The *contents* of this experience are also not pre-existing objects, neither spiritual, which would be "transferred outwards", nor "external" (the "transfer outwards" would, incidentally, be an act of meaning).
Mally dealt intensively with this work; his estate contains a 20-page excerpt. One of Mally's students (see introduction, part 2) translated the introduction to Principia Mathematica into German: Bertrand Russell and Alfred North Whitehead, *Introduction to Mathematical Logic*. Translated into German by J. Mokre, Munich 1932.
‘0• Alexius v. Meinong (editor), *Untersuchungen zur Gegenstandstheorie und Psychologie (Investigations into Object Theory and Psychology)*, Leipzig 1904, p. 9.
'l* Manuscript: therein.
1°• A note was planned here.
l•• Note planned. Probably a reference to A. Meinong, *on possibility and probability*. Contributions to Object Theory and Epistemology, Leipzig 1915.
• I already sought to avoid these inconsistencies during my object-theoretical period by explaining that "the triangle" is not a triangle, that it does not fulfil its defining characteristics, but rather "has" them in a different, peculiar way, for example as "constitutive characteristics"; a so-called 'incomplete object' is not the bearer (i.e. fulfiller or fulfilment) of its constitutive (defining) determinations, but their 'determinate'. This took into account the usage of language, which allows us to speak of "the triangle" or "the circle" without the impossible assumption that an object "fulfils" incomplete determinations without being completely determined. "The triangle" was then also completely determined, like any object, namely as "the determinant of the determination 'to be a triangle'". The determinant of this determination would be *the* determination that such a "conceptual object" actually fulfils. In fact, it can be said that linguistic usage is an expression of a fiction of "determinants" as objects of a special kind. However, it is necessary to eliminate this fiction, which is an unfillable assumption, by interpreting it correctly: there is no object that is the determinate of a determination. (Editor's note: E. Mally had already established his determinate theory in 1912, cf. his 'Gegenstandstheoretische Grundlagen der Logik und Logistik' [Object-theoretical Foundations of Logic and Logistics], *Zeitschrift für Philosophie und philosophische Forschung [Journal of Philosophy and Philosophical Research]*, supplement to volume 148, Leipzig 1912.)
'°• A note was planned here.

l• Cf. N. Hartmann, Ontology, Ethics, etc. Editor's note: Mally is apparently referring to the works *Ethics,* Berlin 1926 (1962); *On the Foundations of Oncology,* Berlin 1935. l'• In the manuscript, there is a hint of a note that the definition of numbers in the "Principia Mathematica" already presupposes the concept of numbers.

' * "Meaning" added by the editor.

l®• In the manuscript: "meaning".

°°• Reference to P. Hoffmann in the manuscript.

°l• Cf. also E. Mally, *Grundgesetze des Sollens,* in this volume.

°°• Reference is made here to the signs with "systematic ambiguity" in Principia Mathematica. Mally comments: "All logical terms are subject to such ambiguity. Such a term does not appear to be class-forming, but it is permissible and indispensable" (margin note in the manuscript).

°°* Cf. Alexius Meinong, *Über Annahmen,* Leipzig 1902, 2nd revised edition 1910.

°'• For Meinong, feelings of value had judgements as their "psychological prerequisite"; Mally describes this as an "intellectualist prejudice", cf. Kollegheft zur Wertlehre 1935, p. 39 (in the estate). Similarly, Max Scheler, *Der Formalismus in der Ethik und die materiale Wertethik*, Bern 1966, pp. 14 and 44ff. On this, see K. Wolf, *Wirklichkeit und Wert,* unpublished Graz dissertation 1933, p. 64ff.

°5 Unfortunately, the word "judgement" has lost almost all of its specific meaning in the usage of German philosophers. Sometimes, and usually, it is restricted to statements of a particular form, with "subject and predicate"; at other times, it also refers to things other than statements, e.g. to mere assumptions (freely accepted propositions, determinations) and even to questions. In this work, it should always mean only as much as a statement (expressed or unexpressed or not even formulated linguistically).

°•• "Chapter 16" added by the editor.

°'• The manuscript also considers the following title: Deficiencies in meaning: Paradonies. Apparent statements. Apparent determinations.

° On spelling. [If *p* is a statement, namely a statement content, then "p" is the sign that means *p*. Or:] p is the statement (namely the statement content) whose expression is "*p*".

°° The endeavour, noticeable among members of the former "Vienna Circle", to replace philosophical reflection with a theory of (scientific) "language" is limited here. It is not correct that the meaning of an expression can be indicated by nothing other than another expression that translates it, that the question of meaning cannot be derived from the signs; first and foremost, language is supported by the non-linguistic reality in which it is used and understood.

*** "how" inserted by the editor.

• Short version: "Whatever involves *all* of a collection must not be one of the collection; or, conversely: If, provided a certain collection had a total, it would have members only definable in terms of that total, then the said collection has no total." *P.M.,* Vol I, p. 37.

°°• Cf. J. Mokre, *Einführung in die Mathematische Logik,* Munich 1932, p. 56 (see note 9•).

°°• In an (incomplete) handwritten table of contents, the following note is made here: "Later! After step theory!"

°•• In the following, Mally uses the word "autological", while in the heading he uses "predicable".

•5 The paradox in Russell's example of "the class of classes that are not elements of themselves" has exactly the same lack of meaning in the "construction" of this "class"; just as the paradox of the "village barber" does not prove anything against the meaningfulness of "x shaves x", it does not prove anything against the meaningfulness of the expression "z is a class and is an element of x", whether this class may or may not exist.

If only what is accessible to one's knowledge should be true for everyone (Bollnow, op. cit.), then – once the concept of "accessibility" has been defined more precisely, which is difficult in itself – most of what constitutes scientific knowledge would not be true for most people. Healthy thinking tends to be modest in this regard: I don't understand that — it is inclined to trust those who understand more and refrains from making its own judgement. In addition, perhaps particularly frequently in our day, we observe an attitude of presumptuous narrow-mindedness that rejects and dismisses everything it cannot comprehend. But that must not be decisive for the concept of truth. The truth is "true for everyone" in the simple sense that everyone who asserts it is right, everyone who denies it or violates it is wrong, and everyone who does not comprehend it simply does not know something: that, if anything, is the meaning of the once much-discussed "universal validity". (Editor's note: The above refers to 0. F. Bollnow, 'Zur Frage nach der Objektivität der Geisteswissenschaften' [On the Question of Objectivity in the Humanities], *Z. f.d. Ges. Staatswissenschaft* 97 (1937) 335—63.

®• In a study on decidability and meaning (meaningfulness validity), he concludes that what matters is logical decidability, i.e. that a decision is not "logically impossible". However, the examples he cites for logically undecidable and at the same time "meaningless" sentences ("This tower is one hundred feet and at the same time one hundred and fifty feet high" and the like) are entirely decidable, namely false and self-contradictory sentences. A contradictory sentence (fine contradiction) proves to be not meaningless, since its negation is a tautological, i.e. self-evidently true, sentence, whereas in the case of undecidability, both the assertion and the negation are meaningless. (Editor's note: Mally is obviously referring above to Schlicks' essay 'Meaning and Verification', published in English in *The Philosophical Review* 45 (1936).)

°®• The manuscript erroneously has "refer".

°° See, for example, 0. F. Bollnow, op. cit. See also my essay: 'Zur Frage der "objektiven Wahrheit"' (On the Question of 'Objective Truth'), *Wissenschaftliches Jahrbuch der Universität Oraz (Scientific Yearbook of the University of Oraz),* Graz 1940, pp. 177–97.

^° The absurdity and irrationality of the assumption that the validity of any meaning depends on subjects has already been explained in general terms in Chapter 7.

^1 Designations:/z, *gx, M, ... f ‹x,* y), *(:r! •)- -. f)! --e•. (•! r) --- f• e•! —*
(fx, t3x 8 (-f• •• H)-/ *P••*

'° A simple determination can, of course, also occur in positive form — for example is a simple determination in the universal domain of real numbers x 0 – that it excludes a single element of the universal class is merely the most convenient characterisation of such a determination. This is in response to W. Burkamp's objections in his *studies on the foundations of logic, "Begrißund Beziehung" (Concept and Relationship),* Leipzig 1927.

^° Incidentally, what has been explained above as a defining element has already been elaborated by E. Schröder in his Algebra der Logik (Algebra of Logic) and, in a conscious departure from grammar, designated as "simplum".

E. Schröder, in his Algebra der Logik (Algebra of Logic), and, in a conscious departure from grammar, designated it as "simplum".

•• Let *fx imply (in general) gx,* in *the* sign *fx gx,* or, if one emphasises ben wants '/tir *every x: fx gx', (x).fx gx.* If y is a thinking person, then *'y posits fx'* will imply *'y posits gx (with) in a meaningful way'*, but not generally 'y *posits px (explicitly) (with)'.* , nor does it imply *that* 'y *asserts gx* in the sense that, when asked whether it also applies in the sense of his assertion '/z', he is immediately prepared to affirm this'. If *(x). fx* pz and applies /xi - where xi is a specific, given "value" of x - then *fxi qxp* applies in the sense of an applied determination implication; but if y sets *fxi*, then it again only sets pxi in a meaningful way and does not have to do so explicitly, nor does it have to admit it when asked about pxi. This means that our setting, grasping, asserting, etc. is imperfect: setting a determination – accepting or asserting its validity – does not imply the corresponding setting of an implicate, but it can be said that, with regard to a setting that is carried out, the setting of an implicate is always more probable than a setting that is incompatible with it. 'y posits *fx'* is not an 'extensional function' of /x, but there is, in the specified sense, a 'tendency towards extensionality' in positing. There is no reason to claim that 'intension functions' are meaningless - Carnap They are not sharp determinations; the same applies

of any 'empirical' or 'descriptive' definition, and yet one does not dismiss them as meaningless. One must simply endeavour to give them a useful and precise meaning. (Editor's note: This text was subsequently deleted from the manuscript, either by Mally or by someone else – perhaps Professor Kröner.)

•^* Cf. the editor's addition to note 14.

4®• The manuscript contains the following fragment of a sentence: "shall this one ascending endless sequence of 'forms' " in linguistic expression.

4' Ygl. G. Frege, 'On Sense and Meaning', *Zeitschrift f. Philosophie und philosoph. Kritik,* N.F. 100 (1892), 26ff.

[4] The concept of geometrically unambiguous determination is, on the one hand, not unambiguous and not absolutely defined. In a geometric consideration, positional relationships, e.g. of triangles among themselves and to other 'structures', can of course be included, in which case 'triangle 1,1,1' is no longer 'geometrically unambiguously' determined.

4• See below. (Editor's note: Mally explains this most clearly in the letter to Dr. Laurin dated 14 November 1943, reproduced in this volume.

volume.)

[5] ° Cf. the infinity postulate of Principia Mathematica.

°'• Note planned.

••• Note planned. Meinong's corresponding views are best understood in his self-portrayal in: Raymund Schmidt (editor), *Philosophie der Gegenwartin Selbstdarstellungen,* 2nd edition, Leipzig 1923, Volume I.

[5] ° It is as if the statement 'There are regular octahedrons' or 'There are no regular decahedrons' transfers the 'there are' and 'there are not' to the determinate: 'The regular octahedron exists, the regular decahedron does not' – even though 'the regular decahedron' also 'exists' as a 'concept' (content of object form). Principia Mathematica explains: 'We say that a class *exists* if it has at least one element' (Mokre, op. cit. p. 45).

• [4] In the sense of Principia Mathematica, one would say: Zero 'exists' in the sense that there is an element of the class of empty classes – which is zero; in every universal domain, indeed above every universal domain, there is the empty class; the class whose only element is the empty class – of the universal domain under consideration – is zero. However, the sole element of this class class does not itself 'exist' in the sense that a one-membered or two-membered class 'exists', namely in the sense that its defining determination could be fulfilled. Actual fulfilment – as defined by the

P.M. seem to demand, if they do not merely accept it – is not required. The **statement 'There** is not *a* single object' could be true; but the definition 'It is a set containing exactly one element' would remain meaningful, and thus the number one would be 'mathematically existent'.

°^• The manuscript is dated 7 February 1942 at this point.

°°• Cf. E. Mally, *Erlebnis und Wirklichkeit,* pp. llff., esp. p. 16.

°'• "stands" inserted by the editor.

•• Because there is no separate object ("class a") that a class name "a" refers to or at least requires for its usability, Principia Mathematica explains that such a name is an incomplete symbol. This cannot mean that it is *meaningless* in itself – species names have specific meanings, even if they *denote* nothing other than things of that species in given applications. The same applies to a clearly "descriptive" name, such as "the author **of** Waverly", of which *P.M.**, p. 70) claims: **"The** author of Waverly' cannot mean the same as 'Scott', or 'Scott is the author of Waverly' would mean the same as 'Scott is Scott', which it plainly does not; nor can 'the author of Waverly' mean anything other than 'Scott', or 'Scott is the author of Waverly' would be false. Hence 'the author of Waverly' means nothing." The expression used here ("means"), which is crucial, does not distinguish between meaning and designating, which would be essential. "The author of Waverly" serves only to designate the writer named Walter Scott, without meaning the same as this name – for the purpose of the proper name, which is essentially a designation, the meaning remains irrelevant, as it may be completely unknown –; but *it is* precisely what the "descriptive" name *means that* makes the sentence "Scott is the author of Waverly" a factual, non-tautological statement.

°° P. Weiss (Mind ...) points out that the propositions of the doctrine of stages, if they are to apply without restriction, either refer to themselves and thus violate the doctrine of stages, or must be restricted.

°° My description of the "determining element" as the exclusion of an individual and, from there, of determination in general, is indeed extensional and "atomistic"; however, it is only intended to be an accurate description and in no way demands that the individual take absolute precedence over the general in terms of meaning.

•'• Cf. note 49, addition.

••• More correct in meaning: "the same as by S".

°® At the very least, the perception of signs is involved, which will be discussed later. (Editor's note: See, for example, chapter 24.)

• 4* Hilbert-Bernays, *Foundations of Mathematics,* Vol. I, Berlin 1934 (1968) p. 20f.

^• ibid., **p. 20 (**Editor's **note**: Note 65 obviously **refers** to the work mentioned in note 64•).

27 October 1943. The "digit", "figure", e.g. '11', is defined by stipulation ("axiomatic") according to its type – for which the number of strokes in it is decisive –; but such a stipulation is not an axiom of arithmetic, but only concerns its representation. It is an incomplete formalism of representation that is defined "axiomatically." It does not belong to the content of arithmetic propositions, not even insofar as they become "intuitively evident" in the digits.

•° Hilbert-Bemays, op. cit., p. 32.

°®• "it" added by the editor.

•• Note: P. Bernays, 'The Philosophy of Mathematics and Hilbert's Theory of Proof', *Blätter für Deutsche Philosophie* **(1930/31).** 'Two **things' is a concept** that is free from the concept of class and is already presupposed in the idea of a two-element set. However, "two things" always presupposes "things of a certain kind" without specifying the kind. Without the idea of the *one* type to which the things belong, one only gets ideas such as "umbrella and keys (I want to take with me)" and adds: "These are two things I must not forget", so the common type to which they belong is already 'thought'. Left undefined, but always presupposed in the idea of "two things of some kind".

'°• This sentence was inserted by Mally in the very distinct handwriting of the last year of his life (1943/44).

" Hilbert and Bernays, op. cit., p. 32.

'° op. cit., p. 33.

'° Ibid., p. 32.

'• Cf. Brouwer, Weyl (Editor's note: On a neighbouring page of the manuscript there is a reference to Hermann Weyl, 'Über die neue Grundlagenkrise der Mathematik' [On the New Foundational Crisis in Mathematics], *Mathematische Zeitschrift* 10 (1921).)

'5 Hibbert-Bernays, op. cit., p. 34.

• Ebendort. - This is not intended to be a critique of the "Fundamentals of Mathematics," especially since this work does not remain within the finite conception; moreover, a theorem (p. 35) in the book shows that the authors are indeed concerned with the method of justification from the "finite standpoint"; the theorem reads: "In all cases where the aforementioned prerequisites for the finite justification of the principle of the smallest number are not fulfilled, the 'tertium non datur' must be used for the integers in order to justify this principle." The emphasis on the difference between "finite content" and finite justification is rather necessary in relation to intuitionism.

Let G be the determinant of "digit", and let the following apply in addition to G: G —(n) W(n),

therefore G • (n) 91(n), G • (ii) H(ii), then *{n) 91{nj* applies and we have (n) W(ii) —(f R — fu) 91(n)) —• G}. This means not only that the assumption that G applies and G implies not (n)'lt(n) together with the assumption that G implies (n) 91(n) implies the non-applicability implies G; it also means that, given G and a given false proposition (n)'it(n), the assumption G —(n) 91(n) would lead to the conclusion that G is invalid. The only thing that is not self-evident is that such a conclusion, as a procedure to be carried out in a foreseeable series of steps, is also available to us. '• See above.

On the one hand, neo-positivists explain that the meaning of a statement is given when it specifies what must be the case for it to be true; it is easy to say that for *"p"* to be true, *p* must be the case. On the other hand, they demand that a method of "verification" must be specified, which usually means the presentation of "facts," and in formal terms, proof, for finitists, using prescribed means. This is arbitrary (cf. also Carnap, *Logical Syntax of Language,* Vienna 1934 **(1968),** p. 114). It is not a question of a method of finding the decision, but only that there is a decision that makes sense, that it is possible according to the meaning of the sentence, which is always the case if the sentence is such that it must either be true or false. One must only be able to specify what finding the decision would "look like".

^°• Cf. L. Wittgenstein, *Tractatus Logico-Philosophicus,* 4.46ff.

•'• A note was planned here. Mally apparently refers to *Principia Mathematica,* Vol. I Part II, Section E, Part **III** Section C; possibly also to Chapter VII 'Mathematical Induction' in the introduction to the 2nd edition.

^•• Mally apparently did not write the planned notes.

•°• This apparently means: 'nm means "unambiguously backwards" if every value of y that satisfies $y is assigned to a value of z that it satisfies by nry and no other'.

•• H. Scholz, *Metaphysics as a Rigorous Science*, Kamp-Lintfort 1941. Even if, in my opinion, what H. *Scholz* offers here is not metaphysics but a purely formal theory, it is nevertheless, precisely for this reason, a theory consisting of propositions that are valid "in every possible world" or "for" them, because they are independent of the empirical facts of the "world".

••• In the manuscript, there is a reference to Hilbert without further details.

°• Before "des" in the manuscript, there is probably an erroneous "in".

®' An indication of this in Carnap, 'Mathematics as a Branch of Logic', *Blätter für Deutsche Philosophie 4* 1930) 298-310.

®® A. Ambrose, 'Stipulations on Proof'.

® *Principia Mathematica Hol.* I, p. XXXIII, also e.g. H. Behmann, *Mathematics and Logic* (Mathematical-Physical Library Vol. 71), Leipzig-Berlin 1927, p. 18: A general statement can apparently be regarded as a conjunction of as many singular statements as there are things in the domain of reference, and the particular statement as a disjunction of the same singular statements.

®° The formula is of the same type and justification as the sentence "If every human being is mortal and Cajus is a human being, then Cajus is mortal"; only in it, at
Replace the class of humans with the universal class, which, according to the meaning of the respective provision *91‹x),* is to be assumed as the domain of x. Just as the sentence "If every human being is mortal, then Cajus is mortal" becomes the valid statement "Cajus is mortal" – and "Cajus is a human being" also expresses the (in the hypothetical case only assumed, not executed) judgemental designation of an individual named 'Cajus',
— thus fu) 91(x) is only supplemented by the assumed designation of a z-value o to become an implicant for 91(a). The second premise is no more "self-evident" here than it is there. The "class of humans" is replaced here by the universal class to which the set provisions can be meaningfully applied.

°' A. Ambrose. Hilbert also understands it this way, which seems difficult to reconcile with his view of the positing of being as disjunction.

• How does this relate to *Carnap's* "Language I"? Anything that falls within the realm of linguistic determination and expressibility with the means determined is always lifted out of an unrestricted, prior, all-encompassing realm — that of meanings, not "things". The formalism of "language", especially *Carnap's,* tends to disregard this. However, it must be taken into account in logical theory. (Editor's note: It is not clear from the manuscript where this note belongs.)

°° When we speak of "logical theory" here, we mean a "system of logic". The term "logic" is avoided as far as possible until the question of whether there is only one logic or whether different "logics" are to be recognised is discussed. If it does occur, it is with the understanding that its justification will be added later.

°4• The manuscript refers here to an earlier chapter without further details.

8'* Cf. note 94*.

"° In the sense of Chapter ... (Editor's note: Cf. note 94').

8' Cf. note 94*.

••• This planned second main section was never written. In the manuscript, an editor (probably Prof. Kröner) notes: "Found as the last fragment on Peace Day, 15 August 1945".

°°• The manuscript lists as an alternative heading: Truth relations. Meaning relations.

'0°• Refers to Alois Hößer, *Logik* 2. Außl. Vienna/Leipzig 1922, p. 204.

*°** Cf. note 94*.

'°°• Cf. note 94*.

'0•• "It" added by the editor. '°^• Cf. note 94*.

0' Cf. note 94*.

B. Russell, Introduction to *Mathematical Philosophy,* London 1919, p. 153. R. J. Nelson in ... *Mind* NS 39 (1930) 452.

'0® o q' means in *Nelson's work* '*p* and q are consistent', and p o p applies to every *p*, even to a 'necessarily invalid' one (as the expression 'im-possible' would probably have to be translated here). '*p* E q', 'p entails q', is explained by 'p/-9', i.e. "p excludes the negation of q", "is incompatible with it". Ibid. pp. 444ff. '0® Against E. J. Nelson, op. cit.

"°• "sufficient" is missing in the manuscript.

'11• A note planned by Mally but not executed apparently refers to *Principia Muthematica* Vol. I, p. 20f.

FORMALISM I

Elementary propositional logic

By formalism, I mean a system of rules governing the use of certain figures. Each figure is either a single figure or composed of individual figures according to specific rules. The figure is a specific shape, e.g. a typographical one. The individual case of its embodiment or representation is not part of formalism; it is only part of a case of its handling. The individual figures are chosen arbitrarily. Their design features are only important insofar as they serve to reliably identify the individual figure and to reliably distinguish it from the others in the system. The same applies to composite figures. The assertion that the figures of formalism are meaningless or insignificant is only correct in a certain sense. A figure in chess, which is often referred to in this context, is not a sign for another object or fact; it does not have the kind of meaning or significance that a word, especially a name, or a sentence in language has. But in the formalism of chess, it has a specific "functional meaning" assigned to it by the rules of its use. Through this, one piece is designated as the "king", another as the "bishop", and a third as the "pawn", whereby the meanings that these names have outside the game are, of course, irrelevant. In the game, such a name is only an expression of the 'function' of the individual piece; it indicates, as a shorthand symbol, its functional meaning. It is through this, and not through the random form of its representation, that the piece is essentially determined, just as in *Hilbert's* axiomatic system of Euclidean geometry, 'point', 'line' and 'plane' are determined only by the definitions of the system, for which the 'structures' usually associated with these names are also only special forms of representation. In the physical figure-things and in the activities of their handling, as well as in

In terms of their design characteristics, formalism, i.e. the content or meaning of its definitions independent of the illustrative, only presents itself; they are not formalism. Formalism can be described as a system of axioms. Its axioms are the determinations of the formation and handling of its figures. A formalism whose figures serve to represent and derive the propositions of a theory is called a calculus.

Here, a formalism of logic, a logical calculus, will be presented. There are several such calculi, but I know of none that is sufficiently general. For the presentation, I will use the forms that *Hilbert* and *Ackermann* use in their *Grundzüge der theoretischen Logik* (Principles of Theoretical Logic), with a few deviations. I will refrain from "content-related interpretations" of the figures in the sense of logical concepts and "operations" as presented by H. and A. for didactic purposes — without making use of them in the derivations. No other meaning shall be attributed to the figures than that *which* formalism itself assigns to them by determining their use. It will become apparent that this "functional" meaning is already the most general logical one.

The formalism to be presented here consists of three types of specifications.

The first type of stipulations determine which figures appear in formalism and are to be regarded solely as figures of formalism. These stipulations are called rules of form. The remaining stipulations determine which of the figures contained in formalism are to be regarded as 'correct *formulas'*, usually referred to briefly as 'formulas'. They do this in the following way:

The second type of stipulations list a limited number of figures as correct formulas: these figures are called *'basic formulas'* (in H.u.A. 'axioms').

The third type of definitions — called 'transformation rules' — determine how other correct formulas can be obtained from correct formulas: figures formed in this way are called 'derived formulas'.

A formalism that accepts every figure that occurs in it — i.e. every figure permitted by the rules **of form — as correct would be a borderline case of formalism and a 'game' in a different sense than a game of rules**.

rules — as correct would be a borderline case of formalism and a 'game' in a different sense than a game with rules, such as chess. Such a formalism would not constitute a calculus. If it is to be a calculus, it must have the means to distinguish between what is correct and *what is incorrect*. The specifications of the second and third types, in our case, will achieve this as far as possible. It should be noted here that the formalism of logical calculus H.u.A. does not do this. A distinction is made there between 'true statements' and 'false statements'; but the fact that certain figures signify 'statements' cannot be deduced from any formal definition, it is determined by an 'interpretation of content'.

The formalism presented here will be presented as a three-stage structure. Formalism I provides a general and elementary *'calculus of the correct'*, which corresponds to what is commonly referred to as 'propositional calculus' – but only in the sense of a content-based interpretation. Formalism II takes up I and develops a *'calculus of determinations'* beyond it; it replaces the 'predicate calculus', which is usually also understood in terms of content and burdened by a presupposition that is foreign to formalism (and foreign to logic). Formalism II is more precisely described as a logical *calculus of the fulfilment* (of determinations). Formalism **III** introduces the concept—but not the prerequisite—of existence and results in a *logical 'calculus of fulfilment'* (of determinations), containing I and II.

BASIC RULES OF FORMALISM I

A. Rules of form

Where the text refers to a figure and quotes it, it is placed in quotation marks; the quotation marks are not part of the figure.

The following *characters* shall appear in I:

1.) the *individual figures 'X', 'Y', 'Z'...,* which are referred to as *'basic figures'*

(not as 'basic formulas');

2.) *composite figures,* in which any of the 'connecting signs' appear with basic figures: *' ' ('overline'),* v ', a ', 'm'; these signs are *not* called 'figures'.

Compound figures are: 'P', P', ... the overline is here an 'improper connecting sign';

'dv *Y', 'X n Y', Ys Z', Pv Z', 'P n P', ... ,* -the overline can be placed above stand for a composite figure and then serve as the 'actual connecting sign': 'dv *Y'*, *'X n Y'*, *'X v Y v Z'*, 'Yes xv Ya *Z'*, ... *Brackets* are used, as in mathematics, to structure : '(dv *Y) n 2'*, '(dv *Y) n(Yv Z)'*,... To *save on brackets*, it is stipulated that v binds more closely than a', and this more closely than '''. For example, one writes 'dv *YmZ'* instead of '(xv *Y) xZ'*, *'X v Ys Z'* instead of '(dv F) *Z'*, *'X n Y'''Z'* instead of *'(*Y *n Y)'''Z'*.

Each figure in I can be formed by a finite number of applications of the stated or implied rules of form.

Interchangeable figures

'Y» *Y'* is always interchangeable with 'dv *Y'*, *'X n Y'* is always interchangeable with 'dv f'.

Based on these two rules, each of the figures in which '»' or ' a appears can be replaced by a figure in which only the connecting symbols v and ', with basic figures.

Symbols for figures

When the text refers to characters without naming them, German capital letters, 'R', 'B', 'C', ..., are used to designate characters. These characters are not characters in I, but *names* for characters.

B. *Introduction to the Gnind formulas of I*

The basic formulas of I are:

a) '$X \vee X \supset X$'
b) '$X \supset X \vee Y$'
c) '$X \vee Y \supset Y \vee X$'
d) '$(X \supset Y) \supset (X \vee Z \supset Y \vee Z)$'.

C. *Transformation rules*

e) *Insertion rule:* If R is a correct formula (r.Fo.) and B is created from R by replacing a basic figure in R wherever it occurs with one and the same other figure, then i is a r.Fo.

§)t *'Schluflregel':* If R is a r.Fg. and the figure consisting of 91, followed by '»' and followed by B is a r.Fg., then B is a r.Fg.

Regarding the designation. The cumbersome expression "the **piece** consisting of R, followed by '»' and followed by i8 — i.e. followed by the piece i8" will continue to be replaced by the designation 'Rci8', which is, however, incorrect. It is incorrect because 'R' and 'B' are not pieces, but names of pieces. A corresponding abbreviation will also be permitted in other cases where reference is made to pieces that are identified by their structure but not mentioned themselves.

Thus, §) is shortened to:

Q)$_2$ If 9Ici8 is a r.Fg., then the following applies: "If R is a r.Fg., then i8 is a r.Fg.".

Since 'If R is correct, then i8 is correct' is equivalent to 'G is not correct or B is correct' (see below) and, according to the explanation *'Interchangeable figures'* (p. 192), Rmi8 may always be replaced by H v i8, the following version also applies:

Q), If W v B is a r.Fg., then W is a n.-r.Fg. (not-correct figure) or B is a r.Fg.

Version §) is particularly useful for the following applications. Since all three versions are equivalent, the final rule can be briefly stated as *'Rule* Q)'.

§') Negation rule: Not every figure in I is a r.Fg. Rule

$') does not occur in H. and A.

Rule Q) deals with 'formulas' in H. and A., not, as here, with figures in general. What can be thought of as a r.Fg. (or as a n.-r.Fg.) that is not a formula will be shown by the functional interpretation.

THE FUNCTIONAL INTERPRETATION OF FIGURES1f I N I

From substitution rule e) it follows that

1.1. A figure that is a correct formula has this property for every case of substitution that meets the conditions of «); *the correctness of a formula is independent of substitution.*

1.2. The basic figures 'Y', *' Y', 'Z',...* have the meaning of *Variables',* indicators of empty spaces in a figure.

Applying the final rule Q) yields:

2.1. The basic formula a), which is a r.Fg., can be read as: 'If 'dv T' is a r.Fg., then 'Y' is a r.Fg.', b): 'If 'Y' is a r.Fg., then

'X v Y', for any x' a real function'; c) 'If 'dv *Y'* is a real function, then
Y v J' is a rational
function', etc. From b)
and Q) it follows that:
2.2. if H is a r.Fg., then W v i8 is a r.Fg. From
b), c) and Q) it follows:
2.3. if i8 is a r.Fg., then fi v i8 is a r.Fg. From
2.2 and 2.3 it follows that:
2.4. If H is a real function or B is a real function, then W v B is a real
function. From 2.2 and #), it follows that:
2.5. If H is a regular figure, then fl is a non-regular figure or B is a regular figure.

The theorem applies to any figure fi, even a non-correct one, if there is one in I. That this is the case has been established in Q'). Now we have: "If G is a r.Fg., then fl is a n.-r.Fg. or any B, even a non-correct one, is a r.Fg.", a disjunction whose second member is omitted as contradictory. We note: From
2.5 and §') follows:
2.6. If W is a r.Fg., then 91 is a n.-r.Fg.

Figures of equal correctness

For brevity, let us say that 'H is a r.Fg. if and only if i8 is a r.Fg.' is 'fl is *true-equivalent* to B' and written 'W rgl. i8'.

From H.u.A. formula (4) and formula (5) it follows:

2.7. R rgl. 91.

The following *rule* applies:

A figure does not change its correctness value ('r' or 'n.-r.') if a sub-figure within it is replaced by one with the same correctness value.

This rule follows from the basic rules, see Rule VI below. From 2.6,
by substituting f(for 9J, after e),

2.8. If Ü is an r.Fg., then fi is an n.-r.Fg.

From this it follows, since according to 2.7 91 may be replaced by R:

2.9. If W is a r.Fg., then H is a n.-r.Fg. From
2.6 and 2.9 follows:
3.0. W rgl. " is a n.-r.Fg'.

Accordingly, in 2.4, the expression 'W is a r.Fg.' can be replaced by 'R is a n.-r.Fg.' to obtain:

3.1. If R is a normal function or B is a rational function, then H v i8 is a rational function.

From §), and 3.1 follows:

3.2. H v i8 is an r.Fg. if and only if G is a n.-r.Fg. or i8 is an r.Fg.

This theorem yields the "complete inference rule":

Q+) The conclusion from '9J is a r.Fg.' to 'B is a r.Fg.' is then and only then correct ('formally correct') if fi v i8 is a r.Fg..

From 3.2 and 3.0 it follows: (using 9J for W according to «)):

3.3. R v i8 rg1. '9J *or* fI' is an r.Fg'.

4. Based on 3.0 and 3.3, every figure in I can be interpreted functionally. A *formula* (in I) has the meaning of a *relationship of correctness, and* every basic figure in it has the meaning of a *position in the relationship of correctness.*

Since H is always true with 'R is an r.Fg.', one can read: 'Y' as "Y' is an r.Fg.

'P' as 'Y' is a non-r.Fg.', or more *briefly 'X not', 'non-X'*

'X v Y' as 'Y *or* r'

'Xn Y' as 'if Y, then *Y' 'X n*

Y' as 'Y *and* r'.

'X n Y' is introduced as an abbreviation for 'dv F', which is to be read as the negation of 'Y not or *Y* not', and this is true if and only if 'Y' is true *and 'Y'* is true.

5.0. In H.u.A., and in other representations of *the 'calculus of propositions'*, it is assumed that for every true proposition there is a false one that is its negation, and the propositions are represented by figures of a formalism. The propositions on 'normal forms' (H.u.A./§§ 4, 5, 6) also specify means of determining whether a figure is 'always true' or 'always false' (or only 'satisfiable'). But that assumption and these important propositions are not part of formalism. They are peculiar to a substantive interpretation that is an *application* of formalism. The formal determinations do not compel the assumption that the figures are representations of 'statements'. They do not exclude the possibility that every figure in formalism is 'true'.

5.1. The *question of consistency* is therefore "asked in a figurative sense" in H.u.A. The question is asked whether it is "impossible, with the help of calculus, to derive two statement combinations ... which are obtained from the pair of statements Y, P when Y is replaced in the same way both times" (p. 31). This indeed proves to be impossible (p. 32f.).

This proves that a false statement cannot be derived from a true statement using the formalism assumed by H.u.A. But this is a matter of application, not of formalism itself. It lacks the stipulation that there are non-correct figures; therefore, the question of 'whether a derived formula contradicts the basic formulas', i.e. whether the stipulations for any formula fl allow us to conclude that both '91 is correct' and 'G is non-correct', cannot arise at all.

5.2. In formalism I, which contains the negation rule §'), this question does arise. It has already been decided by the proof of H.u.A. (p. 32) in such a way that one can state: If R is a r.Fo. in I, then it cannot be concluded from the basic stipulations that the negation of fl is a r.Fo.

Due to the determination of I, neither "J' is a r.Fg." nor "Y' is a n.-r.Fg." can be asserted. But each of the two truth values (R-values) can be "assigned" to the function 'Y'. However, this occurs in a temporary determination, one that is not constitutive for formalism. But it is inherent in the constitutive determinations that such assignments of R-values to the basic variables occur. It is inherent in formalism, for example, that the formula 'dv J' has the functional meaning of 'P' is a r.Fg. or 'Y' is a r.Fg.', and at the same time that of 'Y' is a n.-r.Fg. or 'J' is a r.Fg.'. That formula is regular with each of these statements, and each expresses a relationship of correctness that applies independently of substitution.

A compound function that is not true independently of substitution is like a basic figure, a variable. Its R-value depends on the R-values of its basic variables. Examples of this type are 'P', 'dv *Y'*, *'X» Y'*, *'X > Y'*.

The relationship between the R values of the basic figures and the R values of the composite figures is usually represented in a "value table". Such a table has the significance of a mathematical formula; it provides a necessary and sufficient condition that the composite figure imposes on its basic values, i.e. the R values. We therefore call a composite figure a *determination* for its basic figures. If this composite figure is correct regardless of the R-values assigned to the basic figures, it is called 'unconditionally *valid'* or *'valid'* for short. The composite figure is then a r.Fo.

If the compound phrase is independent of insertion n.-r., it is called °* *'ungültig' (invalid)* or *'widergültig' (contrary)* or *'unerfüllbar' (unfulfillable).4* No insertion makes it a r.Fg.; it is then a n.-r.Fo.

In these two cases, the compound phrase can only be called a definition *in the broader sense*.

A compound function whose R-value is not independent of substitution represents a *determination in the narrower sense,* an *actual determination*. Such a determination is not necessarily valid, but *it is fulfilable.* An *unfulfillable* determination is contradictory; if a determination is unconditionally valid, its negation is contradictory.

5.3. What is presented in H.u.A. (p. 35) as proof of the *completeness* (‘completeness in the stricter sense’) of the system of basic formulas is proof of the theorem: If R is a formula that cannot be derived from the basic formulas, then W can be derived from the basic formulas. This line of reasoning only becomes proof of the completeness of the system of basic formulas under the conditions B of I, i.e. proof of the theorem: If 91 is a formula that cannot be derived from the basic formulas a)—d), then it can be proven in I that the negation of R is a r.Fo. The set of these basic formulas cannot be enlarged by any formula independent of them without resulting in a contradiction.

6.1. *Formulas and variables*

It can be decided for each figure in I, e.g., with the help of the normal forms (H.u.A.
§§ 3—6), whether it is a correct formula, an incorrect formula, or neither.

A basic figure 'J' is not a formula. 'Y' is not an r.Fo., because if 'Y' is an r.Fg., substituting 'P' results in an n.-r.Fg., which violates the rule «); 'Y' is not a n.-r.Fo., i.e. 'T' is not the negation of an r.Fo., because if 'Y' is a n.-r.Fg., substituting 'P' results in an a.Fg. A basic figure 'J' is not a formula. If 'Y' is correct, substituting 'P' for 'J' results in a non-r.Fg., and vice versa: the correctness value is not independent of substitution. — In H.u.A., every figure is called a "formula" — (p. 54, p. 23); the correct formulas are called "provable". The limited use of the word "formula" as defined above is closer to the usual usage in mathematics.

remain.

A figure whose correctness value is not independent of substitution is a *"variable"*. The correctness value of a variable depends on the correctness values that are "assigned" to its basic figures. This "assigning" or "allocating" of an R-value (correctness value) means a determination, a "value choice", it is not one of those determinations (basic determinations) that constitute formalism, but it is part of formalism, as a consequence of its basic determinations, that one can assign an R-value to a figure that is not a formula on the basis of free assumption. These circumstances mean that a figure — R is always rg1. with the statement '9J is an r.Fg.' — if it is not a formula, cannot be "asserted" with meaning, neither it nor its negation, and that the formulas "can be asserted", the correct ones rightly, the incorrect ones wrongly, i.e. in the sense of an incorrect judgement. The 'variable' figures can be 'accepted', set 'in free position'. Even if, for example, a basic figure is given the meaning of a true or false empirical statement, it is a free choice: a substitution in which, for the purposes of formalism, only the assignment of an R value to the variable is significant.

6.2. *'Determination'*

The variable *'dv Y'* is assigned the value 'true' if and only if at least one of the basic variables has the value 'true'; it is assigned the value 'false' if and only if both 'T' and 'F' have this value. We denote the value 'true' with '9t' and the value 'false' with '9t' and represent the relationship just stated in the usual form of a truth table.

Tafel I

X	Y	$X \vee Y$

The fact that there is a conditional relationship between the R values of the composite figure and those of its basic variables — for which the example given above is an illustration — is expressed by the statement: The composite figure is a determination for the R values of its basic variables. For each composite figure in I there is a value table.

— is expressed by the statement: The composite figure is a *determination* for the R-values of its basic variables. For each composite figure in I there is a value table that contains its complete functional interpretation. Our table I has the meaning
the definition: ‘If ‘Y’ is real and r’ is real, *or ‘X’* is real and x’ is non-real, *or ‘X’* is non-real and *Y’ is complex,* then ‘dv *Y’* is a real function; and if 'Y' is non-rational and F' is non-rational, then 'dv *Y'* is a non-rational function.' It can be seen that, in general, a figure *A* consisting of *n* basic variables 'lt', 'Y ... ‘Y by the disjunction of 2” assumptions, each of which is the conjunction of n correctness or incorrectness assumptions, corresponding to the n basic variables, receives its functional meaning.

'dv Y' is an example of a consistent determination for its basic variables, 'Yes P' is an example of an inconsistent determination: no possible choice of the R value of the only basic variable occurring here satisfies it. The *contradiction* lies in the fact that 'Y P' as a determination presupposes the value 3t on the one hand and the value 9t for 'Y' on the other.

6.3. This results in the following appropriate versions

6.3.1. A composite figure (in I) is a determination of the R values of its basic variables — determination in the broader sense.

6.3.2. A determination that is consistent is called *'fulfillable'*, a contradictory one is called 'unfulfillable'.

6.3.3. A determination whose negation is unfulfillable (i.e. contradictory) is called *'unconditionally valid'* or *'valid'*, an unfulfillable (contradictory) determination is called *'invalid'*.

6.3.4. Valid and invalid determinations are correct or incorrect *formulas*. The R-value of a formula is independent of its use: formulas can only be called 'determinations' in a broader sense.

6.3.5. A determination that is not a formula – i.e. fulfilable but not necessarily valid – is a *determination in the narrower sense, an 'actual determination'.**

FORMALISM II

Logic of the fulfilment of determinations

This formalism contains formalism I and is an extension of it same; it adds new figures and rules to those of I. There are

1. *basic figures: 'X', 'Y', 'Z', ..., 'F(x)', 'G{x j', 'H(x)'...*, for which, if clarity permits, *Fx', 'Gx',...*, can also be set; instead of *'x'*, 'y' or *'z'* ... can appear in these figures; '(x) f(x)', *{Ex)* f(x)', ...

In 'f(x)', to which no "bracket" — '[x)' or *'[Ex}'* — refers, *'x'* is a "free" variable, in the other case it is a "bound variable".

2. *Connecting symbols:* v', '—', '»', ' ', '—•', '•-+', brackets.

3. *Composite figures* are formed from the basic figures using the connecting symbols, as indicated in I. One writes *[x} Fx'* for '[xj *Fx'* and '§AJ§ *Fx'* for Ax§ *Fx'.*

Naming: 'x', 'y', 'z', ... are called 'object variables'.

BASIC RULES OF FORMALISM II

A) *Introduction of the correct basic formulas*

a) ‘dv TmY’ is an r.Fo.
b) ‘JcJ vF’ is an r.Fo.
c) ‘dv Fc xv Y’ is an r.Fo.
d) ‘(xm *r)c(Z* v *JcZ* v F)’ is an r.Fo.
e) *x} F(x)*z*F(y)*
f) *F(y jm{Ex} F(x)*

B) *Transformation rules*

a) Substitution rule ii

el) If 91 is a correct formula and B is a figure that arises from W by replacing in R a basic figure at every position where it occurs is replaced by one and the same arbitrary figure — if this does not contain an object variable that was bound in G — then B is a correct formula.

e2) If R(x) is a correct figure containing 'x' as a free variable and not bound at any point, and if *A(y)* is created by replacing 'x' in R(x) with 'y' at every point where it occurs, then *9J(y)* is a correct figure.

a3) This rule, found in *Hilbert* and *Ackermann*, op. cit., p. 56f., is not essential for the functional interpretation undertaken here.

§) Conclusion rule

§) If R is a r.Fg. and R»B is a r.Fg., then B is a r.Fg.

§') Negation rule

If W is r.Fg., then 91 is n.r.Fg., and vice versa.

)) Rules for parentheses

y1) If R»fi(x) is a r.Fo., where i8(x) contains the free variable 'x', which does not occur in R, then fl»[x) B(x) is a r.Fo.

y2) If i8(x)»R is a r.Fo., where i8(x) contains the free variable 'z', which does not occur in R, then *{Exl* (•)»fl is a r.Fo.

6) *Renaming rule for bound variables*

6) If R(x) is a r.Fo. in *which 'x'* occurs as a bound variable, and 91(y) is a formula that results from 91(x) when 'x' is replaced by *'y'*, then 91(y) is a r.Fo. (The addition in H.u.A. p. 57 does not apply to the functional interpretation undertaken here.)

DI E F UN KT IO NAU MEANING OF THE FIGURES IN I I

1. The meanings that follow from the determinations in I remain unchanged.

2. From "2) and 6) it follows (H.u.A. p. 58)

Rule 6'): The truth value of a formula does not change if all its free and bound object variables are replaced by other variables, provided that only in places where the same or different variables stood before are the same or different variables standing after the replacement. Rule 6' is to be understood as follows: If

X(x, y,...) is a r.Fo., then H(u, r, ...) is a r.Fo. and 6'): If *91(x, y,...*) is a r.Fo., then its correctness is *independent* of the filling in (of its empty spaces, which indicate the variables). This means not only that the formula is correct "for any filling", but also that it is correct even without filling its empty spaces.

3. From y1) it follows (H.u.A. p. 58)

Rule y'): If R(x), which contains the free variable 'x', is a r.Fo., then *{x} A(x}* is a r.Fo.

4. According to e), if [x fl(x) is a r.Fo., then *R(x),* which contains the free variable 'x', is a r.Fo.

5. From 3., 4. and 6": (x] R(x) is an r.Fo. if and only if *X(x),* which contains the free variable 'x', is correct regardless of its value.

6. From the definitions in II, it follows (H.u.A., *formula* (33) p. 62): x] *F(x)* is a r.Fg. *if* and only *if '[£x] F(x)'* is a.Fg. Thus, '[£x] *F(x)'* is the "negation" of '[xj *F(x)'.* However, according to 5., the fact that *[x} F(x)'* is incorrect means, according to 5., that *'F(x)'* is not independent of filling in is correct; one can also say that *'F(x)'* "allows a correct filling in (of the blank 'x')". Accordingly, *[Ex} F(x)'* is a r.Fg. if and only *if 'F(x)'* allows a correct filling, and *{Ex} F(x)'* is correct in the same way as *"'F(x)'* allows a correct filling".

7. The functional interpretation 5. of *{x} F(x)'* by " 'J(z)' *applies independently of filling"* (as one can also say briefly) or *"'F(x)' applies unconditionally",* and that of *[Ex] F(x)'* by *"'F(x)'* allows a correcting completion", or briefly *"'F(x)' is fulfilable",* under 6., characterise *'F(x)'* as an expression of a *"determination".* It has the character of a proposition, is a decision in the general sense as discussed in I. However, it has the special feature of "determination for any cases or things", "determination *for objects* (regardless of whether they exist or not), which is expressed in the "blank" of *'F(*)'[5]"; an object variable 'x' *or 'y', ...,* serves only to designate this place, in its functional meaning, and is not a "filling in".

The interpretation does not say anything about whether there are "objects" to which the provisions refer to, nor can it, as an interpretation of formalism (logical calculus), say anything about this. An "existence axiom" is not an analytical proposition; it is not a component or a prerequisite of logic. This must be noted in contrast to the so-called "existential logic" of *Principia Mathematica* and similar systems; it is not logic, but a system of statements about reality, insofar as it asserts the existence of objects in all its propositions.® The Hilbert-Bernays formalism can indeed be interpreted as *a "*pure *predicate calculus"* (H.u.A., p. 90); However, the formalism-alien assumption of a non-empty "domain of individuals" is introduced, translating the symbol *'(Ex) F(x)'* as "There is (a value of) *x* such that *F(x)* holds" and '(x) *F(x)'* as "*F(x)* holds for all (values of) x"; that "sign of being", this "all-sign". To indicate that I do not understand the two parentheses in this (here unjustified) sense, I have replaced them with "square parentheses" – in **III,** I will use the "round" ones, where they are appropriate in the usual sense.

Analytical propositions are absolutely valid; thus, *' [x] (Fx* v *Fx)'* is a r.*Fo. of the pure predicate calculus: 'Fx v Fx'* is an analytical proposition.

H.u.A., formula (21) — a r.Fo. of pure predicate calculus: *‘Fx* v *Fx’* is an analytical proposition. In order to distinguish correct formulas by naming universal propositions, propositions “about all objects”,

they are referred to here, as in I, as "unconditionally valid determinations" or more briefly as *"valid determinations"*. A determination is — as in I — "valid" in this sense if its negation is *contradictory*, and vice versa. A contradictory determination becomes *"unfulfillable"*.

called; thus, *Ex) (F(x) v Fx)' *** expresses unfulfillability, whereas *[Ex} (Fx* v *)'* expresses *fulfillability*, i.e. consistency.

‘F{x) v *F(x)’* is, like every valid determination, satisfiable – which follows from the basic formulas e) and f). Examples of determinations that are "only satisfiable" — without being (unconditionally) valid at the same time — later *[F]* (£Tj (x) *(lx* v *Gx)).* All of this is completely independent of the existence and non-existence of objects, "values" of the "object variables". The propositions of "pure predicate calculus" develop only the meaning of "determination"; that meaning which results in the functional interpretation of formalism II as the meaning of its formal determinations. That the determinations themselves are consistent, and what conclusions they yield, is apparently no longer determinable.

If one objects that the determination of consistency in the given case nevertheless takes place in forms of language (the "syntax language" in Carnap) which are themselves fixed, albeit only by convention, this is not refuted by the fact that every fixation has the "general" character of a determination "for arbitrary cases of a kind," and that the individual case must always be decided in accordance with its meaning and cannot be settled again by a new determination. (Once again: at the end of arbitrariness, and above it, stands logic, which is correct in terms of meaning.)

8. A distinction must be made *between '{x} F(x)»{x} G(x)'* from '[x) *(Fx»tSx)'*. The first expression means *"If 'F(x)'* is a r.Fo. regardless of fulfilment, then *'G(x)'* is a r.Fo. regardless of fulfilment", or "If *'F(x)'* is valid, then *'G(x)'* is valid"; the second expression means *means "Fx»Gx'* is true regardless of the values assigned to x", or "*'Fx* v Gx' is valid".

It is easy to see (H.u.A. Formula (312)) that

[x] (FxmtSx)»({x} Fxm{x] C'rx),

but not the reverse: '[x) *(Fx»tSx)'* or *'[x]* (dv *Gx)'* becomes *{x} (Fx v Fx)'* through substitution, which is a r.Fo. This formula contains two variables: the base variable *'F(*)' and the "object variable" 'x'. The correctness of the formula is independent of the values assigned to both the place indicated by *'F(*)' and the place indicated by *'x*'.
shows. *'F(x)'* is a sign of a "designation for an object", an expression of the meaning "designation for an object". This meaning (this sense) is independent both of the existence of an object whose designation would appear as a value of 'x' and of the existence of a determination of a "given content" whose expression would appear as a value of *'F(*)'. The validity of the formula is solely a matter of its functional meaning (its sense): "designation for an object". This meaning alone implies that *'F(x)* v *F(x)'* necessarily implies that the negation is contradictory ("in itself"). One consequence of this is that 'J(x) v *F(x)'* is satisfiable, i.e. free of contradictions, meaning that *'[Ex] (Fx* v *Fx)'* is a r.Fo. More precisely, this last formula states that *'F(x)* v *F(x)'* is satisfiable at the position 'x'. If one wants to express that *'F(x) v F(x)'* is independent of the filling of the position

'*F{)*' and regardless of whether the position '*x*' is filled, one would have to write *{F]* x) *(Fx* v *Fx)'*, at the same time *'[EF] [x) (lx* v *Fx)'* applies and

'[EF) [Ex) (lx v *lx)'* would apply. Neither *[EF}* nor *[Ex}* indicate existence; both symbols only indicate the satisfiability of the subsequent "operand" *'F(x)* v *Fax)'* at a specific position. - *F} {EG}* [x) (v Gx).

9. As in this example, where a predicate *variable* appears instead of an object variable, in other figures a *sentence variable* (basic variable of formalism I), a 'Y', *Y'*, ..., or several of them may appear. Thus, 'P' is a sentence determination

- "predicate *for* propositions" — and could, for example, after being defined, be written as *'F(Y)'* or as 'Neg(Y)' — "negation of Y"; the predicate 'dv *Y'*, for example, as 'Imp(Y, *Y)'* - "Implication between Y (as implicate) and *Y* (as implicate)" - etc. A formula in I, with two sentence variables, is of the form *[X} [Y] F(X, Y)'* or one of the forms that arise when one of the brackets in the specified, or each, is replaced by a satisfiability sign, *'{EX}'* or *'{EY]'*.

10. Consequential relationship. Domain of meaning.

It is now possible to clearly and precisely define the fundamental relationship of succession that is used in all reasoning, and in particular in the construction and application of formalism. We declare:

F(x)—+G(x) Dt *} (FxmGx),*

where *Fax)mG(x) —-gF(x)* v *G(x)*, and read *'F(x)-+G(x)'* as *'G(x)* follows from *F(x)'*, or as *'F(x)* has *G(x)* as a consequence', *'F(x)* includes G(x)'. Instead of the 'object variables' *'x'* — or *'x', 'y', 'z',...*, when extending the definition to multi-digit determinations — basic variables 'Y', *Y', 'Z',...*, of formalism I or basic variables *'F', 'G', 'H',...*,of formalism II can occur.

The definition states: (1) A consequential relationship or inclusion is a relationship between provisions; (2) it is a special type of correctness relationship known as 'implication'; (3) its special feature within implication relationships is that its validity *is independent of* the *fulfilment* of the provisions that are its members, formally speaking, *independent of the filling* of the empty spaces in the basic figures '*F(*)', 'Er(x)' of our notation. This "independence" does not mean that, in the case of inclusion between determinations

that the correctness relation $F(x) \vee \overline{G(x)}$ applies to every permissible substitution of "values" for the variable 'x' — this is the meaning of "formal implication" in the "logic of P.M." — but *that it applies regardless of whether or not there are any "values" for this variable at all.* This is what the expression *"for* every value of *'x'"* is meant to indicate. Here, the "hypothetical judgement in the casus irrealis" acquires a certain logical meaning. If there were a person who could lift 1000 kg, there would be a person who could lift more than 900 kg. This is a logical truth that remains unaffected by the "irreality" of the case. It is not limited, as in the "existential" conception, to the content of the sentence "There is no person who lifts 1000 kg". Inclusion, consequential relationship, is purely a matter of definitions (of "concepts", even if they are not "pure" — "lifting (at least) 1000 kg" includes "lifting (at least) 900 kg") it is not only valid "in every possible world", but "for every world", whether there may be one or not. Herein lies the legitimate meaning, and in my conviction the correct one, of the much-invoked "a priori".

The case in which there is a value of *'x'* in *'F{x)*—• *G{x)'* whose substitution then provides an *"applied inclusion"* and consequential relationship between "statements" (this is the case usually considered) will be dealt with in detail later (in **III)**.

11. The implications stated (asserted) by the formulas in I and II are consequential relationships. We are justified in writing the basic formulas as follows, in accordance with the definition in 10. (and in this interpretation deviating from H.u.A.):

a)	dv T—+N	or	a)	[J] (dv Y»Y)
b)	**Y—+Tv** *Y*	or	b)	*{X}* *Y]* **(J»Jv F)**
c)	dv Y—+ *Yv* *X*	or	c)	*{X]* *[Y}* (dv *Ys Yv X}*
d)	(J» r)-+(Z v *YmZ* v *Y)*	or	d)	*{X]* *Y] [Z}* ((Yn F)c
e)	*[x} F(x}-+F(y}*	or	e)	*[F}*) (x] *F(x)mF{y))*
f)	*F(y}—+[Ex} F(x)*	or	f)	*[F} [y} (F(y)m[Ex] F(x))*

In d), as in every formula of II, the main implication sign ‘»’ is to be replaced by the inclusion sign ‘-+’, but not the ‘»’ contained in the terms of the main implication, because this does not necessarily mean inclusion and does not mean it, for example, when substituting “empirical implication”.

12. At this point, some may find it obvious to introduce "modality" terms such as "necessary" and "possible" and their negations. This presentation does without them. However, there will be an opportunity to give the terms just mentioned a clear meaning.

FORMALISM II

Logic of the fulfilment of determinations

11 November 1943

The formalism of Hilbert-Ackermann's narrower predicate calculus (which includes propositional calculus) applies, but in an "existence-free" interpretation. Instead of

'e) (x) *F(x)-+ F(y)'*
'Pj F(y)-+(Ex) F(x)'

is written (for the purposes of interpretation, i.e. to indicate the deviation)

'e} [x] F(x)mF(y)'
'f) *F(y)m[Ex) F(x)',*

where '[x) *F(x)'* means *"F(x)* applies unconditionally (i.e. regardless of fulfilment)", *"F'for* every x J(x) — regardless of whether there are values of x —", and *[Ex} F(x)'* means *"F(x)* is — at the point x — fulfilable". The predicate calculus understood in this way is a *"**pure predicate** calculus", a calculus of determinations.*

This formalism is *extended* by new symbols and rules, corresponding to concepts and propositions of the theory.

It means *'(Ex] F(x)'* as much as "There are objects ("values of *x")* that satisfy *F(x)*", "There is (at least one) satisfaction of *F(x)".* However, in the handling of formalism, in the derivation of formulas — after it has been established according to its meaning — it is of no significance. Occasionally, a comment will point out the interpretation where it seems important.

The *formal definitions* follow.

1 DEFINITIONS

DI: *{Ex) – – p (Ex) (F(x) v* $\overline{F(x))}$*,*

DI.1: *(Ex) F(x) -— p (Ex) ((Fx v* $\overline{F(x))}$ *n F(x)).*

Definition DI introduces only an abbreviation for a composite symbol. Where it appears in an expression without a specified operand, '(Ex)' will always refer to *'F(x)* v *Fax)'*.

Interpretation. - In our logic calculus, *'(Ex j'* will mean 'There is fulfilment of *F(x)'*, 'There is at least one object that fulfils *F(x)* as the value of *x*'.

Neither *"(Ex)"* nor *"(Ex)"* is a correct formula. D2:

!z= *p {Ex) (x* ——z)

Interpretation. - The symbol 'z' is understood as "designation of an individual object", "value of x". Then, according to D2, 'z' is an expression of the designation (judgement) of i as a value of *x*. - According to the remark on D1, neither '!x' nor '!x' will be considered a correct formula of our formalism.

D3: *F(i)* Df *(Ex)* (x=x. *n .F(x)) "F(x)": "F(x)* does not apply*"*

D4: +( )'Df !z *n F(x)* "f(x)": "x satisfies *Fax)"*

D5: *(x) F(x) -— g (Ex) n(Ex) F(x)*

D5, together with D1 — and the *interpretation of '(Ex)'* — gives '(x) *F(x]'* the meaning of the "universal proposition". It must be distinguished from the meaning of *{x} F(x)'*. An unconditionally valid determination is not a universal proposition in the sense of D5.

'(x) F(x)' can — in terms of "content" — be read as "Every value of x, and there are some, satisfies *F(x)"*. In short: "With every *x F(x)"*, as opposed to the reading "For every z *F(x)"* in *{x} F(x)'*.

D6: *(Ex) (F(x)* v *G(x))* ——» *(Ex) F(x)* v(Ex) *G(x).*

Correction (21.11.43)

Dl: is only the definition of *the abbreviation '(Ex)'*, in which the undefined symbol *'(Ext'* reappears in the definiens. Its *functional* meaning is derived from the basic formulas.

The definition !i=p_f (lx) *(x* ——*x)* Revealed. The term "!x" is used to define "x=x", the identity.

New count:

D2: *F(x) — — p (Ex) (x -—x. n . F(x))*

D3: f(z)=pt !z *n F(x)*

D4: *(x) F(x) ——p (Ex) n(Ex) F(x)*

D5: *(Ex) (F(x)* v *G(x)) – – p (Ex) F(x)* v (rx) G(x)

The definitions should follow the basic formulas.

---

2 BASIC FORMULAS

A: (lx) *F(x)»{Ex} F(x)*

*"*There is fulfilment of *F(x)"* implies (presupposes) *"F(x)* is fulfilable at the point 'x'", *"F(x)* is consistent".

B: x) *F(x) m(Ex) F(x)* Because *A2 B* means *An B,* i.e.: *[x] F(x) n (x) F(x).*

> The second basic formula says, in our interpretation, that the validity (unconditional validity) of a determination does not imply its fulfilment. What it denies is the statement "What is necessary is actual," insofar as 'necessary' is to be understood as meaning 'logically necessary'. No other tenable concept of "necessity" has been proposed to date. Without formula B, our formalism could be supplemented by including the opposite formula and would then coincide with that of pure predicate calculus, whereby the interpretations of DI—D6 would of course change. The significance of this fact...

This will be discussed.

Correction (21.11.43)

Basic formulas

g): !z»(rx) *(F(x)* v *F(x))*
Interpretation: ‘!x’ is interpreted as an expression of the judgement, designation (or supposed designation) of an object “ ” as a value of x.

h) *(Ex) (F(x)»[Ex} Fax)).*

i): *{x] F(x) z(Ex) F(x) ... [EF] [x] (F(x) n(Ex) F(x))* (better version)

None of our determinations presents a formula as correct that could be interpreted as an assertion or axiomatic prerequisite for the existence (or non-existence) of objects, as fulfilments in the sense domain. Only the *concept* of fulfilment is used in logical theory and developed in analytical propositions.

AB GEL EI TET E FOR ME L N

(1): !z»(lx) From D2, Dl.

The identification of an object includes the determination of a non-empty object area that belongs to the semantic range of *F(x)* v *F(x)*.

(2): !xmF(z) v f(x) From D2, Dl.

The object identified belongs to the domain of the definitions being used.

(3): f(z)c!z From D3, D2.

The predicative individual statement includes or presupposes the demonstration (judgement) of the subject object.

If *'F'* in (2) '*, which stands for any predicate, is replaced by 'f', the immediate consequence is:

(4): f(z) ! x This also follows from D4.

The predication of the negation of a determination in an individual statement also asserts fulfilment.

(5): ***F(x)»(Ex)* f(x)** From D3.

One application of (5) is the sentence: "The fact that there are cases, i.e. fulfilments of *F(x)*, is justified (proven) by demonstrating a z that fulfils *F(x)*".

(6) f(x) !x v f'(x)

From D4 by "reversal" (contraposition). - A statement about Heracles — not about a concept "Heracles" — is false if Heracles fulfils the negation of the determination attributed to him, or does not exist (nor has ever existed), if the name "Heracles" *does not designate* anything, regardless of what it *means*.

(7): *F(x)»E(x)* v *(Ex)* (x=z) From (6), D2 ®*.
(8): (x) *F(x)m(Ex)* From DS.
(9): (x) *F(x)»(Ex) F(x)* From D5.

The implication (9) is not reversible.

(10): (x) *F(x)m(Ex) n (Ex) F(x)* From (8), (9).
(11): *(Ex) F(x)»(x) F(x)* v (lx) From DS.
(12): *!x (x) (xy x) v (Ex).*

From D2 follows !z= *{Ex)* (x=z), from which and from (11) follows the theorem.

(13): *(x) F(x)»(Ex j F(x).*

From D6 follows (£x) *(F(x)* v *F(x))* —*{Ex) F(x)* v (£x) *F(x);* from this and from
(10) the theorem. — Formula (13) has a significantly different meaning than the corresponding one in pure predicate calculus.

Formulas (8), (9), (10), (13) are decisive for conclusions from universal propositions to existential propositions. Through inversion, they give rise to the consequential relationships that underlie the conclusions from existential propositions to negations of universal propositions. They are not listed here.

(14): *(Ex) F(x) n* !x *m F(x).*

From (6) follows *F(x)»* !z v f'(x); from this and from *F(x)m(Ex) F(x),* which is a consequence of (5), follows *F(x)»* !z v *(Ex) F(x),* from which, by inversion, follows the proposition.

(15) *(x) F{x) n !x m F{x).*

From (14) and (9). The proposition expressed by the formula in our interpretation is based on the application of a universal proposition to the individual object (case) being judged; the substitution of 'x' in '(x) *F(x)'* only acquires the meaning that justifies the conclusion in the individual case through the indication '!z'.

(16) ( ) *(F(x) v G(x))* > !z» *(F(x) v G(x)).*

From (15) by substituting *'F(x)* v *G(x)'* for *'F(x]'.*

(17) *(x) (F(x)» G(x)) n* !x»(F(x)» *G(x)).*

From (16) by substituting 'f' for *'F'*. A further assertion results in (18).

(18) (x) *(F(x)»* t7(x)) !z »fi(z) v G(t).

The logical meaning of this formula is the causal relationship that justifies the conclusion that "Cajus is mortal" based on "All humans are mortal" and "Cajus is a human". It is only incompletely stated in the usual representations, l°

(19): (x) *(F(x) n G(x)) =* (x) *F(x)* a *(x) G(x).*
(20): (x) *(F(x)u lS(xj)»((xj F(x)m(x}* G(x)).

The claimed theorem can be reformulated as

(x) (F(x) v G(x)) *n(x) F(x)m(x) G(x).*

Now, because of (19),

(x) ($\overline{F(x)}$ v *G(xy n(x) F(x)m(x) ((F($\overline{x)}$* v *G(x)) n F(x)),*

and

(x) ((F(x) v G(x)) *n F(xj)»(x)* (G(x) *n F(x)),*
(x) (IS(x) n F(x))m(x)'r(x),

therefore the theorem.

Here are some formulas that can be interpreted as expressions of the relationship between satisfiability and satisfaction.

(21): *[Ex] Fax) m(Ex) F(x).*

From A by reversal. "If *F(x)* is unfulfillable, i.e. logically contradictory, then there is no fulfilment of *F(x)".*

(22): [x) *Fax) m{Ex) F(x).*

From (21) with regard to {x} *F(x) [Ex] F(x).* Interpretation for (22): "If non-F(x) necessarily applies, then there is no fulfilment of *F(x)".* The interpretations given for (21) and (22) are proposed instead of the contestable phrases "What is impossible does not exist", "What is necessarily not is in fact not", and similar "modal-logical" sentences.

(23) x) *Fax) x{Ex)* f(x) From (22).

The negation of an unconditionally valid determination is never fulfilled.

(24) : *[x} F(x) n (Ex)»(x) F(x).*

From (23), by conjunctive addition of *'(Ex)'* on both sides of '⊃'.

(25) : *{x} F(x) n* (x) *F(x).*

From [x] *F(x)»(x) F(x)* and the correct formula (x) *F(x)»(Ex) F(x)* it would follow that (x] *F(x)»(Ex) F(x)* against B.

(26) : x] *F(x) n* !I » *(x) F(x)* a !z.

From (1) and (24). — If *F(x)* is a valid definition, then the universal proposition *(x)* J(x) follows from the assignment of a value to x.

(27) : [x) *F(x) n* !z » f(z).

From (26) and (15). — The theorem is based on the application of a valid definition to the individual case.

(28) : *[x} (F(x)» G(x))»((x) (F(x)»* G(x)) v *(Ex))*

From (24) by substituting *'F(x)* v G(x)' for *'F(x)'.*

(29) : (x) *(F(x) G(x))»((*) N( )»(x) *G(x)).*

From

({x} (F(x) v G{x)) n (x) F(x))»{x) ((F(x) v *G(x)) n F(x))*

on the one hand, and

(x) ((F(x) v *G(x)) n F(x)) x(x) (G(x) n F(x))*

and

{x) (G(x) n F(x))»{x) G(x)

on the other hand, it follows that

({x} {F(x) v *G{x)) n (x) F(x))»{x)* 6(x);

and that is, in other notation, the theorem.

(30) : *({x} F(x)»{x} G(x))»((Ex) n [x} F(x) m(x) G(x)).*

From

$$([x]\ F(x) \supset [x]\ G(x)) \supset ((Ex) \wedge [x]\ F(x) \supset (Ex) \wedge [x]\ G(x))$$

and

(Ex) n [x} G(x) x(x) G(x).

In contrast,

’({x] F(x) x {x] G(x))»((x) F(x) z(x) G(x))’

is not a correct formula, as can be seen from its form after conversion to Hilbert's conjunctive normal form.

ADDITIONS AND CHANGES [118]

21 January 1944

Own basic assumptions from III

g) *Fx* —• *(Ex) Fx* r.Fo.

h) *{EF}* ([x) *Fx n(Ex) Fx)* r.Fo.

Rule e): If *'(Ex) Fx'* is a r.Fg., then there is a r.Fg. *'Fx'*.

Definitions

D.: f’(z)=p, ([x] *Fx) n Fx...[x] Fx v Fx*

D.: !1=pt *{F} (({x] Fx n {x] Fx) m(Fx* v fz))

22 January 1944

Basic formulas

g) !z- *(EG)* ([x) *Fx n Fx)* r.Fo.

h) !z *Fzm(lx) lx* r.Fo.

i) *{EF]* (x) *Fx n (Ex) Fx)* r.Fo.

25 January 1944

$$!\mathfrak{x} \equiv (EF)\,(F\mathfrak{x} \wedge [x]\; Fx)$$
$$\overline{!\mathfrak{x}} \equiv (F)\,(\overline{F\mathfrak{x}} \vee [x]\; Fx)$$
$$\overline{!\mathfrak{x}} \rightarrow (\overline{[x]}\; Fx \rightarrow \overline{F\mathfrak{x}})$$
$$\overline{!\mathfrak{x}} \rightarrow (\overline{[x]}\; \overline{Fx} \rightarrow \overline{\overline{F\mathfrak{x}}})$$
$$\overline{!\mathfrak{x}} \rightarrow ((\overline{[x]}\; Fx \wedge \overline{[x]}\; \overline{Fx}) \rightarrow (\overline{F\mathfrak{x}} \wedge \overline{\overline{F\mathfrak{x}}})$$

27 January 1944

!z- *(EF) (Fx n {x] Fx)*

(!z Jz)—• (lx) *Fx*

f'z= D t !i *n Fx*

!z-+ *(Fx* v fz)

5 February 1944

A new system

Postulates: g), h), i), j); Definitions: D1—D4*°*

g) *(Fx n fx} Fx) m !x*

h) (!x *n Fx] x(Ex) Fx*

Dl *Px* - *z Fx n* !z

(1) *(Fx n [x) Fx) m[Ex) Fx* from g), h)

(2) !z *(Fz»(£x) Fx)* from h)

(2') *!x»(Fx v (Ex) Fx)*

(3) *(Fx n {x} Fx)» Fx* from (1) follows *(* *n* [x) *Fx)»(!x n Fx);*
From this and from DI, it follows that (3)

(4) $(\bar{F}\mathfrak{x} \vee F\mathfrak{x}) \equiv (!\mathfrak{x} \vee F\mathfrak{x})$ aus D1

(4') $(\overline{F\mathfrak{x}} \supset \bar{F}\mathfrak{x}) \equiv (\overline{F\mathfrak{x}} \supset !\mathfrak{x})$

(5) fz= *(Fz* v !z) from Dl

(6) *!x»(Fx v Ex]* from (4): !z»f'z v *Fx*

(7) *(Fx* > JI) !I

(8) !x» *Fx v fx} Fx* from g)

(9) *!x»((Ex) Fx* v *{Ex) Fx)* from h) according to theorems

D2 of *F (Ex) (lx* v Crx) ——» *((Ex) Fx v (Ex) Gx)*

D3 *(Ex) — — p (Ex) (Fx* v *Fx)*

(io) *!x m(Ex)* from (9) and D2, D3

(i i) *((Ex) Fx n* !z)»fx from h) by turning: *(Ex) Fx x(Fx* v !z) from this
(Ex) Fx n !x»(Fx n !z) v (!z !z) and further
(Ex) Fx n !x» Fx n !x, and because of Dl, the theorem

(12) *((Ex) Fx n !x) m Fx* after substituting 'f' for *'F'* in h), can be derived as (11)

D4 *(x) Fx — — p* (Ex) *n{Ex) Fx*

(13) (rx) *Fxm((x) Fx v (Ex))* from D4

(14) *(Ex) Fx x((x) Fx v (Ex))* from D4

(15) ((x) *Fx n !x) m Fx* from D4 and (12)

(16) *{x) Fx* *(Ex) Fx n (Ex) Fx*

(17) (x) *{Fx m* the) *!x m(Fx mGx)* from (15) by substituting
‘*lx* v *Gx’* for *‘Fx’*

(18) (x) *(Fx»Gx) n !x»(Ex* v *Gx)* from (17) and Dl (18) is more meaningful than (17)

(19) (x) *{Fx n Gx)= (x) Fx n (x) Gx*

6.2.1944

(20) *(x) (Fx» Gx)»((x) Fx»(x)* Gx)

i) *(Ex) Fx»[Ex} Fx*

(21) [£x) *Fx»(Ex) Fx* from i) by turning [x)

(21') *Fxm(Ex) Fx* from (21)

(21’) *[x] Fxm(Ex) Fx*

(22) [x) *Fx n (Ex)»(x) Fx* from (21’) after conjunctive addition of ‘(£z)’ on each side of ‘o’

(23) [x) *Fx n !x m(x) Fx* from (22) and (10)

(24) (x) *Fx n !x» Fx* from (23) and (15)

(25) (x] *(Fx» Gx)»((x) (Fx»* Gx) v(Ex)) from (21‘) and (13)1°‘

(26) (x) *(Fx» Gx)»((x) Fx* m(x) Gx)

(27) ( ) *Fx* §x§ *Gx) ({Ex) >* § *Fx* *(x)* Gx)

7 February 1944

Designations

If (x)R(x) is a rational function and R(z) is a rational function, then 'z' is called a *"correctly defined function", "rational function"*, for 91(x) — with respect to the position indicated by 'x'; 'z' is called an "individual character", as are 'p', '3', ... If [x) R(x) is a r.Fg., and therefore R(z) is also a r.Fg. — because in this case G(x) is r. regardless of the filling — then 'x' cannot be called a "r.m.Ausf." for R(x) in the true sense and may then be called a *"correct filling", "r.Ausf.", for B(x).* A function that is r.m. or r. is called an *"at least correct function", "min.r.Awf."*.

Basic rules

Rule ej: If 91(x) is a function that contains the free object variable 'x', and *(Ex) X(x)* is a real function, then there is an individual symbol that is a minimum real execution for R(x) at the position 'x'.

Rule q): If fl(x) is a regular function containing the free object variable 'x', and if R(x) ch is a regular function where B does not contain the variable 'x', then *(Ex) A(x) z B* is a regular function.

9.2.1944

Extension to determinations:

Rule t'): If *91(F)* is a function that contains the free determination variable *'F'* - or 'f( )', *'F(x)'* - and *(EF) A(F)* is a r.Fg., then there is a determination individual character — Bestg.-Ind.-Z. - which is a min.r.Ausf. for *91(F),* at the position *'F*'.

Rule 9): If *X(F)* is a random function that contains the free determining variable *'F'* - *'F(*)', *'F(x)',* and if *R(F)mi8* is a random function where B does not contain the variable *'F'*, then *(EF) A(F)»B* is a random function

(28) *(EF) (Fx n* [x] *Fx)»* !x from g) and q')

j) !x»(£F) *(lx* [x) *Fx)*

(29) !i- *(EF) (Fx n [x} Fx)* from (28) and j)

(30) !z- *(F)* (Ü v [x] *lx)* v *(EF)* from (14) and (29)

(30') !z- *(F) (Fx m [x} Fx)* v *(£F)*

NOTES (Formalism I, II, III)

D. Hilbert-W. Ackermann, *Grundzüge der theoretischen Logik* (Fundamentals of Theoretical Logic), 2nd edition, Berlin 1938; hereinafter referred to as "H.u.A."
•• 'jene' was added by the editor.
°* A short word in the manuscript is illegible.
^• A short word in the manuscript is illegible.
• The terms 'valid' and 'invalid' are used here as in R. Carnap, *Logische Syntax der Sprache*, Vienna 1934, p. 126f.
°•• 'is expressed' was added by the editors.
• Cf. also B. Russell, *Introduction to Mathematical Philosophy*, Munich 1923, p. 270.
• The manuscript reads: '(6xJ *{F(x1 F(x))'*, but it is clear from the context that Mally cannot have meant this negation of the contradictory proposition.
® 0. *Becker* has applied formalism II, which he calls the "classical (logical) calculus", to a modality logic: 'Das formale System der onto-logischen Modalitäten' (The formal system of onto-logical modalities), *Blätter für Deutsche Philosophie* 16 (1942) 287—422.
®•• It should probably read: (3).
•• In the manuscript, D2:(4) appears instead. '° Cf. H.u.A. p. 82.
'1• This heading was added by the editor.
l°• This heading, beginning with 'A new system ...', was added by the editors.
'°• 'from (21$ and (13)' was added by the editors.

LETTER

To Dr. Laurin l‘ Schwanberg, 14 November 1943

The finished child is a formalism of general logic ... A particularly characteristic formula of "existential logic" is:

(x) *F(x)= (Ex) F(x j,* its counterpart:

(Ex) F(x)= (x) J(x).

The first reads, *"*Not for all x (does) *F(x) apply*" is equivalent to "There is (at least) one x for which non-F(x) applies," and the second reads, "There is no x for which *F(x)* applies" is equivalent to "for all x non-F(x)". *These are not logical propositions.* For they presuppose (but do not state) that there *are* things x, "values of x" at all, and that is not self-evident; i.e., that things exist is not a logical, analytical proposition, but a matter of experience. I will now write the above formulas somewhat differently, namely as follows:

$$[x]\, F(x) \equiv [Ex]\, F(x),$$
$$\overline{[Ex]}\, F(x) \equiv [x]\, \overline{F(x)}$$

and *interpret them as follows:* 'The fact that *F(x)* is not analytically valid – or that *F(x)* is not logically necessary – is equivalent to the fact that *F(x)* is satisfiable – that the assumption non-F(x) is not inherently contradictory' – or also

“That *F(x)* does not have to apply is equivalent to saying that *non-F(x) can* apply*"* (but the first version is more precise). The second: "That *F(x)* is unfulfillable (contradictory) is equivalent to non-F(x) necessarily applying (having to apply)". With this interpretation, the formulas are correct: expressions of pure "self-evident truths" or "tautologies," as befits logical formulas. They are no longer dependent on a prerequisite of existence, do not deal with things, *do not* deal *with the fulfilment* of any determination *F(x)*, but *enter* only *the determination itself* (the conditions of fulfilment and *non-fulfilment).* Now the whole "classical logical calculus" can be i.e. to paraphrase the entire formalism of the prevailing "existential logic" (which is not pure logic) in this way — with "[x)" instead of *"(x)", "{Ex}"* instead of *"(Ex)"* and, which is *of* course *the* only correct *interpretation.* In this interpretation, it results in a *"pure predicate calculus", a "pure determination calculus".* This is the first fundamental observation: (I). This calculus yields an *essential part* of *logic* that is unknown to "classical logic". Of course, there has always been talk of "necessity" and "possibility", and a theory has often been developed on this subject under the name of "modal logic". But the prevailing exact logic believed it could do without them. Above all, however, the *connection* with the logic that was being pursued *was not clearly established*. It is clear that logic, as *general logic*, needs a theory of fulfilment in addition to the theory of fulfilability. It is not its business to assert (or deny) fulfilments, to establish or deny reality; but it must analyse the *concepts* of fulfilment and non-fulfilment, i.e. develop the *logical* laws governing the assertion and negation of fulfilments, and establish the connection between assertions of satisfiability (and their opposites) and assertions of fulfilment (and their opposites).

And now to my observation (II). The conventional statement "What is necessary is also actual" is inaccurate and contestable if "necessary" is to mean "logically necessary". What is "logically necessary", i.e. *the validity of a determination* such as "x = x", *"x' —- x or*

x'4x", "F(x) v F(x)" and the like, *has nothing to do with actualities.* There need not be any things as values of x, no fulfilments of *F(x),* and logic itself cannot determine whether they exist. An *unconditionally valid provision does not have to* be a fulfilled provision, from which it follows that even a *fulfilable provision* is independent of fulfilment. Example from the realm of purely geometric provisions: If we say that 'Ku(x)' means 'x is a sphere' and 'Ro(x)' means 'x is a body of revolution', then 'Ku(x) c Ro(x)' is *unconditionally* valid, and I can write: (x§ (Ku(x) m Ro(x)), which is equivalent to (x§ (Ku(x) v Ro(x))
"x is either not a sphere or it is a body of revolution" - "if x is a sphere, then x is a body of revolution". This is what "classical

calculus" as a proposition *"about all real things":* "A real thing is either not a sphere or it is a body of revolution". Let us assume (for which there is good reason) that a real thing is never an *exact sphere,* then the proposition is always true, because in the

disjunction '$\overline{Ku(x)}$ v Ro(x)' the first term applies. The "necessary" would be "actual", the *absolutely valid* determination Ku(.x) cRo

(x) would *always be fulfilled,* because every thing would be a non-sphere (which, according to the theorem, does not mean that it must be a body of revolution, but may be anything it wants to be, salva veritate of the theorem).

On the other hand, it must be said that there is possibly (and almost certainly) no object in reality that is a perfect sphere, which is true; but there is also no object that precisely fulfils any other geometric definition

, a case of Ku(x) within the realm of geometry. However, "Ku(x)" can only ever be understood as *a definition of this same realm* to which Ku(x) belongs. In this realm

, pure geometry, the following applies

(x) (Ku(x)» Ro(x)) or

[x] ($\overline{Ku(z)}$ v Ro(x))

A determination of the type $\overline{Ku(x)}$ is always a geometric determination that is incompatible with *Kti(x),* e.g. 'x is a cylinder' etc. would fall under this category

, i.e. representing a "type", an implicans, of $\overline{Ku(x)}$. Every calculus presupposes that the negation of a determination it considers belongs to the same domain of meaning as the determination itself. In geometry, there can be no talk of cooking dumplings.

It follows that if a determination is not fulfilled — such as our Ku(x) — its negation — our Ku(x) — does not necessarily have to be fulfilled. *Both can remain unfulfilled;* whether they remain so, or whether one of

is fulfilled is a matter for reality alone and not for

deciding based on logic. "Necessity implies actuality" is a false statement. — It is entirely in line with the development of my basic approach that I arrive at these conclusions. From the outset, the distinction between *questions of meaning and questions of being* was decisive. Questions of meaning are, in this context, questions about what a purpose includes (in terms of meaning), what it excludes, and what is achievable and what is unfulfillable according to the meaning of a determination. This is completely independent of *fulfilment.*

And now *general logic,* which contains as its *first part* the logic of fulfilment, i.e. pure determination logic, but as its *second* part (without ever asserting fulfilment) examines the fulfilment propositions in their meaning and (also) in connection with the fulfilment propositions.

I have now completed this part (after establishing that the *first* part results from the specified reinterpretation of "classical calculus", which has already been accomplished) ... Following on from this, we can examine the formalism, which is defined in six definitions and two basic formulas, and enjoy seeing how further propositions emerge from it. I have derived 27 of the most important ones; of course, one can deduce endlessly, but everything lies in the basic formalism. Its definitions are, of course, only *definitions of notation, axioms of formalism, not* axioms of logic, which for its part remains free of any axiomatic definition. Because it consists entirely of "tautological", i.e. analytical, propositions. But choosing a formalism *of representation* has the advantage that (1) one is more precise and (2) one is more confident in developing further propositions — which can become quite difficult as the complexity increases. Indeed, as in mathematics, without such an aid, one could not go beyond the most elementary propositions. -

Here, in the purely logical sense, you can already see how the basic principle that reality as the fulfilment of determinations is an extra-logical matter applies strictly. At the same time, however, the transition to the doctrine of reality can be seen: it will have to begin with the fact that reality does not fulfil any precise determination (as logic assumes and considers it), namely, it does not fulfil it strictly, but rather "lives" in an unfinished state of being seized, whereby the applicability of precise determinations in the sense of approximation results. But I am not there yet. There is still much to be done in the purely semantic sense. Everything has become bigger than it first appeared.

I am still compiling an overview for consideration, evaluation and selection of names. What I have referred to "above" as "Part I" is elementary propositional logic; I am now separating *this* out and calling it I. *So:*

General logic (or simply: *logic)*
I. Elementary propositional logic
1) Formalism 2) Interpretation
II. Logic of satisfiability (of determinations)
(Pure predicate calculus)
1) Formalism 2) Interpretation
III. Logic of fulfilment (of determinations)
1) Formalism 2) Interpretation

Each subsequent part takes up the concepts and sentences of each previous part and adds new ones: structure.

To Dr. Laurin Schwanberg, 1
December 1943

One question kept cropping up last week, once during work and then in conversation with Krug°, and I also remember that it haunts Kerns°*'s lengthy discussion letter (which I still haven't read through). But I already know the answer, at least in principle. I will briefly elaborate on this, following on from the long theoretical letter from

recently4'... It states that *(Ex) (Fx* v *Fx)'* is by no means a logical proposition. It may be, i.e. the assumption is free of contradiction, that a given determination *Fx* is not *fulfilled*, but its negation *Fx* is

. As an example, I cited the geometric definition "Ku(x)",
“Let *x* be an (exact) sphere”. We find nothing in reality that is a sphere, and nothing that fulfils a geometric definition incompatible with Ku(x) of the type Ku(x). If *Fx* is a geometric

determination, then we must assert *(Ex) (Fx v Fx)*. It follows that

, however, does not follow *(lx) (Fx* v *lx)* or, which would be the same thing, *(Ex) (Fx n Fx),* or (£x) *{Fx n Fx).*

It cannot be said that *there exists anything* that is neither non-spherical still be a non-non-sphere (i.e. a sphere); this would violate the law of (excluded) contradiction, because it would mean that it would be both a non-sphere and a sphere at the same time. But what do we do with "real things" then? Are there not conical balls and well-rounded dumplings that can be said to be "approximately spherical"?

Shaped things" and billiard "balls" and others that are even closer approximations? And surely other things, such as dice or needles, which are not even remotely spherical? In short: what are we to make of the "things of reality" and what of the approximate application of precise definitions and thus of logic to them? The question is poorly phrased. It does not ask thoroughly enough. For since it asks "about these things" and what to make of them, or what to say about them, without examining the assumption that talking "about these things" is justified — that is a genuine prejudice. But if one asks philosophically, one must ask thoroughly. And I have already acquired this questioning; it appears in the 1935 book Zauberbuch 5, and is handled more sharply in the 1938 book Wahrscheinlichkeitsbuch. And now even more sharply. But how must we take care that what we have achieved does not slip away from us! Nothing is stronger, more convenient, more useful and more dangerous than habits of thought. And language is their powerful guardian. So I reflect again and ask: To what extent is our talk "about the things of reality" justified? What is the lasting meaning of our statements about reality? (This is already stated in the book on probability, but in the context of "general logic" the question takes on greater significance.) I must now say: the experience that it is not possible to find anything real that precisely fulfils strict definitions leads to the conclusion that what is logically possible...

In fact, the case of non-existence of "objects" in the strict sense, as logic understands the word, does indeed exist. More precisely: the assumption of the existence of objects as precisely and unambiguously defined is zero-probable ("infinitely improbable") in relation to all experience. But as soon as we "talk about things", we make claims to the correctness and justification of such judgements in statements "about them"; in judging, we submit ourselves to logical lawfulness and must tolerate that our words are taken at face value. We must therefore logically say: our statements "about things of reality" are not to be understood literally. And I propose to declare: when I speak "of things," I do not claim that things exist as fulfilments of precise determinations, and that they fulfil these and those determinations, e.g. spatial-temporal ones. But I experience reality, or reality experiences itself and becomes clear as a meaningful striving, and its meaning — the "direction", not a

The objective goal of this endeavour is expressed in my statement of reality. At the same time, since I make use of more vivid or illustrative "empirical determinations", this expresses a *way of fulfilling* that pure direction of meaning. Herein lies the vagueness and the "approximation", as in the coarser or finer line drawn on paper, the supposed straight line in the sense of geometry. A more precise, sharper and "logical" meaning is expressed in the ways in which the phenomenon is fulfilled, which also includes the statement of reality. *It is to be understood as such an expression.*

However, expression is understood by clearly experiencing that which is being expressed, that meaningful, meaningful reality ("directed striving"), so that a second clear expression of it arises in this experience of understanding.

The meanings of reality expressed in statements are now more or less essential to reality, the ways in which they are fulfilled more or less clear ("transparent in meaning"); this determines the rank of the statement in the order of meanings, and thus its justification. Please note: there is not only theoretical meaning. In a theoretical-practical context, a rather imprecise statement ("about dumplings") may well be justified and more appropriate than a physically very "precise" one, i.e. one that is theoretically — and in a certain sense — more accurate. Fundamentally, however, none of them should be understood as statements "about" something; each should be understood only as an expression of aspiration and meaning. The same applies to my statements "about our speeches" that I am making here. I must make use of language and its traditional forms, even in linguistic criticism and interpretation. So we need to continue using it, but understand it correctly! Always as an expression.

As an expression of meaning, without regard to reality, logical language must also be understood and interpreted according to what it says "about determinations, statements, fulfilments". Here there will be something to say "about determinations in the logical sense". More on that another time. Questions still arise. There is still work to be done on them.

However, once again, reality is not an object and not a collection of things and cases. But the fact that language has developed in such a way that in our speech "reality presents itself as a thing" is probably due to the fact that, experiencing striving in ways of fulfilment and becoming absorbed in it, we "take for fulfilment" what is not

: instead of directed striving, we take "objective goals". And as soon as we want to grasp "the goal" precisely, we always encounter difficulties and impracticabilities. Taking everything even more precisely does not help thoroughly; it only helps to reflect on the meaning.

NOTES

• Dr Gertraut Laurin, librarian at the Styrian Provincial Library in Graz.
•• Dr Josef Krug, secondary school teacher in Vienna, co-author of the textbook for introductory philosophy lessons (cf. bibliography p. 331, no. 18).
•• Dr Fritz Kern, universal historian in Bonn (d. 1950), who worked for decades on an unpublished philosophy.
^* Cf. p. 2l9ff.
•• Humorous term for "experience and reality" (cf. p. 325).

ERNST MALLY

ERNST MALLY

AL PRINCIPLES OF THE OBLIGATION TO ACT

ELEMENTS OF THE LOGIC OF THE WILL

FOREWORD

In 1919, the word self-determination, which was on everyone's lips, prompted me to attempt to form a clear concept of the word. Of course, I soon encountered the difficulties and ambiguities of the concept of oughtness: the problem changed. As the basic concept of all ethics, the concept of oughtness can only provide a useful foundation for its structure if it is defined in a system of axioms. I present such a system of axioms here.

Will and judgement aim at facts, will in particular at the factuality of something real, in which the will itself plays a decisive role: a factuality that, at least in the sense of the will, ought to be. Judgement and will are materially correct when they correspond to facts. The laws of their material correctness prove to be strictly conforming; the logic of judgement is joined by a logic of will or deontics. The fact that we think in terms of determinations and the incompleteness of comprehension that this entails means that, although material correctness is always striven for, only formal correctness — in which this striving takes place — is actually required. I believe I have demonstrated the essence of this correctness, which is at the same time the necessary and sufficient condition for value and oughtness that can be grasped purely intellectually. The essence of value and that of oughtness, of course, remains accessible only to direct, intuitive comprehension that makes use of emotional presentation. With this in mind, one can arrive at an exact, pure ethic on the basis of our laws of oughtness without resorting to unauthorised rationalisation.

In my presentation, I have reduced the purely formal part to the smallest permissible extent and have also designed it in such a way that, in the end, no one is forced to follow formulaic derivations in order to understand the work. In addition to serious external reasons, the main internal reason for the limited consideration of the literature was that

I wanted to provide only a positive foundation for the time being. So I have limited my citations almost exclusively to the writings from which I learned the essentials for this work — it is not surprising that these are mainly works by my esteemed teacher —; mere agreements are rarely pointed out and polemics are avoided altogether.

Graz, 13 September 1925

INTRODUCTION

There is a risk involved in judging and deciding: the facts may prove us right or wrong. Those who judge correctly and those who will successfully have hit the mark; their behaviour is proven by the facts. It is in harmony with them in a way that goes far beyond the obvious fact that everything that happens is in accordance with the laws of fact — because it is indeed fact. Nevertheless, we do not rate such merely external proof very highly, either in judgement or in volition: both may have been blindly correct. On the other hand, even if a judgement, such as a diagnosis or prognosis made to the best of one's knowledge, fails to be proven correct, this does not alter its value in terms of logical justification; and a desire may miss its target and yet find and deserve recognition as a correct and right desire. If, on both sides, justification is not bound to success, it is certainly not unrelated to it: logically correct judgement and correct volition have a very significant relationship to success; by virtue of their very nature, they possess an inherent tendency to succeed, and their justification is based on this. If what is inwardly right proves to be wrong and what is inwardly wrong proves to be outwardly right, we attribute this to chance and do not change our assessment. The value of justification transcends risk; it carries with it the courage of conviction and will that fears no outcome.

Judging and willing are specific ways of taking a position on objects. There are conditions for the correctness of judgement, laws of correct judgement, which must be called objective because they have their basis in the nature of judgement as this particular type of stance towards objects, and therefore, given the nature of the stance, depend only on the nature and essential relationships of the objects. It has long been customary to assign them to *logic*, finding them in more recent exact logic and clearly articulates these entirely rational, strict laws, distinguishing them with certainty from the approximations to certain regularities in the actual occurrence and course of correct thinking, as psychological observation may empirically determine. We have not yet reached this stage in our understanding of the will. The will is also a specific stance towards objects, and it is clear that here too there are essential laws of correctness: laws of correct willing, which have their basis in the nature of the will as this specific type of behaviour towards objects. And it is clear that these essential laws of right willing are objective and rational in the same sense and for the same reason as the logical ones, and that they must be sharply distinguished from all empirical laws of a psychological nature that are only approximately valid. Hardly anyone would seriously and clearly consider replacing the laws of logic with observations about how people really think under certain conditions — however important and informative such investigations may be in their own right and especially for logicians. But there is still far from similar clarity and unanimity about the meaning of ethics. It is not the worst thing that there is disagreement about whether ethics should determine rational laws of right volition or empirical findings about right volition, or perhaps only what is considered right, or perhaps both. But the rational laws of volition have hardly ever been developed with sufficiently clear and simple intent, and even less so with any useful success. This task is undertaken here. The logic of thought is to be complemented by something that can be called a logic of volition; but since it is not a subfield of logic — such as the logic of the concept or the logic of judgement — but rather the essential laws of behaviour towards objects that is not thinking, this counterpart to logic might be better given an independent name, such as *deontics*. The relationship between deontics and ethics will be easier to assess later.

In order to recognise the essential laws of judgement and those of volition, one must first consider the objects to which these behaviours are directed. One can, of course, judge any object, but the objectivity that is initially and essentially conveyed by a judgement

In the strict sense, *what* is judged (about any object) is of a fairly uniform nature: we judge that something is or is not, that something is or is not, or something that can essentially be reduced to one of these main forms. What we grasp through judgement is therefore the being or non-being, the being this way or not being this way, of something; in short, a *state of affairs*. But it is also facts that our will is directed towards: we want something to be, not to be, to be this way or not to be this way. At least, that is how one can express it if one disregards, as in the case of judgement, finer distinctions in the form of the fact, which are irrelevant to our investigation.

Facts are concrete entities, objects, as one might say in the proper application of this word, with clearly defined characteristics that come to the fore in a set of common properties, general laws of facts. Since an object of any kind is naturally characterised by the facts that apply to it as its properties, the laws of facts also generally specify the properties of the properties of any objects and thus apply to all kinds of objects: directly, insofar as they are facts, or indirectly, since they are always carriers of facts. In this sense, therefore, the laws of facts are laws of the most general kind. They apply in their own way to any arbitrarily defined subject area and are thus facts with which science, which selects such a subject area for its work, must in any case reckon, whether it explicitly observes and establishes them or not: the most general and primary prerequisites.

It is now understood that logic must begin with a general *theory of facts* or take it as its basis. After all, it aims to specify the general laws of correct thinking, and all thinking is the comprehension of facts and, if it is to be correct, must be in accordance with the laws of facts. Whether one describes these laws as logical, as is usually the case, or bases them on logic, as a theory of thinking, in a separate theory of facts

- that forms a major part of general object theory — without including them in logic is a matter of delimiting logic and is of secondary importance.° In any case, the laws of facts are distinct from the laws of thinking.

to distinguish between them, even though, if one is to think in accordance with them, they are in a certain sense laws for thinking. Since willing, insofar as it is right, must also be in accordance with them, they are at the same time laws for willing, although they are as little specific laws of willing as they are specific laws of thinking. If we assign to logic the specific laws of thinking and to deontics those of willing, then the general laws of facts fall to a separate theory that provides a common basis for both by dealing with the objective, towards which thinking and willing, each in its own way, are directed.

I. FUNDAMENTALS

§ 1. PRELIMINARY STATEMENTS

The prerequisites of deontics, which do not belong to it itself, are therefore the concepts and propositions of the theory of facts. I will not present them in detail, but will only refer to what is indispensable, partly immediately, partly where it is needed.

I. Facts as objects of volition

Neither judgement nor volition can be directed at completely arbitrary circumstances. In order to arrive at a clear distinction between the possibilities, it is advantageous to differentiate between two types of circumstances. A few examples may help to illustrate this. The fact that the Thirty Years' War ended in 1648, that gold is heavier than iron, that 2 + 3 = 5, are very *specific facts*, as are their negations, the facts we think of when we deny them, or, for example, that the Earth is larger than the Sun, that the number of prime numbers is finite, and so on. Every definite fact is either a true *fact* or a false *fact,* and therefore a judgement that asserts it is either true or false. — However, in addition to specific facts, there are also those that lack the character of certainty. As I write this, I am making the judgement that "tomorrow is Monday", and I have thus grasped a specific fact, namely a fact. If I were to assert the same thing tomorrow, I would also be grasping a definite fact, but this time a non-fact, and accordingly, in every case where "tomorrow" refers to a definite, present "today", such a judgement will grasp a fact or a non-fact, each time a definite fact. But if, without such a reference to a given case, one merely considers the meaning of the sentence "Tomorrow is Monday" thinks — as happens, for example, when we encounter the sentence in a grammar book as an example —, he certainly also thinks of a fact, but one that, in contrast to those considered earlier, has a certain vagueness attached to it; we call it an *indefinite* fact. The facts that constitute the meaning of the following sentences — forms of utterance, but not utterances [4] — are of the same kind: x is (be) red, $x < l$, $x + y$ ——*5*. It can be seen that the indeterminate facts are at the same time *determinations* for the indeterminate or variable elements that occur in them. Obviously, such an indeterminate fact is also indeterminate with regard to its factuality: it is neither fact nor non-fact. Let *A(x), B(x)* and the like be signs of a *determination* for the variable or indeterminate x.

If we now consider the behaviour of judgement and volition towards these two classes of facts, we see first of all that indeterminate facts can neither be judged nor desired. It is impossible because it makes no sense to assert or even to suppose, and equally impossible to desire, that *x* is red or that $x<y$, as long as *x* and *y* are indeterminate objects, variables. However, in "x = x" an indeterminate fact seems to be asserted, and in a demand such as "let (x) do its duty" such a fact seems to be desired. Meanwhile, what is judged there and demanded here is not an indeterminate fact, but an arbitrary one — left indeterminate, which — and thus implicitly every *case* of determination.° To judge "x= x" for any case, to demand "x do its duty" for any case, is correct because the general judgement "for every value of x, the determination *x* — — *x* applies" is correct, as is the general demand "for every *x* for which the determination 'x does its duty' makes sense at all, it should be fulfilled". However, the idea "for every x, x= x applies" obviously covers a very specific situation; the indeterminacy of x is, so to speak, eliminated for it, because it is no longer a variable value of x that is considered, but the entire range of values for which the determination makes sense at all. And correspondingly in the general requirement. The particular judgement of the form "for some (at least one) x, the determination *B(x)* applies" and likewise the corresponding requirement also refer to a specific situation, despite the x that appears in it. For such a judgement asserts as much as "there is at least one x for which *B(x)* is fulfilled".

the requirement amounts to "there must be at least one such x". Thus, indeterminate facts are not accessible to judgement, volition or desire.

There is now no discernible limitation to judgement; it can encompass specific and apparently *all kinds of* specific facts. With volition, it is not quite so simple; indeed, there seems to be a difficulty here. That gold is heavier than iron, that 2 + 3 = 5, are certainly specific facts, but they cannot be willed because they are facts anyway, nor can facts that are incompatible with them, because they are non-facts. Now, however, every definite fact is a fact or a non-fact, and so it might seem that we were wrong to deny that volition has indefinite facts as possible objects, because now it seems that there are none at all. A somewhat closer examination of the circumstances of volition brings clarity. Caesar decided to cross the Rubicon. That Caesar crossed the Rubicon in 49 is indeed a fact, and indeed a timeless one; nor was it an indeterminate fact when Caesar first contemplated the act. Just as the judgement that captures this fact is "always" true, or more correctly, true independently of the time of judgement, so the fact itself is independent of any time, "always" a fact. If two observers of earthly events had bet against each other before the event whether it would occur, one and only one of them would have spoken the truth, not something that only became true. So it is indeed a specific fact that is the object of volition. In fact, when Caesar had made up his mind, indeed at the moment of deciding, he could also judge: I will cross the river. Not with complete certainty, of course, but with an approximation of it and with a subjective certainty that was all the more complete the more certain he was of his cause. Of course, he could not do this before making his decision. But not because the fact was uncertain at the time, but because it was not yet known to him with sufficient certainty. The facts to which the will is initially directed, which are thus predetermined for the will, are defined as facts or non-facts, but are not grasped in this certainty; no position has been taken on them, since they are thought without any obligatory element of conviction, without judgement, i.e. they are merely *assumed*. It is true that facts can already exist before the will enters into play with conviction, but then only to be judged in a reduced sense, i.e. in the sense of presumption: we may want to bring about something that we believe would happen even without our intervention, but we want to "ensure" that it happens. In any case, what we want is a *specific* situation, only conceived in a particular form, namely as *the actualisation of a determination* – which is then also called a requirement – in a specific case, in some cases, in all cases of a specific kind. But these cases are not grasped in their complete determinacy, not in such a way that the applicability or non-applicability of the required determination in the sense of certain judgement could be read from the perceived given determinations. And here is another essential point: we believe — at least implicitly — as willing beings, to supplement the given determinations of the case (or cases) in such a way that it now also actualises the required determination. The willing person, who does not will unsuccessfully, is, through his behaviour, *co-constitutive* of the case he has in mind and in which the desired determination is fulfilled. It is irrelevant whether the psychological act of willing itself or something else, of which it is the experiential expression, plays the role of the partial cause in which this co-determination is realised.

2. *General object-theoretical prerequisites*

Anyone who studies the laws of thought or volition cannot do without the concept of consistency. However, this concept presupposes — something that is often overlooked — a certain relationship of a purely objective nature between facts: the relationship of *implication.* If *A and B* are facts — capital letters in our symbolism always denote facts — and if the relationship is *"if A* (exists), then *B",* we say *"A implies B"* and write

$$A \quad B.$$

Here, the "if-then" is understood in its broadest, most undemanding sense. "If *A*, then *B"* should mean no more than "it is not *A* without *B"*, "that *A* exists and *B* does not exist does not exist", whether it be that *A* is not possible without *B*, or that *A* simply does not actually exist without *B.* An implication can be rational or a priori recognisable, such as the relationship "if 864 is divisible by 9, then 864 is divisible by 3".

However, it can also only be determined empirically, such as "if Peter comes today, he will be at my place at 5 o'clock" or "if an electric current flows through a conductor, heat is generated in the conductor". The first example provides further examples of implications.
— and indeed rationally recognisable ones — if one replaces 864 both times with x or with "a number"; then instead of an implication between *definite* facts, one has one between *indefinite* facts. Another correct example is: "if 17 is divisible by 9, then 17 is divisible by 3", which, as we said, means nothing other than: "that 17 is divisible by 9 and not divisible by 3 does not exist, is untrue", and that is true. Our thinking even makes fruitful use of similar cases of implication. For example, we establish that 865 is not divisible by 3 because the sum of the digits of 865 is not divisible by 3, and in doing so we have tacitly or explicitly recognised the relationship: "if 865 is divisible by 3, then the sum of its digits, i.e. 19, is also divisible by 3" — precisely because this is not the case, we recognise that the former is not the case.

The relationship *Am B*, i.e. "*A* is not without *B* being", obviously always applies when *B* is a fact, because no circumstance *A* can exist without the facts existing; it is not possible for *A* to be, but for it not to be a fact. In fact, if anything is, then the facts always are; they are implied everywhere. — The relationship *Am B* also always applies when *A* is not factual. Whatever circumstance *B* may mean, it is always true that "it is not possible for the non-fact *A* to exist (i.e. to be a fact) and for *B* not to exist". In this sense, therefore, a non-fact implies anything. This is also used in everyday thinking and speech when, in order to describe something quite emphatically as untrue, one says, for example: "If that is the case, then 2 x 2 = 5".

We denote *facts, i.e. what is true,* with *V,* and *untruths, i.e. what is not true,* with d, and write the two sentences just stated as follows:

ff c *Y*   *A rv M*

read *"M* implies K, for every value of 3f" or "for every M – i.e. for every fact *M* – '3f implies K' applies", or "d implies *M,* for every value *of M"* or "for every *M*, 'd implies *M'* applies*"*.

If *A and B* are facts, then *"*d *and B"* or "that *A* exists and *B* exists" is again a fact, which we refer to as *the conjunction* of *A* and *B* and as

A n B or as *AB*

. So if *"A"* means "it is raining" and *"B"* means "it is cold", then *"A n B"* or *"AB"* means "it is raining and it is cold".

With the help of implication and conjunctive linking, the relationship of *equivalence* can now be defined. Facts *A and B* are called *equivalent* if they imply each other; we write this as

A B or *Be A.*

According to this explanation, *"A B"* means *"(A m B)* and *(B» A)"*, i.e. *"(A m B) n (B» A)"*.

Equivalent statements are, for example, the provisions "x is divisible by 15" and *"x* is divisible by 3 and by 5". Among the specific facts, all facts are equivalent to each other and all non-facts are equivalent to each other. For if *A and B* are both true, then neither *A* is without *B* being true, nor is *B* true without *A* being true, because on the one hand *B is true* and on the other hand *A is* also *true.* If both are false, then the same applies again, but this time because on the one hand *A is not true* and on the other hand *B is not true.*

These two propositions justify retrospectively that we have used K to denote "the facts", "the factual" or "the fact" per se, and N to denote "the non-facts", "the non-factual" or "the non-fact" per se.

Finally, we introduce the concept of *the negation* of a fact. If *A* is a fact, then *"A"* is obviously equivalent to *"A* exists" — for example, "this is red" is equivalent to "that this is red exists", because if the first is true, then the second is also true, and vice versa. If *A* is a fact, then "*A* does not exist (is not, does not apply)" is also a fact, and this is called the *negation* of fact *A* and is denoted by

non-d or *A'.*

Now we can express the previously introduced description of *the implication Am B* very simply and concisely using the terms and symbols presented. We said: *"A* implies *B"* means "that d is and *B* is not, that is not"; this can now be written as follows:

$(A \quad B)=(A \quad B')'.$

3. *The ought and the demand*

The peculiar essence of desire, which cannot be grasped or eliminated by any attempt to analyse this experience and reduce it to different elements, the moment that determines the peculiarity of this attitude towards an object, eludes any direct description. One can only refer anyone who wants to know about it to their own experience, with the warning, if anything, that they should not mistake all sorts of accompanying phenomena for the essence, which lies solely in the unanalysed and unanalysable moment of striving. Beyond this reference to the given, there is only the possibility of indirect characterisation. One such possibility is the statement that this moment gives the experience a *meaning*, the meaning that a state of affairs *should* exist, that a determination *should* be realised — in the given case or in certain cases. The volition directed at state of affairs *A* is expressed in the sentence: *A should be* (be a fact).

This oughtness, or more precisely, *the oughtness* of a state of affairs, corresponds to the Wanting as a concrete counterpart: it is attributed to the object, namely the facts, to which the wanting is directed. Now one could say: that *A* should be means nothing other than that *A* is wanted by someone — perhaps only wanted in a dispositional sense — the relationship that exists in that someone wants *A* provides the reversal: *A* should be. This is contrasted by the fact that in many cases of oughtness, we will be somewhat at a loss to identify the subject or subjects of the corresponding volition. But what is more decisive is that, precisely in these cases — and these include the most important ones, those of ethical oughtness — the unbiased person does not miss such a subject at all, because he does not think of a wanting and a wanting person at all. If the naive view is correct here, then not every oughtness will correspond to a wanting. But conversely, not every volition corresponds to an actual ought. This view, which, remarkably, even those who think differently in theory repeatedly assert in serious cases, distinguishes between cases where something ought to be merely in the sense of a volition and cases where it ought to be in reality, and then *ought to be* without qualification.

Consideration for any desire. When I demand work from a person, they should perform it in accordance with my desire; whether they *should* actually perform it is not yet decided. If they have entered into a contract with me and accepted the work, and the terms of the contract are met, then they *must* perform it, and no will can create this obligation; every will — that of the law, the state, the community, a deity — that desires it is merely in accordance with this obligation, corresponds to it and, in a certain sense, does it justice, but is completely uninvolved in its existence. Similar to how an act of recognising a fact does justice to it in its own way, but does not create or constitute it. Justified wanting is opposed by an obligation that is actual; unjustified will also refers to an ought — it is not meaningless, it also has the meaning that something ought to be —; but it refers to an ought that is nowhere actualised — i.e. there is no case where it actually exists — just as a false judgement refers to a fact that is not a fact.

Our endeavour to describe the peculiarity of desiring objects has led us to the meaning of wanting and thus to what ought to be. Of course, something so irreducible is characterised by something else that is equally irreducible. Nevertheless, this step will not be without benefit. For now, instead of the psychological, which remains so peculiarly intangible despite all its "actuality," we have before us its objective counterpart, an objective entity that, despite its unanalysability, is nevertheless relatively easily accessible to systematic knowledge: oughtness can be described by specifying the lawful *relationships* that exist between the facts of oughtness. So let us undertake to recognise oughtness by seeking out the laws of oughtness. These will at the same time yield the essential laws of right or reasonable volition — precisely the deontic laws —; for volition is essentially characterised precisely by what its meaning is, and to satisfy the laws of this meaning is its own essential lawfulness. It is in the nature of things that the insights we will gain in this way have a "formal" character — it is no different in the case of the laws of logical thinking.

The concept of *oughtness* – the oughtness of a state of affairs – is a fundamental concept of deontics, and indeed the only one peculiar to it, i.e. not already belonging to the general theory of facts, that we introduce. If *A* is a fact, then "A ought to be", "let it be *A"* is again a fact, but of a different kind; one might call it a demand, of course in a purely objective sense of the word, which does not take into account any desire or desirer.

One can always substitute "it is valid (exists) that *A* should be" for "*A* should be", because one obviously does not apply without the other, and so one has again substituted a normal, one might say theoretical, circumstance for the demand, something that one can *think,* judging or merely assuming, without wanting anything. From this point of view, the theory that considers the demands and seeks to recognise their laws is opposed to them. The demand *"A should be", "let it be A",* is expressed by

The theory will have to deal with implications in which demands appear as elements; in particular and initially, relationships such as "if *A* (exists, applies), then *B* should be" will be considered.
i.e. *"A* implies that it should be *B*", or in our notation *"A x* ! *B".* For the sake of brevity, this relationship can be expressed as *"A demands B"* — as one might say "guilt demands atonement" — and written *as "A fB".* By virtue of this explanation

(A fB)   (A » ! B).

§ 2. THE BASIC PRINCIPLES OF THE SOLLENS

By starting from the recognition of the essential lawfulness of volition, we will attempt to derive certain fundamental laws from the meaning of volition, which is present in the essence of obligation, from which, by purely logical means, namely deduction, the entirety of the remaining laws that characterise it should emerge as far as possible. That the principles we establish will sound quite self-evident is, in turn, almost self-evident, especially if we want to develop a "natural" system that should begin with propositions that are immediately obvious from the nature of the subject matter. If propositions are then derived from these principles, some of which

Although this is quite obvious, our endeavour is more than a mere logical exercise: the purpose is to recognise how the laws are interrelated and through which of them the entirety of the essential laws of volition is implied in a rationally recognisable manner, i.e. its internal legality is already completely given. The considerations that precede the formulation of a principle should only point to the aspect of the facts that must be taken into account in order to draw attention to the fact expressed by the principle; they should by no means represent derivations of these propositions — which, as principles in the system, cannot be derived —; these considerations stand outside the system. The system begins only with the finished principles and comprises only those assertions that emerge from them through strict deduction.

An essential characteristic of correct thinking and correct willing is consistency. Those who judge not only take a position on the facts they are judging, but also on other facts, namely the implications of what is being judged. Not equally, one might say: equally narrowly. Anyone who asserts the validity of a determination A(x) for a given case, but believes that a determination *B(x)* that is rationally implied in the former does not apply in the same case, will be accused of violating the most necessary and "self-evident" consistency—*as when someone asserts that the present number is not divisible by 3*, but then claims that it is divisible by 4. it does not apply in the same case, will be accused of violating the most necessary and "obvious" consistency—as when someone claims that the given number is divisible by 15, but then or at the same time claims that it is not divisible by 3. The violation is perceived as less serious if there is only an empirically recognisable implication between the two determinations. Of course, if the person making the judgement is aware of this implication, the error will be judged differently than if they are not aware of it: in the first case, it reveals a logical defect, at least thoughtlessness or carelessness of thought; in the second, merely a lack of empirical knowledge, of observation, which is all the more serious the more difficult the experience was to gain, but in any case remains an error. The fact underlying all this can be summarised as follows: anyone who judges a situation has, "implicitly", as they say, also judged every implication of that situation. Therefore, they will only behave consistently if they treat each of these implications as if they had judged them themselves, indeed, as if they had

Let him judge. Thus, the consistency of a sequence of thoughts lies in the fact that the later ones capture the current ones, or at least do not contradict what was implicitly captured in the earlier ones, in terms of their meaning and essence, merely implied, even if not thought. And the reason why consistency must be demanded of judgement, i.e. why it is part of its correctness, lies precisely in the fact that judging a situation is a behaviour towards that situation of such a kind that it *affects all its implications.*

The same applies to the consistency of volition. If someone wants to see a determination *A(x)* realised in a particular case and is then made aware of a consequential determination *B(x)*, with the question of whether they also want it to apply in the same case, they will logically have to answer in the affirmative or abandon their original volition, just as in the case of judgement. However, a violation of this only occurs if the intellectual behaviour is correct, the judgement is in accordance with the implications at hand, is actually and purely attributed to the will, and it is not right to decide whether a distinction is made according to the same or different criteria as in the case of judgement — always assuming that there is a genuine implication, that it is believed with certainty, that the person willing is certain and does not merely suspect to a greater or lesser extent that d(x) entails *B(x)*. The reason why such consistency belongs to the correctness of volition is again that volition is directed at the implications of what is willed, that with one state of affairs, all the states of affairs that it implies are also willed. This lies in the meaning and essence of volition, both reasonable and unreasonable, correct and incorrect. But the one remains in accordance with that first meaning in further acts, the other does not. It is noteworthy how the investigation of correctness in the realm of will as well as in the realm of judgement immediately leads to a multitude of acts, to contexts in which it manifests itself.

The peculiar character of reaching into the implications
— on everything that is "contained" in the desired situation — is not found in all forms of desire. One may well desire something, provided it is pleasant or otherwise valuable to us, but not desire the unpleasant flip side of the situation at all. The fact that one wants the desired situation in all its particulars, with all its consequences, is a distinguishing feature of wanting.

Those who seriously desire something will be made aware of the undesirable aspects of what they desire, and will also consent to them, even to the unknown that may follow, accepting it with a "so be it" or "come what may". They have decided to do so.

Anyone *who* wants a fact also wants everything that is necessary for that fact to exist. That is inherent in the meaning of wanting. In other words, the following objective relationship exists: if a fact is to exist, then every implication of that fact must also exist. Therefore, the following relationship also exists: *if, under the condition of fact A, fact B is to exist – if,* as we said, *A requires B – and if B implies fact C, then it also applies that, under the condition A, C is to exist – that A requires C.* This relationship, which is more general in nature than the one first stated, is expressed by our first deontic principle. It is expressed more concisely and clearly in the following symbolic notation.

Principle I [A f

B) n(B x C)c{A f C)

or in more detail:

(A» ! *B)* *(B»* C) (A » ! C).

The proposition can be called *the principle* of *co-requirement* or *the principle of consistency*, because the will that takes it into account is consistent.

A second fundamental characteristic of volition is closely related to the issue of consistency. If a person or body has made a decision in response to a given situation *M*, and if a second demand is later made in relation to the same situation, either with or without explicit reference to the first decision, the question immediately arises as to how the two demands are compatible with each other. The question is particularly pertinent if both demands originate from the same person or persons, because the answer then sheds light on the internal rationality of their volition, but it is equally meaningful and important even without this prerequisite. The reason for this lies in a circumstance that is very obvious.

appears: if, in case *M*, *A* is required in one instance and *B* is required in another instance for the same case, then in case *M*, both *A and B* are required, which is why it is so important how the two circumstances are compatible. It is in the nature of volition that the requirement of *A* and the requirement of *B* for the same case result in *a* requirement for the coexistence of *A* and *B* or the conjunctive composite circumstance *AB*. And this is entirely independent of whether one requirement is thought of in relation to the other: the meaning of volition alone is decisive for this. This is the basis for the validity of the statement: *if, under the condition M, a circumstance A is to exist and, under the same condition, a circumstance B is to exist, then under the condition M the circumstance AB is to exist.*

As obvious as this statement may be, it is nevertheless an observation of a characteristic of volition that distinguishes it from other types of behaviour towards facts. Only in judgement, and only in *certain judgement,* can this combinability still be found; if I assert that, under condition *M*, a fact *A* will occur, and if I assert on another occasion that, under condition *M*, a *fact B* will occur, then I have implicitly asserted that, under this condition, *both A and B* will occur — this is implied by my assertions, regardless of whether I think about it or not. If the two assertions are replaced by assumptions, their meaning is no longer that of an assumption of the coincidence of *A* and *B.* When a coin is tossed, it can show "heads"; when the coin is tossed (the same coin in the same case), it can show "tails", but this does not mean that when the coin is tossed, it can show heads and tails at the same time. "It can be *A*" and "it can be *B*" does not mean "it can be *AB* (i.e. *A and B)*", but rather "it can be *A* or *B*"; in this respect, possibility and necessity are essentially different.1' Just as supposition is the weaker form of judgement, so desire, as the weaker form of craving, differs from the stronger form in that it lacks the property of combinability considered here. One can desire *A* for case *M* and, on the other hand, desire *B* for case *M*, without it being in the sense of these desires that both *A* and *B* are then desired for case ff. *A* may be desirable in itself, and in the same case *B* may also be desirable in itself, but their coexistence may not be. This then becomes apparent in the transition from desiring

Wanting will result in sacrificing one desire—and one value—to the other, if one wants to be reasonable; for if one were to hold on to both, one would also want them to coexist. The demands that are linked to *a* prerequisite thus combine to form *a* demand with composite content and can be replaced by such a demand: *they all form a single demand.* This applies to the acts of demanding, in terms of their meaning, and therefore also to demands in the objective sense, i.e. requirements. We demand...
We formulate this sentence as the second of our fundamental laws of ought.

Principle II (M f

A) n(M f B)>(M f AB)

or

(M z ! A) >(M» !B) z(M» ![AB]).

The theorem may be called *the principle of composition or union.*

.

The first two principles concern conditional requirements, as we often encounter them in decrees and orders of a general nature, in state laws. Here, the obligation is linked to a condition that is not always necessary, but always, in the strict sense, i.e. if the decree is meant precisely, a sufficient condition for its occurrence. It should now be noted, and it is formally important for the derivation of certain corollaries, that every conditional requirement can be replaced by an equivalent unconditional one. For example, a law that stipulates that anyone with an income of m crowns should pay ii crowns in income tax requires that the implication relationship should exist in future – as if it were a law of nature: if someone in the state has this much income, they pay this much tax on it. *The conditional requirement "if A (is), then B shall be" is equivalent to the unconditional "it shall be true: if A (is), then B (is)".* This is stated in our

Principle ***III***

(d *fB)*   !(A » *B)*

or

(A» ! B)   *!(A*   *B).*

This transformation is not only of theoretical significance. Someone may have received a series of instructions, perhaps at longer intervals, which complement and restrict each other, so that in order to clarify his duty, he will ask himself: what *am* I actually *supposed to do*? The answer will be: in this and that case, I should behave in such and such a way. In this way, they have freed the various conditional "I should" statements from their conditions and placed them, as an unconditional statement, before a fact that has the form of a conditional connection (an implication).

Since Principle **III** allows a requirement to be detached from its conditions, separating the ought on the one hand and the ought-free situation on the other (which is required), it may be called *the principle of separation*.

The first three principles merely state what is required in terms of *each* demand; no distinction is yet made in terms of justification. What these statements say is of the form: if this is to be, then that must also be; but none of them claims that anything actually *must* be. Yet all justification depends on this. A demand – even in the subjective sense of the word – that is justified is obviously itself somehow in accordance with what is demanded, corresponding to what ought to be; thus, it can only be actually justified if this ought to be actually exists. Without actual ought to be, there is no actual justification. Every "sense of justification", every purely psychological characteristic of justified will that is not matched by such an objective, actual ought, is a claim to justification without justification. The theory must therefore make the following statement: *there is (at least) one fact that ought to be the case.* We write for this:

Principle IV

{KU) !*U,*

to be read as: "There is (a fact) V of which the following applies: *U* should be". *U* is called *the unconditional requirement*, ! é/ the unconditional requirement, and the principle *of the actuality of ought* can therefore also be called *the principle of the unconditional requirement*.

Just as logic allows contradictory judgements, so deontics may allow contradictory

We do not exclude demanding and therefore unreasonable claims from our consideration, for both exist. But theory must establish that no actual facts correspond to a contradictory judgement and no actual obligations correspond to a contradictory claim. We achieve this with a sentence that states *that what must be unconditionally – our U – does not require its negation – non-U.* According to the agreement in § 1, 2, this negation of 9 would be designated with t/", It will play an important role in our investigation alongside *U*, the unconditional requirement, the ought, as the *contrary of the ought*, and may be emphasised by a particularly conspicuous designation, namely d ("inverted t/"). The symbol for *"does not require"* shall be *f'*, that for *"does not imply"* shall be »'. Then our proposition takes the following form:

Principle V

v f'n

or

U⊃' !*Ω*.

It is referred to as *the principle of the inconsistency of (actual) oughtness* .

Overview of principles

I. *(A f B)> (B C)»(A f* C)
II. *(M f A) n(M f B)m(M f AB)*
III. *(A f B)=* !(A» *B)*
IV. *(UU)*!*U*
V. *U f'R*

A, *B*, *C*, *M*, *U*, *R* are facts.

I. (Principle of co-requirement or consistency) If *AB* requires and *BC* implies, then *A* requires *C.*

II. (Principle of union) If *MA* demands and *MB* demands, then ff demands *A* and B (their coexistence).

III. (Principle of separation) The conditional requirement *"A* requires *B"* ("if *A* is, then *B* should be") is equivalent to the unconditional "it should be such that *dB* implies (that if *A* is, then *B* is)".

IV. (Principle of the actuality of oughtness or unconditional demand) There is (at least) one fact t/ of which the following is actually true: t/ ought to be. ((/ is what ought to be.)

V. (Principle of the inconsistency of actual oughtness). What ought to be (what is absolutely required, f/) does not demand its opposite (what ought not to be, /l).

II. **NEXT** CONSEQUENCES

§ 3. OVERVIEW OF THE FOLLOWING PROPOSITION

From the principles that can be gained from considering the nature of volition and oughtness and put into precise form, one can now derive a system of deontic propositions by purely logical conclusions and represent the process of derivation purely and transparently in all its steps by means of the few simple symbols we have introduced. But the logical passion of deduction is foreign to many people who are nevertheless capable of thinking well; they are content to recognise immediately what can be recognised, to deduce the rest in an informal derivation, and do not ask how all this can be derived precisely from the smallest possible set of basic facts, which ultimate premises are necessary and sufficient to justify the whole system of assertions. This overview is written primarily for them; it is intended to present the most important laws of ought that emerge from our five principles, without giving more than a hint of the deductions. The following paragraphs of this section, which provide these deductions, can thus be omitted for those who are not fond of formulas.

(From § 4)

Principle I (principle of consistency) – If, in the case of *A* being true, a fact *B* is required, then every implication of *B* is also required for the same case – taking into account that the facts are implied everywhere, i.e. they are always implications of *B*, this results in the following statement:

1. If, in the case of *A* being true, a fact *B* is required, then the facts are required for the same case. The proposition allows for the brief, albeit not entirely accurate, formulation: The facts are *required* everywhere. We record it because, in relation to the facts,

seems to lose its meaning, as *a strange conclusion that needs to be checked out.*

Considering that a non-fact implies anything, it follows from I:

2. If, in the case of *A* being true – in short: through d – a non-fact is required, then *A* requires any arbitrary fact.

— This sentence is also one of the *strange* conclusions.

From *Principle II (Principle of Union)* — If fact 3f requires fact *A* and ff requires fact *B*, then *M* requires the coexistence of *A* and *B* (the reverse of which can also be proven, incidentally) — using I, we get:

4. If fact *M* requires fact *A* and fact *N* requires fact *B*, then the coexistence of 3f and *N* requires the coexistence of *A* and *B*. The sentence allows for the *combination* or *unification* of requirements in a more general sense than Principle II. According to this principle, a system of provisions that links different requirements to different prerequisites must always take into account the possibility of the prerequisites coinciding and must anticipate the coexistence of the requirements in this case.

(From § 5)

*Principle **III** (principle of separation)* states: The fact that circumstance *A* requires circumstance *B* is equivalent to the implication "if *A*, then *B"* being required. It states that the conditional requirement "if *A* is, then *B* shall be" is equivalent to the unconditional "it *shall* apply: if *A* is, then *B* is*"*. However, it leaves aside the question of whether any requirements actually exist.

5. The meaning of an unconditional requirement *that "P* should be" — which, incidentally, may or may not exist, rightly or wrongly — is the same as that of a requirement linked to any given circumstance that demands *P* "under all circumstances"; "that *P* should be" means "that any given circumstance *M* is required".

From this and from I, it follows that:

6. If *P* should (unconditionally) be, then every implication of *P* should be; then, using **III** (through a somewhat longer deduction):

8. If *A demands B* and *B demands C,* then *A demands C.* The demand thus extends or transfers not only to the implications of what is demanded, but, as this sentence states, also to its postulates, i.e. to what is demanded in the case of the applicability of the demanded circumstances or, as we say, "by these circumstances".

Sentence 9 also states this fact for the postulates of an *unconditionally* required fact *P*.

An obvious error in application can make sentence 8 — and then also 9 — appear incorrect. Under the conditions *(A)* stipulated in a contract between x and y, x is to perform something for *y* (let it be *B*), and once the performance *(B)* has been completed, *y* is to provide consideration (C) in return; according to our proposition, it seems to follow that as soon as condition *A* is met, y must perform his C, which is obviously incorrect. A precise formulation of the case immediately reveals where the error lies. If *A* applies, then *x* shall perform *B* for y, i.e. there shall be a time (to be determined more precisely) when *B* is performed by x; when this is fulfilled, there shall again be a specific time when C is performed by *y*. And from this it follows that if *A* applies, there shall be a specific time when C is performed by *y*. This is in accordance with the contract: it requires that this consideration be fulfilled; and this is all that can be asserted according to our proposition, which says nothing about *how* this is to be achieved and in no way imposes on y an unconditional obligation that can only be fulfilled through the performance of the other party and his subsequent consideration. It is noteworthy how the phrase "if *A* applies, then y shall perform *C*", to which the application of the proposition has led, is immediately understood as a demand on y, as an expression of an obligation on y, whereas it says and can say no more than "if *A* applies, then it shall be the case that *y* performs *C*". This indicates that what is to be done by me is not necessarily my duty; what is to be done by me is only my duty insofar as it depends on me, i.e. on my will, that it happens. A contract according to which, if *A* applies and *x* performs *B*, *y* should perform *C*, would of course not result in "if *A* applies, C should perform", but it would also not be an application of our theorem, because in such a contract there are not two demands, as the theorem presupposes, but only one.

Sentence 10 establishes the composition or combination of unconditional claims into *one* claim — the coexistence of the individually claimed circumstances — in the form

! A ! *B* *!(AB).*

By analogy with the concept of equivalence — § 1, 2 — the concept of *"equal claims"* or "*equivalence of claims"* is introduced. The facts *A and B* are called equally demanding if *A* demands *B* and *B* demands *A* — then everything demanded by *B* is obviously also demanded by *A*, and vice versa — we write *A‹:oB* for this. It is easy to see that

11 ∞

i.e. that equally demanding circumstances are also "equivalent in terms of demand" in the sense that they are required to be equivalent. The following sentences present *equivalent transformations of the demand relationship*, the most important of which is the counterposition expressed in 14.
contraposition expressed in 14:

14 *(A fB)* *(B'f A')* -,

"if *A* is, then *B should* be" is equivalent to "if *B is* not, *then A* not be".

When applied to situations that take the form of a provision being applicable in a specific case, it must be ensured that the contraposition does not change the time specifications and, accordingly, the time relationship between the elements: If I have borrowed a book, then I should return it; the reversal is, of course: if I do not return a book, then I should not *have* borrowed it, which is the same as: it should be true that if I do not return a book, I have not borrowed it.

(From § 6)

Principle IV (principle of unconditional learning) states: There is a fact *U* that *is supposed to* exist. This unconditional requirement, *U,* the ought, is now — according to the meaning of unconditional requirement, according to 5 — that which ought to be in fact under all circumstances, under all conditions:

15 ! tf or *M f U ,*

i.e. *M requires U "for every M"*, or: any given fact *M* requires *U*. In this respect, *U* conforms to the actual, K, which applies and exists under all circumstances and under all conditions. While sentences 6 and 8 stated *that if* something should be (unconditionally), then the implications or postulates of this should also exist, it is now *unconditionally* asserted:

16. and 17. What is implied or required by what is absolutely required is absolutely required, it should be.

In the next context, this results in the proposition

18. A claim that is to exist ("apply") does exist; and the reverse of this, hence the equivalence:

19 !!d = !d.

Sentence 20 states *that all unconditionally required facts are equivalent in terms of requirements*. This sentence emphasises a concept of requirement that knows *no degrees of obligation*: everything that should be, should be equally, namely under all circumstances, unconditionally. This proposition must also be contrasted with *pr'ii]'en*, namely an undoubtedly existing concept of oughtness that allows for the distinction between stronger and weaker demands (§ 11, 6).

In l it has been established: if *A* demands any fact *B*,
Thus, *A* demands the facts; in 7: if something is absolutely required, then the facts are required; if something is to be, then the facts must be. Since, according to IV, it has been established that something is to be, it can also be asserted that:

22. The facts ought to be: ! K.

This sentence is, of course, one of the *"bizarre"* ones. It states that at least what actually is should be, but leaves open whether the reverse is also true. Of course, it does not say that a determination that applies at the present moment should also apply in the future, but only that it should apply at this very moment. Nevertheless, this is strange enough and seems to run completely counter to an actually existing concept of oughtness.

The following sentence, however, is again completely in line with this concept. Its formulaic inscription is

23 *V‹:o U* or *!(Um V).*

This means: the facts and what is unconditionally required are equivalent in terms of requirements; or: what actually is and what ought to be ought to be equivalent. - It is in the nature of requirements that what is ought to coincide with what is required.

Part of this factual situation is described in 23'

V f U

: The facts demand what absolutely ought to be. — One could say that "the facts themselves" demand it, that which absolutely applies, and that there is no need for special conditions to which its demand would be linked. The idea of the "self-evident fulfilment of obligations" is hinted at.

(From § 7)

By contrasting sentences about what ought to be (/, a series of sentences about what ought not to be (A) results.

One of the *strange* sentences is again

27. The contrary of what ought to be demands any given state of affairs. — If what ought not to be is, then anything ought to be or anything ought not to be, which amounts to the same thing. — The sentence presents the contrary of what ought to be, A, in analogy to the non-factual, d, which implies any given state of affairs.

An obvious consequence of this proposition is

28. The contrary of what ought to be demands what is contrary to what ought to be. — For which, despite its "strangeness," certain feelings of retribution and revenge seem to speak.

In contrast to these two propositions, it is now important to note that even the contrary to what ought to be does not negate the unconditional requirement of what ought to be. The requirements associated with the contrary to what ought to be are contradictory; in addition to

28 *R f 11*

and

30 *11 fA*

,

29 *K f U,*

that which is contrary to what ought to be demands what ought to be. Even if what ought not to be is, what is absolutely required ought to be. Just as a false assumption in the theoretical realm may yield false conclusions, it does not negate the existence of facts: these remain implicit everywhere, even in the unfacts.

In demanding *U* and K, there is no difference between what ought to be and what ought not to be. The essential difference is highlighted by *Principle V (the principle of consistency)*, which states *that what ought to be, U, does not demand what ought not to be, R.* - Now it immediately follows that:

32. What ought to be does not demand what is not factual; and
33. What ought to be does not imply what is not actual.

Therefore, the unreal is not oughtness. From this and from 22., the real is oughtness, it follows *that the real and the oughtness, in the realm of specific facts,* which contains only facts and non-facts, *are equivalent,* and therefore *the unoughtness and the unreal are* also *equivalent:*

34 u= P,
35 d =J.

These last sentences, which seem to identify what ought to be and what actually is, are probably the most alienating of our "alienating conclusions". It is necessary to examine the conclusions that lead to them and the principles from which these conclusions originated. It can be anticipated that these laws, if they are correct, will modify the usual concept of ought in essential respects, or reveal a duality of coexisting concepts of "ought" whose mutual relationship will have to be clarified — a task whose solution must essentially involve establishing the relationships between ought, want, and the facts. — First, the formal derivation of the corollaries.

§ 4. CONSEQUENCES FROM PRINCIPLES I AND II

If one sets in

I *(A f B) (B C) >(A f C)*

for C the special value None, then we obtain

(A fB) (Bm K)m(A f V),

and since the second condition, *B x* K, always applies according to § 1, 2,

1 *(A f B)»(A f V).*

Sentence l states: If *A* demands anything — any *B* — then *A* demands the facts. Or: if, under the condition *A*, any circumstance is to exist, then under this condition the facts must exist*. The facts are implied in every conditional demand.* If one substitutes the special value d for *B* in I, i.e. assumes that some fact *A* demands something that is not factual, i.e. unfulfillable, then one has

- with regard to § 1, 2, N » *M* -

(A f A) s (A x M)»(A f M),

i.e.

(A f A) »(A f M) .

In addition, the reverse is of course also true

(A f M) x(A f A),

so the equivalence

2 *(A f A) (A f M)* .

Dafl A demands what is *absolutely impossible and unachievable*, *which is equivalent to dafl A demanding any arbitrary fact M.* Just as a premise that contains a false consequence leads to arbitrary conclusions — if something false is true, then anything is true — so a demand for something that is contrary to the facts leads to arbitrary demands.

Principle II reads:

II *(M f A) (M fB)»(M f AB).*

It is easy to see that *the reverse* of this statement also holds true. Since it is clear that

AB is e A, AB» B,

then, according to I,

(M> ! AB) (ABS A) (M» ! *A)!**
(M» ! *AB) (AB B)»(M» ! B),*

therefore, by combining,

(N» ! *AB}»(M» ! A) (M» ! B)*

and from this and from II:

Ht *(M f A) (M fB} (M f AB).*

The fact that ff requires fact *A* and *M* requires fact *B* is therefore *equivalent* to *M* requiring the coexistence of *A* and *B*. However, as the derivation shown above demonstrates, not the entire statement made here is a deontic basic fact, but only the partial statement made by II, while the reversal of this — and thus also the validity of the equivalence relation H — follows purely logically from II and I.

The proposition H, about the *conjunctive* connection of demands, is not opposed by a similar proposition about the *disjunctive* connection, i.e. the connection made by *"or"*.

Is

d v *B*

the fact that *at least one* of the facts *A and B* applies, i.e. the fact *"A or B"* — without the elements necessarily being mutually exclusive — applies, taking into account I —

(M f A)(A» A v *B) x(M f A* v *B),!** , i.e.

(M f A)m(M f A v *B)*

(M f B) (Be A v B) m(M f A v *B),* , i.e.

(M f B) m(M f A v B)

and therefore – purely logically deduced –

3 *(Äffd)* v *(MJB)* *(Äffd* v *B),*

a counterpart to II; but the reverse of this proposition does not hold. Demanding that *A* exist *or* demanding that *B* exist implies demanding that *A or B* exist, but not vice versa.

Principle II provides a combination of requirements which, even when set independently of one another, are linked to the same prerequisite. Indirectly, in the corollary to be presented here, it also provides *a combination of requirements that arise from different prerequisites when these prerequisites coincide.*

For it is clear, as can be seen purely logically — i.e. without regard to the special nature of ought — that

(M» ! A) (N» ! B) (MN» ! A > ! B)

and likewise

(MN»! A > ! B) (MN» !A) (MN» ! B),

on the other hand, according to II

(MN» ! A) (MN»! B) (MN ! AB),

therefore also

4 *(M» ! A) (N ! B)»(MN ! AB)*

or

(M f A) (N f B)»(MN fAB).

$. CONSEQUENCES OF PRINCIPLE III (AND I, II)

Principle III states:

III *(A fB) !(A» B).*

The observation that the implication relationship is transiti , i.e. that

(1) *(A B)(B C)»(A* C),

and the further fact that the demand also extends to the *implications* of what is demanded, as I note, raises the question of whether the demand also extends to what is *demanded* of the demanded, to the *postulates* of the demanded, one might say, i.e. whether the demand relationship is also transitive, whether

(A f B) (B f C)»(A f C)

also applies.

According to III,

(A fB) (B f C)=!(A x B) n !(Be C).

Now, the assumption that (absolutely or unconditionally, i.e. under any circumstances) *!P* applies is, as can be seen purely logically — cf. § 1, 2 — equivalent to the assumption that *P* is required by any given fact *M*, i.e.

5 *! P (M implies P) (M » ! P).*

Therefore, with regard to I, it follows that

$$!P \wedge (P \supset Q) \equiv (M \underset{M}{f} P) \wedge (P \supset Q)$$
$$\supset (M f Q)$$
$$\supset !Q,$$

asO

6 *!P > (P z Q) z* ! *Q.*

If K is substituted for Q here, the result is

$$!P(P \supset V) \supset !V,$$

and, since *P»* is satisfied, further

7 *! P» ! V,*

i.e. *if anything is (absolutely) required, then the facts are required* — a supplement to 1.

Applying this to our assumptions, we have

(A f B)(B f C) - *!(A x B) n !(B z* C), as above,

[M f (A c *B)) [M f (B»* C)) according to 5,

3f M

[M f (A m B)(B m *C)]* according to Hr.

! *[[A » B)(B x C)]* according to 5,

n !*(A » C)* according to (1) and 6 ;

therefore

(A fB) (B f C)» !(A x C)

or according to III,

8 *(A f B) (B f C)* *(A f* C).

If *A* requires *B*, then not only are all implications required by *A*, as stated in I, but also all *postulates* of *B*; *if A requires that B applies, which in turn requires C, then C is also required by A.*

If, in particular, *each A* requires fact *B*, which requires *C* requires *C*, then every *A*, i.e. scMechthin, requires *C*:

(A f B) n (B f C) »(A f C)

& SO

! B>(B f C)» ! C

or, in other symbols,

9 *!P (P f Q)» ! Q.*

This sentence is a counterpart to 6 and says: *If something is required by what ought to be, then it ought to be.* However, since the assertion of *the existence* of unconditional requirements is not yet taken into account by lV, this does not claim that anything ought to be in fact.

From the proposition

Ht *(M f A)(M f B) (M f AB)*

it can be inferred

(Ltg *A)(My B) (M f AB)*

or

10 *! A> !B* !(NB),

unconditional claims are combined to form a claim in the same way as conditional claims.

If two facts *A and B* are mutually dependent, they may be called *claim-equivalent* or *equally dependent*, and this relationship may be written as

Then ∞

$(A \infty B)$ $A \supset B)$ $!(B \supset A)$

$\equiv ![(A$ $B),$

so

11 $(A \infty B)$

Principle III allows the negation sign to be "removed" from a claim relationship *A f B* or *A» ! B* and placed before an implication relationship: !(d *cB)*. Now the implication *x B* can be replaced by var[redacted]ous equiv[redacted]ents. "If *A* is so- , according to § 1, 2, as much as "that *A* is and *B* is not, is not"; [redacted] as can be easily recognised, also means *"A* is not or it is *B"*, i.e. "at l[redacted] one of the facts *A', B* exists". — So we have

(A m B) (A n B') *(A'* v *B),*

therefore

12 *(A fB)* *(A m* ! *B)* !(A *m B)* !(A *n B')' = !(A'* v *B)*

and on the other hand

13 *(A m* ! *B)* *[A n(! B)')'* *A'* v ! *B.*

"If *A* is, then *B* should be" therefore means "that *A* is and *B* is not required, is not" and "A is not or *B* should be".

A more important transformation of the implication *"*if 3f, then *P"* is "if *P* is not, then 3f is not", which is known in logic as the *contraposition*.

This logical contraposition

(M» P) *(P'* *M')*

we can add a *deontic contraposition*:

(A fB)= !(Am B)= !(B'x A')=(B'» ! *A')=(B'f A'),*

i.e.

14 *(A f B)* *(B'f A').*

"If *A is, then B should* be" is therefore equivalent to "if *B* is *not*, then *A* should not be".

§ 6. CONSEQUENCES OF PRINCIPLE IV (AND I, II, III)

When we have spoken of unconditional demands in our deductions so far, it was only to determine what lies in the nature of demanding,
i.e. in the essence of oughtness, without asserting that an ought actually exists. This assertion is only made in Principle IV:

IV *(U U)! U.*

That *U* is unconditionally required means that V is required under all circumstances, by every fact; one has

15 *M f U* beside *M m V*

and what *ought to be* appears, in that it is "required everywhere", as the deontic counterpart of *what actually is,* which is "implied everywhere". Further correspondences between what ought to apply unconditionally and what does apply unconditionally will be shown in the following sentences.

From

(M *V) (U» A) (M f A)*

it follows, since the first condition may be omitted as fulfilled,

(U A) (M A)

or

16 *{U» A)> ! A* analogous to *(V» A) m A,*

an implication of what is unconditionally required is unconditionally required, just as an implication of what is actual is actual.

According to 8, the following also applies

(3f/ U) (9 *J A)»(M f A)*

and since the first condition, having been fulfilled, may be omitted again,

17 *(U f A) ! A ,*

a postulate of what is absolutely required is absolutely required—what is required by what ought to be—and such a thing exists—*shall be.* One notices how this sentence goes beyond the content of the related sentence 9.

If *A* is required by *U,* and only in this case, one can say, "there should be something that requires *A,*" "it should be such that *A* should be," or "it is required that the requirement *!A* exist." Since, under these conditions, according to 17, *A should* be, it follows that: *A requirement that should exist does exist, what should be required is required,*

18 *! ! A» ! A.*

And since, as will follow from 22, but is also otherwise obvious, a requirement that exists should also exist, let us anticipate here the *equivalence*

19 !! N ! d.

From

$$\infty$$

follows, since the second Yorass condition is always satisfied,

20 ∞ according to *(V» A)* *(A*= P).

What is required by what ought to be is equivalent to him in terms of requirements, i.e., according to 11, it should be equivalent to it — analogous to how what is implied by actual is equivalent to it.

If a fact *A* is to be, and only in this case, *A* is required by every fact and in particular by (/, and the condition —the left side— of 20 applies. Thus we have

21 ∞

All absolutely required facts are equivalent in terms of requirements.

There are characteristic relationships of implication and requirement between what ought to be, *U*, and what actually is, K. In section 7, we already established that if a certain state of affairs, *A*, is to be the case, then the facts must be the case.

Here we now conclude from the sentence

! U (U V) ! V,

since according to IV the first condition is fulfilled in any case, and according to § 1, 2 the second condition is also fulfilled,

22 ! K.

The facts are absolutely required; a situation that actually exists should exist.

Therefore,

according to 21, 23

***V*∞*U*,**

or, with regard to 11,

23 *!(U* U).

It is required that what is actual and what ought to be are equivalent; that what ought to be is, and what is ought to be.

One aspect of this is contained in 23 and can be highlighted from it

23' *V f U,*

the facts demand what must be.

Of formal importance is that because of

*(*f*)•(*»t*»*! »(Uft*»*!)*

also applies *!(A» A),* so, according to III, also

24 *A fA.*

Accordingly, the *relationship between the claims, A f B,* is not only *transitive* – according to 8
— but also *reflexive,* thus possessing both essential properties of *implication.* This is not immediately apparent from the definition, which explains *"A f B"* as *"A » ! B",* i.e. as a (special) implication; this explanation *immediately* yields only that, on the one hand, *(A»* ! *B) (! B x ! C)»(A »* !*C)* and, on the other hand,
!d » !d must also hold, statements that do not repeat our theorems 8 and 24
.

From *;A f A) (A B) z(A f B),*

which applies according to I, one concludes, since according to 24 the first condition is always fulfilled,

25 *(A B)»(A f B).*

What is implied by a fact is therefore required by it.

- A consequence of this theorem is

26 ∞∃

Equivalence is also equivalent in terms of requirements, which implies that the same circumstances also require the same circumstances.

§ 7. FORCE MAJEURE. CONTRARY CONSEQUENCES FROM THE PRINCIPLES I AND V

The contraposition of the implication relationship *A» B,* i.e. its transformation
into the equivalent *B'm A',* provides the counterpart to the proposition Kfz K

A » M' or *A » M,*

which we have already established in § 1, 2. If any fact *M* exists, then the facts always exist; this results from the contraposition: if the facts do not exist, then *M* does not exist, for any arbitrary *M*, and this then amounts to the same thing as saying that any arbitrary ff exists. For in the totality of the facts of a logically possible system, for every *A there is* its negation *A′*, and if all are negated, one obtains *A′* from *A* and *A* from *A′*, i.e. the same totality again. Whoever denies everything also affirms everything.

The counterposition of *M f U*, "if anything is possible, then the absolutely

necessary requirement, i , must always be met", now yields

27 *R f M′* or *R f M* .

What should not be, that which is absolutely forbidden, demands the negation of everything, forbids everything, and that means, again, it demands everything – for to forbid everything is to demand everything. If what should not be is, then nothing should be, or everything should be arbitrary.

What ought to be is unconditionally demanded by everything and everyone, even by what is contrary to what ought to be – which demands everything. It is, alongside

28 *R f R* (corresponding to A

md), which applies according to 27, i.e.

also

29 6 *f U* (corresponding to d» K).

What is wrong demands what is wrong. But it is also true that *what is wrong demands what is right.* This is and remains required under *all* circumstances. This is the deontic equivalent of the propositions: what is untrue implies what is untrue; what is true implies (but also) what is true. The facts remain unconditional under all circumstances.

The contraposition of 23' *V f U,* or 23 Km i7, yields

30 *R fA*

31 ∞ or !(/l d).

If what should not be is, then the unreal – even the impossible – should be.
— be. What is contrary to what ought to be is equivalent in terms of demand to what is not factual; it is demanded that what is contrary to what ought to be be unfactual, that what is unfactual and what is contrary to what ought to be be the same.

The proposition K»'d, that facts do not imply non-facts — expressed in logical terms, that falsehood does not follow from truth — corresponds to our *principle*

V *Uf'Ո,*

which states that what is unconditionally required does not require its opposite, the contrary.

From this it follows, first of all, that t/also does not require d, the untrue. The assumption *U f A* would, together with d»/), result in (U *f A) (A» E) (U f 11),* i.e., since the second premise is fulfilled, *(U f A)»* *"(U f* R). Since *U f R* is false according to V, it follows that

32 *U f' A.*

From this it follows again, applying the contraposed theorem 25,

33 *U» A.*

What is absolutely required does not imply what is not factual – what ought to be is not contrary to the facts. *What ought to be is therefore not unfactual.* It follows from this that, since in the realm of *specific circumstances* – and this is what volition concerns – there is no third category besides facts and unfactualities, *what ought to be is factual.*

This strange consequence, together with the reverse of proposition 22, according to which what is fact should be, yields the *equivalence of what is actually required and what is factual:*

34 t7= K,

and therefore also *the equivalence* of *what is contrary to what ought to be and what is not factual:*

35 N N.

III. THE INTENTIONS AND THE FACTS

§ 8. THE INTENTION OF THE FACTS

1. *The essence of the strange conclusions*

Certain conclusions drawn from our fundamental laws have cast doubt on the concept of oughtness, or at least on its unity. Now, this concept should be derived from the consideration of volition as the concept of that object which constitutes the meaning of volition. It may also be examined by investigating whether those strange conclusions actually arise from what an examination of the essence of the will reveals to us as its meaning.

The train of thought that led to those conclusions can be summarised in its essential steps as follows, understood as a consideration of the will:

(1) Willing refers to specific circumstances.

(2) It is in the nature of volition that the implications of the intended circumstance are also intended.

(3) The facts are implications of every circumstance; the facts are therefore always willed along with the circumstance.

(4) The meaning of volition is that a circumstance should be. Thus, the last sentence implies that if something should be, then the facts should be.

(5) There is justified wanting; there is some circumstance that should actually be. From this and from (4) it follows:

(6) The facts (at least) should be.

(7) Justified desire is consistent.

(8) A non-fact implies any given circumstance, including its own negation.

(9) The desire for a non-fact is therefore not consistent: non-facts should not exist.

In the realm of desire, there are only facts and non-facts — according to (l) —:

(10) Therefore, only facts should exist.

As a summary of (6) and (10), the following finally emerged:

(11) What should be is that and only that which actually is.

The independent prerequisites that occur here are of various kinds. They include:

assertions about the nature of volition, namely in (l), (2), (4), (5), (7), including in particular those about ought as the "meaning" of volition in (4) and (5) — overall, deontic premises;

assertions of a general object-theoretical nature about facts, in particular implication relationships between facts and non-facts, in (3) and (8).

The rest is inference and does not need to be examined.

2. *The general willingness to accept implications*

Here, the deontic premises are particularly relevant, and among them, not (1) at first, because this assertion only plays a role in the last conclusions of our series — where it concerns the equivalence of oughtness and actuality; but it is precisely their critical examination that will provide important insights into the nature of oughtness. Above all others, as can be seen immediately, assertion (2) must be examined, which, together with the general object-theoretical proposition in (3), yielded the first and most important strange conclusion: that facts are universally willed.

Does it really correspond to the nature of volition to claim that whoever wants something wants everything that is implied in the desired situation? There are certainly enough cases where someone has to learn that, precisely through the realisation of what they wanted, they or others encounter something completely undesirable, about which they state with complete conviction: I did not want that. The right to make this statement is also indisputable in a certain sense, namely insofar as it means that the undesirable consequences were not the immediate, explicitly understood object of the desire. This is exactly the same as in the case of the assertion of a fact that is contradicted by the non-applicability of certain implications of that fact and is subsequently refuted. We will probably also say of such an untrue implication that

We would not have wanted to make that claim, but rather, we would not have thought of it or we would not have known that it was not true. In any case, we will not stick to the claim — if we are still judging rationally — and by dropping it, we acknowledge that it contradicts the facts, apparently precisely because of the implications whose untruthfulness we have now recognised. This clearly shows a fundamental characteristic of judgement: whoever judges a situation takes a certain position on every implication of the situation being judged, as if he were judging it. The implication does not have to be explicitly stated in the judgement: in this sense, it is not judged. But it is implicitly judged, co-judged, and it is in the spirit of the judgement to judge the implications of the judged facts in this way. Even the undesirable consequences of what we want can, if we become aware of them early enough, move us to abandon that desire, and we will always do so if we decisively do not want these consequences, i.e. want them not to be. Once what we wanted has been realised, it is of course too **late** to give up that desire**,** and a revocation, as in the case of assertion, seems irrelevant here. However, it is not entirely irrelevant. It expresses that we have changed our position on what we previously wanted, which can be very significant in practice and is also important in theory: as an indication that there is also a difference in our volitional position towards accomplished facts.

The evidence provided by language is not entirely clear in our case because it does not distinguish between explicit desire and implicit desire or consent. Thus, the phrase "I did not want that," which we have just discussed, is contrasted with the phrase "you do not know what you want," which reminds someone of what they want, probably in contradiction to their usual desires, merely implicitly, without knowing or considering it.

There are certainly "closer" and "more distant" implications of what is wanted, depending on the degree to which the implication can be grasped; a subjective or human point of view that comes into play when attributing the consequences of actions and the degrees of responsibility for them: as it were, in a shifting of responsibility from the will to the judgement (cf. above, p. 245). But in purely logical terms, this is irrelevant, i.e. it is irrelevant to and cannot change the fact that, in the sense of

The will lies in fulfilling *every* implication of what is desired. And everyone has already had the opportunity to become aware of this fact. For everyone has experienced having to make a decision in an unclear situation and did so with the thought, "I don't know how this will work out and what will come of it, but come what may, I'll do it this way." Here, the person who wants something has also explicitly taken on something unknown to them in the sense of their desire and declared it to be part of their desire.

3. *"Actual" and "non-actual" implications. The co-willing of the facts*

What proves to be intentional in the cases under consideration is brought about by the realisation of the intentional, as a consequence or in the realisation of the intentional, as a means, and its occurrence appears as "close" or "distant", but always as an *"actual"* implication of the actualisation of the intentional. The relationship that connects it to this is of the kind to which the "if—then" is applied in a completely natural and unforced manner. It is somehow inherent in the *determinations*, the realisation of which are the circumstances under consideration, that one "brings" the others with it; whenever the determinations whose actualisation is desired in the given case are actualised, the implied determinations will also be actualised, regardless of whether this is established a priori or only through experience. However, this type of implication does not extend to what is fact independent of our will.

For the sake of easier understanding, a few names will be introduced. The implication between two determinations *A(x) and B(x)*, which is expressed as "if (any) *x* fulfils the determination *A(x)*, then x also fulfils the determination *B(x)*", is called a *determination implication* – in logic, it is given the less descriptive name *of formal implication* – such as "if x < 1, then *x* < 2", "if x is gold, then *x* is soluble in mercury". An implication such as "if 516 has a digit sum divisible by 3, then 516 is divisible by 3" — also, of course, the one we obtain when we substitute 517 for 516 — or "if this ring of mine is gold, then it is soluble in mercury" is referred to as *an "applied determination implication"*. As can be seen, it is an implication between *certain* circumstances, facts or misdeeds.

things *such as instances of determinations A(x), B(x)* – more generally *A(x), B(y)*, such as "if today is Sunday, then tomorrow is Monday" in application to a given today – and specifically those between which there is a determination implication: one could say an *application case* of a determination implication.

The implications between certain facts that we usually consider, especially those between individual cases, are mostly of this kind. In contrast, the implication that exists between facts and non-facts as such, without regard to the determinations actualised in them, is merely *a "material implication"*. Thus, between the facts "my ring is made of gold" and "my ring is soluble in mercury" there is the specified applied determination implication. At the same time, there is a material implication, but this is reversible — both facts are facts and so one cannot be without the other — whereas the present applied determination implication is not reversible, i.e. the pure determination implication "if x is made of gold, then x is soluble in mercury" is not reversible, and so its application cannot result in a reversible consequence. Applying these distinctions, we can therefore say that whatever is implied in the intended facts in the sense of any applied determination implication is also intended. The determinative implications, pure and applied, correspond perfectly to the usual *"if* - so", as the *"actual"* implication; thus, the *"actual" implicates* of the intended would be intended, and, it seems, only them. But how, in the case of intention, should one draw the line between "actual" and "non-actual" implicates? What I want is not simply the actualisation of a determination *(Box)*, but its actualisation in certain cases, for example in this given case and then under all the conditions of the case, which, insofar as they are unknown to me, certainly do not enter into the determination *B(x)* and are therefore not "actual" but only materially implied in the intended situation. Between these conditions and those clearly grasped in the understanding of the case and included in *B(x)*, however, there are many that are only vaguely or "half" grasped, that are close to being explicitly grasped but are not quite grasped by it. "Because the man was careless with the gun, his friend had to die" — here the conviction of the existence of an actual implication is expressed, but it certainly does not exist between the stated provisions *"x* is a man and handles a rifle carelessly" and "y is a friend of x and dies". That is not the opinion either, but it is clear that in the case in question, i.e. in the stated provision together with certain other provisions of this case, there is an actual implication for the validity of the second provision. But how many of these contributing circumstances are clearly understood, even by the eyewitness? — Now suppose the man wanted to clean the rifle and shot his friend. The determinations he grasped intentionally do not imply the determination that actually materialised in this outcome, and he will not be blamed for wanting the former to be true. But he wanted it to apply under the circumstances of the case, which he did not know completely, but for whose incomplete consideration he is held responsible. I want a determination to be realised *under the existing conditions of the case*; I want something that will certainly not be, indeed cannot be, without these conditions, because they are facts. But whoever wants *A*, which cannot exist without *B*, also wants *B*. And since this formula applies to every fact *B*, every fact is therefore also wanted in every desire.

4. *The explicit desire for facts*

The facts are, of course, not entirely intended in the same sense as the "actual" implications of the intended circumstances. These can also be explicitly intended when attention is drawn to them. However, a fact that confronts us as a fact is beyond our influence and thus, it seems, beyond any volition. And yet, there is an experience in relation to facts that is sufficiently volitional to be included as a borderline case under an expanded concept of volition. It is even occasionally expressed linguistically in the same terms as actual volition. Someone who has expressed their will and who has been given a circumstance to consider that runs counter to their will says, for example, "it may be, it shall be," thereby announcing that they are sticking to their will, incorporating the fact into what they want: "they accept it." The desire has explicitly adjusted to the fact; it is a *consent* to the fact. However, the same "it shall be" is also encountered, sometimes quite clearly, When, in view of the facts, we abandon a goal and "change our will": in such a case, we will make ourselves clear about these facts in an energetic overview, incorporating them, as it were, into the foundations of our judgement, but also of our will — for the future. This is not merely a passive abandonment of a desire that is contrary to the facts, but, even if no new goal has yet been set, a preparatory yet active adjustment of the will. It may be accompanied by the explicit intention "I will take that into account"; this is then a normal, actual desire, only with the particularity that it is directed towards one's own future acts of will; the essence of the process, of which this desire is only an expression, lies deeper and consists precisely in that adjustment of the will to the facts. This adjustment differs from the adjustment to an actual goal of the will: the will is directed towards the goal, but it is guided by the facts, it "pursues" them, "strives for them", it is "based" on them — all images that suggest the contrast and the relationship in the experiences.

5. *The inauthentic ought*

So if it is in accordance with the will that the desired state of affairs should exist, it is also in accordance with the will that the facts should exist, as that without which the desired state of affairs cannot exist. Of course, this oughtness of the facts no longer corresponds to the ordinary, natural concept of oughtness: just as the willing of the facts is no longer actual willing — but nevertheless a volitional behaviour — this oughtness of the facts is now only an *"inauthentic" oughtness.* Nevertheless, it is so closely related to actual oughtness, a fact of the nature of oughtness, that it justifies its inclusion under a correspondingly expanded concept of oughtness.

The inauthentic, one might say fulfilled, ought lies in the consequence of ought: it is implied in every ought. It is very noteworthy here that the step leading from the natural to the expanded concept of ought is of the same kind as that leading from the natural to the expanded concept of implication, and that it is essentially connected with this step. The connection is clear: it is only from the implication of the facts in every state of affairs that ought to be that the ought-to-be of the facts has resulted, but this implication is merely a material implication.

The experience that first brings us into contact with the concrete relationship of implication is that peculiar transition from one judgement to a consequential judgement, expressed in "therefore" or in *"irei/* — so"; from "because *A*, *B* is*"* we arrive, if *A* remains unjudged, at *"if A*, then *B"* — that peculiar transition of thought from cause to effect remains here as well. It corresponds in a unique way to the objective relationship of implication and is the original means of grasping it. The implication that corresponds objectively to this motivation of thought is applied or pure determination implication, i.e. "actual"; only in relation to it does this peculiar positing of one fact "with regard" to another, as a consequence of a reason, exist. But exact logic cannot stop at this relationship of implication. It is not irrelevant to it whether *B* "follows" from *A*, and the cases of implication in which this occurs deserve special theoretical treatment; but beyond this relationship of evidence — evidence is a matter of thought — the general truth relationship of "if — then" must be understood, consisting in the fact that if the d-judgement is true, the B-judgement is also true (regardless of whether this "makes sense" in relation to that or not): it corresponds to the relationship of the facts that *A* is not without *B*; this is implication in its general concept, which encompasses both the actual and the merely material implication. Without this concept, logic would lack strict generality and precision. Without considering material implications, we would not be able to recognise formal implications in the realm of reality through experience and would remain limited to a priori implications. For empirical formal implications arise only from the material implications of the cases from which we derive them through induction.

The experience that corresponds to the actual co-existence of the actual implicate of the demanded originally corresponds is a peculiar transition from wanting to wanting, the motivation of wanting the implicate through wanting the implicant. The demand relationship *"A* demands *B"* presents itself as "actual" insofar as B can be wanted *(actually* wanted) "with regard" to *A*.[*4]

The experience of wanting corresponds to the actual unconditional ought. It has been shown how it is in the nature of wanting to extend to every implication of what is wanted and how, as an "uneigenfulfilled volition, also concerns the facts. Just as the original concept of implication, based on the experience of consequential connection, leads to the necessity of extending beyond the scope of that experience and to a generalised objective formulation when attempting to treat it precisely, so too does the original concept of demand (the demand relationship) and of oughtness. Just as an experience could be demonstrated in the "inappropriate will" of the facts, which as a remnant of will still corresponds to the inappropriate ought, so too can the "inappropriate", i.e. merely material implication of the facts be contrasted with a peculiar psychological situation. In the realm of actual implication, we find ourselves peculiarly bound to an implication by positing a state of affairs, experience a tendency to posit it, and in the given case, its original judgement takes place under this tendency, the "motivation" of the judgement. A remnant of this remains even in the case of mere material implication: through every judgement, and in a certain sense through every assumption, we adapt ourselves to the facts. We posit a fact, judging or assuming, salva veritate, we place it in the context of the facts. To every assumption we make, we readily add facts already recognised in the course of reasoning, with the awareness that this "changes nothing" about the assumption: we thus only explicitly add to the assumed state of affairs a second one that was already implicitly assumed with it, as a material implication of it. After all, the object of the material implication is far clearer and easier to grasp than its psychological counterpart.

With the extended deontic concepts, the opposite seems to be true. The improper will is a clearly demonstrable experience, while the improper ought, which wants to be an equivalent of actuality but not actuality itself, seems to be only indirectly comprehensible, namely through consideration of the concomitant requirement of facts. However, it should be noted that the oughtness of facts is by no means foreign to non-scientific and, as it seems, unbiased thinking. One often hears the expression "it was meant to be" or "it will happen as it should happen". The first occurs particularly in cases that we are justified in interpreting as cases of improper volition: when one comes to terms with a fact, i.e., finds a volitional attitude towards it. The second need not necessarily be an expression of fatalism. Whatever unclear secondary thoughts may be associated with such statements, they clearly show that the concept of improper volition is not foreign to ordinary thinking.

§ 9. THE INTENTION AND THE CERTAINTY OF THE EVENT

I. Desirable behaviour in response to undefined circumstances

The next of the conditions in § 8, 1, which must be examined, is the assertion (1): Willingness relates to specific circumstances. — Specific circumstances are facts or non-facts, and we cannot actually will facts and non-facts as such: it therefore seems that, since there is such a thing as actual willing, we do will indeterminate circumstances. What needs to be said to refute this appearance has already been said in § 1, 1. But, if I may judge from my own experience, this appearance may be like many others: even when refuted, it is still not eliminated and continues to disturb our thinking. It will therefore be useful to contrast ordinary actual volition with a case where there is in fact something like a desire for an indeterminate state of affairs; this will show what kind of volitional behaviour is still possible in such a case. I can judge that $0 < 1$, but I cannot judge that $x < 1$ as long as *x* does not already have a corresponding predetermined meaning; however, I can *accept* this undefined state of affairs as a *determination* for x. The normal expression of this assumption is *"x* is less than l". This sentence does not make a statement, but a determination, and determinations are "arbitrary". In fact, at the same time as the assumption, which is a purely intellectual act, something desirous can also be expressed here. Indeed, it seems that this is precisely where the freest will comes into play; we can dispose of nothing as unrestrictedly and unconditionally as we can the meaning of signs – at least for ourselves. The stipulation that "the sign *x* should mean a number smaller than 1" is in fact an act of will. But this volition does not refer to a determination, but to a specific fact that — within the scope of the stipulation — actually exists or does not exist: the volition does not refer to an indeterminate x, but to the sign x, and that is a specific object, and it is fulfilled when the participants actually use the sign in the stipulated sense.

understood. However, it should be noted that when we say "*x* is less than 1", we do not tend to think about symbols and understanding symbols at all. The analysis of the concept of the "indefinite" or the "variable" and its "determination" is probably one of the most difficult tasks in psychology; but this much can be said: just as the assumption "x is less than 1" is an imaginative reproduction of a judgement that cannot occur here due to the absence of a specific subject, so too is the identical determination of desire – insofar as it does not concern the sign and its meaning, but the "variable" itself – merely a fantasy desire*5, a fantastical reproduction of a volition. This example can easily be followed by others that concern "real" and therefore more volitional determinations, such as "*x* pays *y* one hundred shillings" and the like. They can only show that a pure determination can at most be desired imaginatively, but not seriously — not even seriously wished for; just as one can only "judge imaginatively",
i.e. not judge them, but only assume them in a judgement-like intellectual behaviour.

2. *Objective certainty in the face of subjective uncertainty*

Of course, we do not want facts or even non-facts as such in our actual desires — and that is all we are talking about here. Nevertheless, every circumstance that we desire is one or the other. It is, timelessly and in a certain sense always, even before my decision, a fact or a non-fact that what I have in mind as desirable will come to pass. In view of this situation, the task arises of showing what meaning and significance actual willing and oughtness have. Is it not just an illusion that we determine the course of events by our will, since everything is already determined? And what is the point of another ought, if what ought to happen does happen anyway?

What we want is neither a determination as such, nor a fact or non-fact as such, but the applicability of a determination in certain cases. The cases are completely predetermined, but they are only grasped by the willing person in an incomplete determination. For the sake of simplicity, let us consider a desire that refers to a single case. It is clearly understood by the willing person as *this* case — for example, the next moment in which, in the sense of my

Willingness, my hand should reach out — as the case *that* it is, but by no means in every respect *as* it actually is. Its *predetermined* purpose, i.e. the purpose grasped before the will comes into play, does not normally contain (imply) the purpose that is to be realised in it and which we briefly refer to as the *intended* purpose, while maintaining that it is not the intended purpose itself that is actually intended, but its fulfilment in this case. Before willing, I cannot read it out of the predetermined and thus recognise that it will apply. It may well be that even before willing, I see myself, as it were, stretching out my hand and grasping the fact of this stretching out not purely imaginatively, merely assuming it, but already judging it. But then the judgement will not have the character of certainty; if it did, there would be no actual wanting anymore, but I would only behave as a spectator and at most want inappropriately for what happens to me and with me. Whenever actual volition occurs, the prior grasping of the situation to be willed is, if not mere assumption, at most a supposition: it presents itself to me as *possible*, not as actual and not as non-actual. Herein lies, of course, despite the certainty of the situation itself, a subjective uncertainty. But this only means that it is incompletely *grasped* and that the appearance of indeterminacy of the circumstances to which the will is directed is just that: an appearance. We can say that the circumstance that is willed is a fact or a non-fact, but is only grasped in advance in incomplete modal certainty, as *possible*.

3. *Possibility in provisions and in certain circumstances*

Here, it is essential to clearly understand the key features of possibility. The actual realm of possibility is that of undefined circumstances: "*x* is a real number" and "*x* < 1" are possible determinations. This means that *there are* cases in which these determinations are realised or apply. It is possible for a dice to fall on one of its faces, in particular on the face marked 1. The fact underlying this statement is this: in the total range of (possible) cases in which a dice falls on one of its faces, *there is* a sub-range of cases in which it falls on 1. Since this makes up one sixth of the total range of cases, we measure the specified possibility as }. It is the possibility

possibility of a determination, or more precisely, the possibility of an indeterminate case of the applicability of a determination: the possibility that an (indeterminate) case in which the determination *A(x),* "x is a cube and falls on one of its faces", is at the same time a case in which the determination *B(x),* "x falls on face l", applies. We call the possibility of a determination, corresponding to the fact that there are cases of this determination, *pure possibility,*[18]

But pure possibilities tend to interest us little, except in theoretical considerations. It is much more important for us to know what is possible in a given case and how it is possible. But the given case is completely determined, and the fact that a given determination, *B(x),* will apply in it is simply a fact, or it is a non-fact, even if we do not know it. It is precisely this circumstance, that we only incompletely grasp the case, that makes us ask about possibilities, even though facts are available. Not only do we describe a dice as possible when it falls, especially when it falls on l, but also when I throw this dice again, this result is called possible, and if I also say that it is a fair dice and I will throw it in the usual way, the same possibility
that was found to be a pure possibility for the indeterminate case. We thus transfer the possibility of the determination or the indeterminate case of the determination to the determinate case, to the applicability of the determination in this case: we refer to the possibility of the determinate fact, which we understand in this way, as *applied*.

The question of the meaning and justification of such a transfer is answered once we understand what has happened here. We have understood a given case in the incomplete — pre-given — determination *A(x),* merely "as a case of this determination". Since the determination *A(x)* entails a certain pure possibility for the applicability of *B(x)* — "in the indeterminate case", "in general" — we have also claimed the same possibility for the applicability of *B(x)* in the given case. And now it is clear: this possibility of being a case of *B(x)* also applies to the given case, *insofar as* it is a case of *A{x)* and *only* insofar as this is the case. The case is, of course, completely determined — either in such a way that the applicability of *B(x)* is actually the case, or in such a way that it is not actually the case, i.e. in such a way that it is modally completely determined — but regardless of this complete determination, it also has the aforementioned possibility *relative to its partial determination A* (x). Accordingly, the *determined* fact "*B(x)* applies in this case" has, in addition to its complete absolute modal determination, the incomplete *relative modal determination* of possibility, namely relative to the determination *A* (x) of the case.

This relative modal determination is just as real as, for example, the relative determination that the city of Baden is located south of Vienna, which applies to the city regardless of its complete local specificity, and, like the latter, is not something "subjective". What is subjectively determined is only the incompleteness of the grasping of what is objectively completely determined and the selection it makes among the existing determinations of the same. In this respect, a certain fact that is understood as possible is "subjectively indeterminate".

4. *Relationship of volition to events and facts*

In volition, a peculiar and very remarkable change in the intellectual position towards a situation takes place. The applicability of the determination *B(x)* in the present case appears to me to be possible before I will it, but by willingly doing so, I can now judge "I am doing it," "it will be so," and this judgement has, from a psychological point of view, the character of certainty. This certainty is essential for the fact of decisive volition, and where this decisiveness is lacking, there is not actually volition, but something related to it. It seems, of course, that even the most resolute volition can remain without such certainty. The ambitious person wants to attain a high position, but may not be certain that he will attain it. But he does not really want this goal, the realisation of which depends on so many things; he desires it, strives for it, and only wants to do what seems necessary to him in each case, and in each such case he is also certain that he will do it. The more distant the goal, the more external circumstances influence its realisation, the less certain it will be, all other things being equal, that it will be achieved; but the will always brings the judgement of the completion of this certainty closer, even if only in relation to the immediate goals, one's own actions, until completion.

In this conviction of the actual validity of what is desired — which, incidentally, may become apparent in an explicit judgement or exist only as an intellectual attitude — it is remarkable that it appears to be directly connected with the will. I do not first have to state, as if I were an outside observer, that I want something, in order to then conclude "so it will happen"; rather, simply by wanting it, I say "so it will happen," and this statement expresses both judgement and will. The act of willing itself, and not a judgement that ascertained it, carries the act of judgement as a psychological prerequisite. — In the case of improper willing, it is the other way around: here, the certain judgement carries the willing. Wanting thus completes our judgemental statement about the future; it leads us from an incomplete grasp of the facts to a complete grasp in modal certainty; it allows us, in the best case, to discover facts. Those who think clearly and want strongly know most about the future.

This addition to conviction through volition is not without objective justification. My volition is the experienced expression of a real process within me, which is a real determination of the case to which it refers. The case is co-constituted by this reality — therefore by me. In possession of this constituent of the case, I now know it more completely and can, in the best case, recognise that a possible determination will actually apply to it. In that the will — or that which finds its lived expression in the will — constitutively determines the real event, it enables the judgement to grasp the fact of this event. The fact that this or that happens at this or that time is, of course, timeless and unchangeable, but equally timeless and unchangeable is the fact that it happens in this way through my will. The real function of volition is to play the role of a real factor in real events, and volition is decisive for these events. Of course, there is nothing to determine about the facts; the will cannot create them, it can only help to discover them.

The examination of our premise (1) — "the will is directed towards specific circumstances" — has not only been confirmed, but has also yielded an important addition that removes the appearance of indeterminacy from the circumstances of the will and clarifies them. The idea

— § 8, l —, which led to the paradoxical deontic propositions, is thus legitimised, with the exception of the specifically deontic assertions (4), (5), (7) about oughtness, which probably no longer require further justification. The assertion (8) that a non-fact implies arbitrary facts follows automatically from the expanded version of the concept of implication, which was justified in § 8, 5. — The following investigations have the task of clarifying the concept of actual oughtness, especially its relationship to inauthentic oughtness and to facts, by considering right will and the nature of its rightness. They will also bring the meaning of the three deontic assertions just mentioned to full clarity.

IV. THE RIGHT WIL

§ 10. ACCURACY OF MATERIAL

I. General Laws of Right Intention

The essential laws of volition, which are expressed in our fundamental laws of ought, are these:

(I) Whoever wants something wants everything that is implied in the required circumstances; the will extends to the implications of what is wanted.

(II) If circumstance *A* is willed and circumstance *B* is willed, then it is also willed that *A and B* be.

(III) Anyone who makes a conditional demand, "if *A* is, then *B* should be," implicitly wants "it should be the case that if *A* is, then *B* is also."

(IV) There is a situation *U* that it is absolutely right to want.

(V) Those who want correctly do not want (even implicitly) the negation of what is wanted; correct wanting is consistent.

The laws are consistent with the essential laws of judgement, namely those of certainty and assertion. These can be derived from them if one always substitutes "assert" for "want" or "demand" and "is actually" for "should be" in **(III)**. Here we can clearly see how the logic of judgement, as an essential part of the logic of thought, and how, on the other hand, the logic of will (if one may say so) contrasts with the "logic of objects", namely the general theory of objects — in particular the theory of facts.

Here we must develop the "logic of volition." Its laws emerge from the fundamental laws cited above, just as the deontic corollaries (in Chapter II) emerge from the principles of oughtness, or could be deduced from them. Here, only the most important conclusions will be presented, particularly from the perspective of the correctness of volition. I will continue to refrain from always emphasising the analogies of judgement; they are obvious.

The first three propositions deal with wanting in general: they concern all wanting, both wrong and right. Only the fourth introduces right wanting into the system, and the fifth establishes a criterion of rightness, namely consistency. In the realm of specific circumstances — and these are judged and desired — only the facts are free of contradiction, and accordingly, among judgements, only the true ones, and among desires, only those that are fulfilled, let us say the *materially correct ones.* What is at issue here as correct volition, and whose laws are being considered, is volition that proves itself externally by asserting itself. It will become apparent to what extent this external correctness coincides with internal correctness and how the latter distinguishes itself from the former. In any case, as in logical thinking, the main focus here is on correctness and its conditions.

Principle (II) — the connecting principle — establishes an essential connection between all demands that may be made at any time, either simultaneously or successively: they all constitute *a* demand, and this demand requires that what is to exist in the sense of each individual demand must also exist together.

This becomes particularly significant — also in practical terms — when demands are directed at the same subject or made by the same subject, where the question "what should I do?" and the equally important "what do I want? What, in fact, do I want?" arises. The second case in particular is of obvious importance when it comes to the correctness of one's desires. My desires can only be correct if not only what I want at the moment is consistent, but also the entirety of all the things I have ever wanted or will ever want. Since one *should* desire correctly,[*0] the requirement arises — naturally only in the sense of actual oughtness — to desire in such a way that all our desires can be united into *one* without contradiction.

It is important to keep in mind the reason for this commandment ("imperative") so that one does not seek it where it cannot be found: it simply lies in the fact that all demands together form a single, composite content, are equivalent, and in fact yield their meaning, that this is only correct if it is consistent, and that one should want what is right.

Sentence (I) – the consistency principle – states that with a fact, every implication – every fact without which the former does not exist –

is desired and results in the following statement regarding the correctness of desire: it is right to desire an implication of what is desired — explicitly, expressly. But this does not imply the requirement to also explicitly want every implication of a desired state of affairs, which would obviously be impossible to fulfil; for the statement does not claim that wanting is only correct if every such implication is explicitly wanted. One might want to make a weaker demand: to want in such a way that we could also explicitly want every implication of what is wanted — actually or at least inappropriately want it, consent to it. But such a demand would not be very specific, because it is not stated and it is hardly possible to clarify what this "being able to desire" actually means. What the sentence implies for the correctness of desire is initially only this: a desire is only correct if the (explicit) desire — the actual or non-actual desire — of every implication is also correct. And this corresponds to the requirement: to want in such a way that the (explicit) wanting of every implication of what is wanted — in short, everything that is also wanted — is correct. This "imperative" has practical value in that it points us to a useful — and often used — means of testing the correctness of a desire or a project.

Since, according to (II), all my demands amount to a single one, which, according to (V), is only correct if it is free of contradiction, and since, according to (I), all implications of a desired outcome are also desired, my desire will only be correct if every implication of one demand is compatible with every implication of every other demand without contradiction. This also provides a possibility for testing, which, however, can never prove correctness on its own, but can reveal existing incorrectness. If I want *A*, which implies *B*, and on the other hand I want C, which is incompatible with *B*, then I want *C and B* to be true and want incorrectly. Of course, in this case, d is already incompatible with *C*, but the conflict may be hidden, whereas there is a clear contradiction between *B* and C.

Sentence **(III)** establishes the law governing the correctness of volition: conditional volition, "if *A* is, then *B* shall be," is only correct if unconditional volition, "it shall be such that if *A* is, then *B* is," is correct. Here, instead of "only then," one could also say "then and only then," because the two requirements are equivalent. The "imperative" that could be formed from this is obvious enough, but another one can be deduced from our sentence that is far more significant.

2. *A principle of application. Motivation through desire*

The implication "if *A,* then *B"* together with the fact *"A* is true" results in the fact *"B* is true". This fact, which is simply included in the concept of implication, is nevertheless of utmost importance for thinking; not insofar as it comprehends it, but insofar as it *applies* it. The application takes place in *the sentence,* which has the form: if *A,* then *B; A* is (in fact); therefore *B* is (in fact). All deduction is based on the *principle of implication*: what is implied by a fact (is a fact and) can be correctly asserted. It is accompanied by *the principle of application of the relationship of demand*: what is demanded of something that ought to be (ought to be and) can rightly be desired – cf. § 6, 17. — It is the principle of *motivation,* which is not done justice by thinking it, but by wanting it in accordance with it, applying it willingly.

The principle does not contain a necessary condition of correctness, but only a sufficient one. Therefore, it cannot be expressed as a command, but only as a permission. There is no actual requirement to assert the implications of a fact, to want the postulates of what ought to be — this happens implicitly, "by itself", is in fact already the case and is therefore only required in an improper sense. The necessary condition of correctness — but not a sufficient one — and therefore required is only that an implication of a fact is not denied, that a postulate of what ought to be is not opposed, and this is already expressed in the commandment of consistency (§ 10, l). Only when a position is taken on a co-asserted or co-willed proposition should this be done in the sense of asserting or willing (at least consenting).

The opportunity to will something that is also willed arises in relation to an object that plays the role of *a means*. For example, I want to make an iron wire glow red-hot and use the means of sending an electric current through it. Then this means was willed as a cause, the effect of which is the event willed as an end. The given case of passing a current through the wire is the cause of the given case of heating the wire; cause and effect are real events, completely determined cases. Between these, in their completeness and actuality, there is now, of course, a reversal...

bare (material) implication; it is clearly not what we mean when we assert a causal relationship. In doing so, we understand the two cases only in incomplete terms: as cases of certain (incomplete) determinations *M(x)*, *N(x)*. *M(x)* could be, for example, "x is an iron wire and a current is sent through x", *N(x)* could be, for example, "heat is generated in *x*". And now, in application to the given x, the *applied implication* "if ff(x_1), then *N(x)"* is asserted. Of course, this only happens implicitly here, because, in addition, Äf(x_1) is judged to *be* true, resulting in the judgement *"because M(x), N(x)"*. In all cases of causality, such applied implication is present. The existence of the "cause", insofar as it is a case of *M(x)*, is the implicant of the existence of the "effect", insofar as it is a case of *N(x)*.[°1]

If the existence of the cause is merely an implicant (sufficient condition) for the existence of the desired effect and not at the same time an implicate (necessary condition), it is not required by this desire – and this also applies to partial causes. In fact, I can also achieve the heating of the wire by other means, and the fact that I choose this particular means is not clearly determined by the purpose. But it is determined by this purpose together with the circumstances, or by it under the circumstances in which I want it. Accordingly, there is a duality of motivational concepts; it corresponds to this that one can say that this means was not motivated by the purpose alone, and yet one can also say that it was motivated by it. In one case, we are talking about motivation in a narrower sense: only the will of an "actual" implication of the intended effect is motivated in this way.
- and that is not generally the existence of a specific cause or partial cause. On other occasions, an expanded concept of motivation is applied, according to which the desire for an "improper" (merely material) implication, such as the existence of this specific cause (or partial cause), is also motivated by the desire for the effect. Only the desire for the means is "actually required" and "actually motivated" insofar as they are necessary, i.e. insofar as their existence is a necessary condition of the desired situation, an implication in the sense of applied determination implication. Here, one notices how the actual ought appears to be bound to applied incomplete determinations. But this will be discussed later.

3. *Motivation through judgement*

Wanting can be motivated not only by wanting, but also by judgement: I want *B* "*because A* is", where *A* is a ought-to-be or even a purely "objective" fact.

However, the reasons we usually give for our actions tend to be rather inadequate. I want to go out "because it's so nice"; I give someone a piece of clothing "because they are poor", another person a book "because it's their birthday" — all of these are statements that need to be supplemented. The specific circumstances of the case, both internal and external, always play a part, and it is often very difficult to identify those circumstances which, together with the partial reason given, constitute the full justification for the desire.

It may even seem questionable whether such a justification — apart from the difficulties inherent in our limited capacity for knowledge — is ever or even once possible in this matter. There is probably a kind of dark will, simply directed from the given reality towards the next event, a will to exist that is directed in complete indeterminacy towards the existence of my individual self, which is also not grasped in determinations of being, perhaps simply towards reality. But although this is probably inherent in every will and is the foundation on which every will rests, it will hardly appear as an independent act of will and may be disregarded here.

The actual volition is directed towards the applicability of a determination, and for the applicability of the determination *B(x)* to be desired in the present case, in the sense of the volition, the reasons lie in the — actual or supposed — applicability of a concept *A(x)* of determinations at the moment of volition. Willing is therefore always motivated by judgement, regardless of the influence of other motives. The decisive determinations may be difficult to find in sufficient completeness and even more difficult to state clearly, but some reflection often leads us to clearly grasp the facts "from which" we will or have willed. Then we say "because in this case (x,) the determination *A(x)* applies, *B(x)* should apply", or "because *A{x)* is, *B(x)* should be", and this *ireif* is now an expression of an applied determination im-

Application: the application of a requirement relationship between provisions, the relation *A(x) implies B(x).*** The statement "because *A(x)* is true, *B(x)* should be true" implies that *if A(x) is true, B(x) should be true* – for every x. This general requirement is what is known as the *maxim* of action.°3

Whether we always act according to maxims is not a question of deontics; whether we should act according to maxims cannot be decided at this point in our investigation (cf. § 12, 4). But this is certain: if someone brings about the truth of a proposition *B(x)* because the proposition *A(x)* is true — this "because" understood in the strict sense of a sufficient reason — then they have "acted according to a maxim" and it is implied in their intention that this maxim "if d(x) is true, then *B(x)* should be true" applies generally. In other words, whoever wants *B(x)* to be true in a given case because *A(x)* is true implicitly wants the relationship "if *A(x)* is true, then *B(x)* is true" to be universally valid "like a law of nature" (this reformulation of the previous sentence makes use of Principle **III).** *Kant's* demand that he should "be able to will" that his maxim should have such universal validity is based on the fact that he implicitly, i.e. according to the meaning of his will, actually wants this.

The desire for one provision to apply on the basis of another provision applying is only correct if the general requirement implied therein, namely that this provision entails that provision (in every case), is also correct. This is a very strict condition of correctness. According to this condition, a merely "external" fulfilment of the will, by achieving what is desired in the given case, without the applicability of the implicitly required general circumstance, is only an apparent fulfilment – after all, what was actually intended has not been achieved – and such a will is materially incorrect. - But the whole matter of "maxims" and their co-intention only acquires its essential significance in the realm of actual oughtness, from the point of view of a correctness that is not merely material.

4. *The materially contrary to oughtness*

The propositions about what is contrary to oughtness that appear in our deontic system naturally never claim that it actually occurs. Principle V states that what is contrary to oughtness is not implied by what is in accordance with oughtness.

otherwise, there are sentences of the form "if something is contrary to what ought to be, then this and that is the case," and these owe their truth in the realm of material correctness entirely to the fact that (here) what is contrary to what ought to be never is — such a sentence says, after all, "what is contrary to what ought to be is not, or this and that exists." In this realm, even incorrect volition is not contrary to what ought to be, insofar as it actually is. A materially incorrect volition is directed at something that ought not to be — i.e., in this case, something that will not happen — and only in this sense is it “contrary to what ought to be,” but not in the sense that it itself ought not to actually occur, for it does occur. When we speak of contrary-to-duty volition, we naturally always mean something that demands something that is contrary to duty in the sense of actual duty, and then it is also the volition itself. But volition that only lacks material correctness is also called incorrect, although not always. One would then rather say that someone did not start correctly, but also that they did not want correctly, even if their intentions were good and internally correct.

Our sentence (IV), which states that there is (at least) one circumstance 6f that it is right to want, simply reflects the actual circumstances. Materially correct wanting is based on facts, whether it achieves its goal as actual wanting or whether it recognises a fact as non-actual.

Sentence (V) requires consistency for correct volition. This does not only mean freedom from an "inner" or "logical" contradiction in what is desired, i.e. freedom from implied (applied) *determinations,* one of which implies the negation of the other, but also freedom from material implications, one of which is the negation of the other. A fact such as that Socrates died of old age is not contradictory "in itself", but it contradicts the fact that Socrates did not die of old age, which it (materially) implies as a fact. Thus, it is not materially free of contradiction. Anyone who wants material incorrectness therefore implicitly wants the factual and the non-factual to coexist. If they try to "pursue their intention", they will reach a point where, precisely defined, it ceases to guide the direction of their actions, where everything seems equally good and equally bad for its realisation — because nothing can serve it: false will becomes directionless, ultimately demanding

Anything and nothing at all. — If something is contrary to what ought to be, then anything ought to be, and anything ought not to be. This paradoxical statement, a simple consequence of the fact that what is contrary to what ought to be never is in the material sense, nevertheless finds a kind of reflection in our consciousness: namely, when something happens that, in our view, should not be according to the order of nature, something that seems "impossible" to us. Anyone who has witnessed a great catastrophe is familiar with the feeling of the incomprehensible and the thought that arises despite better knowledge: if something so monstrous happens, if that is to be, then everything ceases, or then everything can and should already be. It takes effort to muster the will to accept such events, to "feel the whole of nature and suddenly say: yes, it is right" (Dostoyevsky, *The Demons).*

11. MEANINGFUL WILL

1. *Material and formal correctness*

The external or material correctness of judgement and volition simply consists in the accuracy of the facts. The materially correct, i.e. the true judgement, grasps a fact; the materially correct volition grasps, as something inauthentic, the fact of the existence of a reality independent of it, and, as something authentic, the existence of a reality that is co-constituted by the will. However, there is a correctness that seems independent of this external verification by facts. First of all, there is what is referred to as the formal correctness of a conclusion and, analogously, the formal correctness of motivation.

A(x), (•) are applied provisions, i.e. of the form "here - in the case of x, — *A(x)* applies*"* or "*B(x)* applies". Then the conclusion *"*because *A(•)* is *Box)"* is formally correct, regardless of whether *A(x)* is a fact, if the relationship "if *A(x)* applies, then *B(x)* applies" actually exists: the formal correctness of the conclusion is based on the existence of the determination implication applied in the conclusion and thus

judged in the reasoning. Under this condition, of course, the desire for *B(x)* would also be formally correctly motivated by the desire for the fact *A(x).* This formal correctness belongs to *the consistency* of thinking and desiring or acting; it is probably what is meant when one pays tribute to someone for recognising that even if he has not done the right thing, he has nevertheless "thought logically" or "acted logically". This kind of inner correctness does not entail the material correctness of the motivated judgement or volition, but it clearly favours it: those who think and will consistently have a chance of also "doing the right thing" in material terms; those for whom consistency is a general characteristic of their behaviour need only, so to speak, be lucky enough to "find" the right conditions.

If it is not the desire for *A(x)* or the judgement that *A(x)* should be, but simply the judgement of the fact *A(xy)* that sufficiently motivates the desire for *B(xt)* — "because d(x) is, *B(x)* should be" — then this motivation is formally correct, provided that the implication *A(x)» ! Box)* holds; i.e. the relationship "if *A{x),* then *B(x)* should be". Then the following would have to apply: "it should be the case that if d(x) is true, then *B(x)* is also true". Now the question is what this "it should be the case" means. If it is understood in the sense of an improper ought, it means something like "it is actually so". And without doubt, in view of a fact or supposed fact *A(x),* the improper desire for its ("actual") implication *B(x),* i.e. consent to it, is also formally correct and internally correct. But that is only consent to what actually *is* (or is supposed to be) or will come, and that is not the case that concerns us here. The "it should be so" could be given by a volition, such as a legal order: "it should be so that whoever has an income of *m* crowns pays *n* crowns in tax". Here we have an actual ought, in the sense of a desire that demands the existence of a determinative implication. Accordingly, a desire that in the given case, "because I have m crowns of income", *n* crowns be paid is also formally correctly motivated *in the sense of that legal will*. However, whether or not formal correctness actually exists depends on whether the "will of the law" is correct, i.e. it depends on whether the ought exists not only in the sense of some kind of will, but whether it actually exists.°4

2. *Judgement-motivated volition*

We are thus faced with the **question:** What constitutes, or, less ambitiously, when and under what necessary and sufficient conditions does the actual ought, which is the formal correctness of the judgement-motivated actual intention? To find the answer, it is necessary to examine the facts of this judgement-motivated intention in more detail. We said that whoever wants the determination *B(x)* to apply in the given case (x,) because *A(x)* applies in this case, wants *B(x)* to apply (always) when *A(x)* applies. This is certainly correct if that "because" is meant in the strict sense. However, it should not be overlooked – and this has already been noted – that the determinations that are given as the basis for a desire, or even just become conscious to the desiring person, are usually so deficient that to understand this "because" in the strict sense would clearly be a misunderstanding. If one attempts to supplement the basic determination, one may succeed in arriving at an *A(x)* that contains enough conditions that one can now say: if A(x) applies, then *B(x)* should indeed apply, i.e., if it is not already fulfilled without our intervention, it should be brought about by us. But if we look more closely, we will find that we can never take this "if-then" too strictly if the definitions it connects are not themselves already of a formal nature, definitions of facts and requirements as they appear in the laws of the material correctness of volition. If we call a requirement that concerns requirements and therefore presupposes other requirements a *higher-level requirement,* it can be established that the actual requirements of a general nature, of the form *A(x)» ! B(x)* or *A(x) f B(x),* which are strictly valid, are always of a higher level. It is not possible to prove that a *primary requirement — a requirement of the first level* — which occurs in this form of "if-then" or, as unconditional, is intended to apply under all circumstances, is a maxim that must be followed without exception.

It has often been pointed out – and it is precisely because of this that it has been believed that the existence of a general moral law can be ruled out — that for each of these commandments, such as: one should return borrowed goods, tell the truth or, when testifying, not tell untruths, not kill, etc., there are cases in which their observance is not right or at least not required.

The situation is different, of course, with the prohibition of lying, stealing, slandering, cheating, and with the commandment to do one's duty, to stand up for oneself, and the like. But lying does not simply mean consciously telling untruths as it is usually defined, but the word also expresses a disapproval of such behaviour, which obviously cannot apply to cases where it is permitted or even required — e.g. when it appears necessary to protect a valuable asset from unlawful access — and so lying actually means telling untruths when one should not. The same applies to the meanings of stealing, slandering, etc., and it is clear that doing one's duty means doing what one should do. Thus, the prohibitions and commandments cited owe their plausibility and general validity to the fact that they contain provisions that already presuppose an obligation: understood in this way, they are demands of a higher order, and indeed rather empty ones.

If the demand "you shall not lie" is not meant to express a simple self-evident truth, then worthlessness and immorality must not already be inherent in the concept to which the word "lie" belongs; it must then be understood as a primary demand, for example: not to consciously utter untruths in order to make another person believe them. This can now easily be put into the form of an "if-then" statement: if x believes that the judgement *u* is false and that *y* will believe it if *x* states it, then x should not state ii to y. But this requirement cannot be upheld: it does not apply without exception, i.e. it does not apply. And yet there must be something in the act of deliberately telling untruths that causes it to be *commonly* referred to as lying and negatively evaluated, as this word expresses. However, the prohibition of such behaviour apparently only applies as a rule that should normally be followed, as does the commandment to "speak the truth", or more precisely, to speak according to one's convictions when making a statement. Such *rules*, to all appearances, i.e. as far as can be determined by examining individual cases, apply everywhere where a primary, actual requirement is established in the form of a general commandment or prohibition; it may come very close to strict validity, so close that the exceptions are practically insignificant — in many cases this is obviously not the case — but there are exceptions everywhere. But then such a commandment, in the form in which it appears, is actually [wrong] and can only be tolerated as "not meant exactly". The task for theory is to work out and precisely define the fact of obligation that undoubtedly lies behind such a demand.

3. *Relative imperatives*

Here, an observation can point the way that one often takes when trying to clarify the correctness of behaviour in a case that is not entirely easy to judge. It is the case that we cannot initially judge the case as a whole with certainty, analyse it and then say: in view of these circumstances, this action should be taken, but in view of those other circumstances, it should not. This shows that — in the case of *primary,* actual demands — the way in which one condition is a prerequisite for the fulfilment of another is expressed not in an *"if"* but in an *"insofar as".* Insofar as the information the doctor gave the patient was in line with his conviction, he acted as he should have; in that he took away the man's last comfort of hope, to which he had clung, there is something contrary to what ought to be done; but insofar as this prompted the patient to put his affairs in order, which he would otherwise have neglected to the misfortune of his family, the action is again correct; the example could easily be continued in this way. — It can be seen that a condition *A(x) requires* the fulfilment of another, *B(x)* — in such primary requirements — in the sense that in a case where *A(x)* applies, *B(x)* should also apply *in relation* to this very circumstance. The fact that in the given case where *A(x)* applies, *B(x)* should also apply is only an *incomplete determination* of the deontic modality of the fact *B(•),* a determination of what ought to be, which only
as relative, in relation to or with regard to the given *A(x)*
.

This incomplete oughtness of the fact *B(x)* in the given case corresponds to the circumstance that this fact has a *value* with regard to the determination *A(x),* insofar as this applies. Informing the patient about his condition had the value of truth or at least sincerity, but also the negative value of robbing him of hope and comfort, and again the value of prompting him to take useful measures. The overall value of the action will be composed of such partial values and, accordingly, the final, decisive ought, as the deontic modality of the situation, will be composed of the individual relative and incomplete ought determinations.

A *principle of independence* can be established here: the incomplete...

The necessary determinations of a situation exist side by side and independently of one another — as is generally the case with relative determinations of the same object — each relative to an actual or supposed partial determination of the case. If we call the totality of the determinations that, according to the conviction of the volitional being, apply to the case the *total aspect* or, in short, *the aspect* (as opposed to a partial aspect) of the case, then we can say that the *resulting decisive ought-determination* applies to the intended situation relative to the total aspect of the case. But the subject to whom this epitome of determination is given as the overall aspect should now, and apparently absolutely, want in the sense of the resulting ought-to-be determination. The still relative datum of the resulting ought-to-be — that, insofar as the determinations of the overall aspect apply, the factual situation ought to be — is the basis of an absolute ought: someone to whom these determinations are given as an overall aspect *ought to* want the factual situation. He will act formally correctly if he follows this ought.

4. *Formal correctness. The unconditional actual ought*

We have now gained a more complete description of the facts of formal correctness in judgement-motivated actual volition, but we still do not have the necessary and sufficient condition we are looking for, namely that the actual ought, on which this correctness rests, exists. However, the discovery of this condition has already been prepared from two sides.

The fact just considered of relative ought-determinations and the actual ought resulting from them shows an unmistakable analogy with the relationships of applied possibilities and probability.°6 The specific fact of the applicability of a given determination *B(x)* in the given case x has its complete and absolute modality of actuality or non-actuality and, at the same time, the deontic modality of improper oughtness or non-oughtness. The same fact $B(x_1)$ has, in relation to any partial determination of the case x, a certain possibility and at the same time a certain relative oughtness, and it has, in relation to the entire given determination *A(x)*, which may be given to someone as the aspect of the situation, a certain resulting possibility and at the same time a resulting oughtness, both still as

Relative data, incomplete modal determinations. Someone who is given the determination *A(x)* as the overall aspect of x behaves formally correctly if he wants in the sense of the resulting relative ought-to-be, and the demand placed on him that results from this is itself *absolute*. There is also an analogue for this: the same subject behaves intellectually correctly only if it bases its decisive judgement as to whether *B(x)* applies here or not solely on the *resulting* possibility that arises from the most complete aspects available, in view of all the partial determinations and partial possibilities that these yield for *B(x)*: when it believes or does not believe in this sense.

This strict analogy is accompanied by a second one, which shows that there is not merely an external similarity here, but an essential connection. The will is directed towards the realisation of a determination: in its innermost essence, it is the striving for realisation. Willing is materially correct when, in the realisation of what is willed, it encounters a fact, just as a judgement is materially correct, i.e. true, when it encounters a fact. Now, the fact that, given the situation as presented to me by aspect *A(x)* in the given case, I will "realise" *B(x)* by willing it, i.e. that I will behave in such a way that the applicability of *B(x)* in this case is a fact, is, despite objective certainty, always only given as a possibility: in *the* possibility that it has relative to *A*(x). This is decisive for me — and for everyone in the same situation —; I behave correctly when, after considering the facts, I make my decision that "*B(x)* will be true" if the possibility of this is greater than the possibility of it not being true. But this judgement is based on the existence of something real in which I myself am involved as an active factor, and this real participation in the event is expressed in the experience of my will: it is only through my will that I arrive at this judgement; the decision in the judgement is based on my resolve. Thus, with the decision, the resolution is also correct: materially, if both prove themselves in the facts; formally, if they have been made in the sense of the predominant possibility, and therefore have *a predominant chance of proving themselves*. This is the necessary and sufficient condition sought for the actual ought, which establishes the formal correctness of the judgement-motivated will. The requirement to want formally correctly is the

Commandment to strive for material correctness of volition. Act in such a way that your volition has the greatest probability of realisation. It is in the nature of volition, which seeks proof, that it is correct insofar as it can be proven. The will, one might say, in seeking fulfilment, gives itself the sole commandment to satisfy the conditions of fulfilment. There is no commandment that is not contained in this one.

The commandment of formal correctness requires what can reasonably be demanded of human beings as creatures endowed with free will: to fulfil the requirements of material correctness "to the best of one's knowledge". These requirements (the most important of which are set out in § 10) represent an ideal: fulfilling them is the goal, but cannot actually be required; striving to fulfil them is strictly and unconditionally required.°° Formal correctness does not guarantee material correctness, i.e. the fulfilment of the will, and is therefore independent of it; but it favours fulfilment and, in a large number of cases, will lead to it in a corresponding majority of cases. If we have once failed to achieve a goal and we can issue ourselves a certificate stating that we have acted correctly to the best of our knowledge, i.e. that we have fulfilled the conditions of material correctness to the extent possible given our knowledge of the situation, then we have acted formally completely correctly.

However, it may seem doubtful whether this correctness is sufficient in all cases; this is certainly true of the many everyday actions that are considered morally indifferent, but what characterises an action as morally good seems to be something other than this simple validity of the will that underlies it. The question remains to be investigated. For now, just one remark to show that the requirement of formal correctness is not quite so insignificant after all. Since all our demands, indeed all demands in general, constitute a composite demand, it is part of material correctness that they are all compatible with one another (§ 10, 1). Thus, in striving for material correctness, individual volition should seek to fit into a system of harmonious desires and help to constitute such a system. In the face of this requirement, some external manifestations of a desire will prove to be mere pretence, and the desire, which has nothing more than the probability of realising a single goal taken out of the larger context, will not be formally correct.

5. *Primary general demands as rules*

Attempts to establish a primary requirement of a general nature that would strictly apply as a maxim for our actions have repeatedly failed; however, this empirical fact does not prove the impossibility of such a maxim. Now, however, the reason for this failure can be recognised and explained based on the nature of volition.

The actual desire — and that is what we are talking about here — is directed towards a future event, albeit one in the near future, the reality of which I anticipate with a certain degree of certainty, but without the strict evidence of this certainty. My knowledge of the situation — given in its entirety — yields, even in the most favourable case, only a probability that what I want will happen because I want it to happen. And my will itself, above all its strength, also plays a significant role in the justification of this probability, in the degree to which it can claim it for itself. Therefore, one can never claim that if the condition *A(x)* — let it be a condition that does not presuppose any requirements — applies, then the will that *B(x)* applies will be successful. And therefore one cannot say: if *A(x)* applies, then the desire for *B(x)* to apply will have a predominant possibility of realisation. For this "if" means, strictly speaking, "in every case where *A(x)* applies". Since it cannot be claimed that in every such case *B(x)* is realised by wanting, there are possible cases where this does not happen, and in such a case there will always be some determinations which, added to *A(x)*, result in a determination that already excludes the realisation of *B(x)*. This means, however, that there are cases of *A(x)* where a corresponding addition to the aspect — and this is indeed possible — results in an arbitrarily small possibility for *B(x)*, and since the possibility from the respective overall aspect is always decisive, the formal correctness for the volition of this determination will then be lacking.[°0] Therefore, it is never generally and strictly required to want to bring about *B(x)* if *A(x)* applies. One could attempt to derive a primary general requirement by including in the prerequisite, in addition to the determination *A(x)* of the situation, the condition that *A(x)* be considered as an overall aspect.

or rather as an overall decisive aspect. One such requirement would be, for example, that when making a statement, one should testify truthfully if there are no known circumstances that would prevent sincerity. However, this would constitute a higher-level requirement and would lose the essential content of the original maxim, which is to testify truthfully. Strictly speaking, the new maxim, with its clause, says no more than, for example, to testify insincerely if no counter-reason is known. The difference — which is lost in this version — is only that in the first maxim a decisive counter-reason is known only in exceptional cases, while in the second it is known as a rule. And so the meaning of the maxims, which appear in the form of general primary demands, is that, although they are literally and strictly speaking false, they are nevertheless, when taken imprecisely, useful approximation formulas or rules. In this respect, the best "laws" are wrong in that they make what can only be a rule into a law.

Such a rule, if followed by all who wish to follow it – and in this Meaning "like a law of nature" — producing more right than wrong, but also wrong and occasionally formally wrong desires. **That is why** there is no single primary requirement that meets the demands of Kant's categorical imperative as a maxim: "Act according to that maxim through which you can at the same time will that it become a universal law." At least, if this ability to will is not to depend on personal inclinations, but means something like being able to will without contradiction in one's demands. A maxim of strict validity is only the demand for formal correctness itself and everything that follows from it. However, this is not merely a maxim, but at the same time the highest commandment: the imperative.

6. *The very essence of oughtness. The contrary of oughtness*

A determination that is a necessary condition of formal correctness must apply; a determination that is a sufficient condition of this correctness is, in this respect, one that *may* apply. Such a sufficient condition is an implicant $C(x)$ of the actually required determination $B(x)$. This $B(x)$ can only be fulfilled by being true in one case, therefore as an implicate of the complete determination of the case, and thus it will always be such among the determinations of the case.

Implicants C(x) — provisions that go beyond *B(x)* — are given. The partial provisions of C(x) that are not contained in *B(x)* are neither required nor prohibited: they *may* apply and nothing more. No obligatory action is so precisely prescribed that it cannot still be carried out in any of an infinite number of ways.

The reason why such deontically *indifferent* determinations exist is easy to see from our definition of the concept of formal correctness. If this correctness lies in the fact that volition has maximum validity with regard to the aspect of the situation, then any provision relating to volition or its execution that has no discernible influence on this validity must also be indifferent to what ought to be done. It is clear that the broad area of what is indifferent in terms of what ought to be done owes its existence to the incompleteness of our understanding. If an indifferent determination constitutes the overall aspect of an action, then the action itself is indifferent. A large number of our ordinary activities belong here; if we knew more, they would be considerably reduced, but in some respects also expanded, as knowledge reveals new dependencies and sometimes also new independencies.

Our ethical judgement recognises a difference between strict obligation and Zero of obligation, which exists in indifference, a kind of transition. A small deviation from the required performance is often tolerated, a larger one is more difficult to bear, as if the more precise approximation to the required performance were less obligatory than the lesser one. This is obviously impossible if the required performance itself is strictly obligatory. It can only occur if the required specification is more precise than intended. For example, a mechanic undertakes to deliver a piece that "should be 10 cm" in length, meaning that the deviation from this measurement, either upwards or downwards, should be less than 0.3 mm, and other specifications allow for much greater leeway, the limits of which cannot be precisely specified. In all these cases, only adherence to the interval is strictly required, with perhaps the achievement of a specific point within it being most desirable. But, if rightly desired, this is then the most feasible in the case, and the fact that it is not strictly required is only due to the imperfection of our abilities. What is strictly required is the best possible approximation to the best, and that is again the greatest achievable reliability of the action; anything beyond that is not required to a lesser degree, but is not required at all.

However, conscious and deliberate deviations from the most proven course of action are considered more forgivable if they are minor, and more serious if they are major; one speaks of minor and major transgressions. If someone has preferred a less proven course of action to the most proven one, they have not violated a lesser obligation, but a strict one. However, instead of the highest value, they have nevertheless realised or strived for a lesser value and, instead of following the decisive resulting obligation, have followed a relative obligation (cf. §11, 3). Only the relative ought allows for different degrees: what has a greater chance of success in view of the given situation is *in this respect* more strongly ought, but only the most proven is wholly and decisively ought. Only this may be called obligatory, and everything that is obligatory is equally obligatory. If we call one breach of duty lighter and another heavier, this only has the justified meaning that in the former a stronger relative ought is fulfilled, while in the latter only a weaker one. But this relative ought is of a different kind than the unconditional ought; what ought to be in a relative sense is actually something that *ought to* be, insofar as this or that partial determination applies, but *ought* not to be in view of other circumstances that are present.

The significance of propositions about what is contrary to oughtness, assuming the concept of improper oughtness, has been demonstrated (§ 10, 4). In contrast to proper oughtness, however, there *is* actually contrary-to-oughtness, and it is noteworthy that in such cases emotions and desires arise that correspond entirely to our paradoxical propositions, according to which the contrary-to-ought demands the contrary-to-ought and, if there is contrary-to-oughtness, nothing or anything should be ought. The urge for retribution and revenge, as well as the "if you are like that, why should I be better" illustrates the first, the sometimes bewildered indignation ("that's where it all ends") and the "demanding" of senselessness ("there should be ...") illustrates the second proposition. In poetry, there are many beautiful examples of emotions in the sense of this sentence (e.g. in Shakespeare, Macbeth **III,** 4, 1—20; King Lear **III,** 1, lff, **III,** 2, 18 and elsewhere). — In fact, even that which is contrary to what ought to be does not negate the unconditional requirement of what ought to be (sentence 29, § 3, § 7), and this is decisive for the correctness of our behaviour. But the contrary to what ought to be, in addition to its opposite, also demands the contrary to what ought to be as a consequence, and thus establishes a *relative* ought to be for what is contrary to what ought to be. Thus, not only does the saying about the curse of evil deeds, which so easily have their counterparts in their wake, apply, but also the abandonment of the consequence of the wrong, the correction of evil, although dutiful as a whole, always and essentially — already as the abandonment of a consequence — has worthless components and thus relative contraventions of what ought to be: it is the worst thing about evil that it does not allow us to do what is right without guilt.

The improper nature of what ought to be comes into play here in that even what is contrary to what ought to be, once it has happened, must be accepted as fact, and it would be unreasonable to oppose it, however much one may wish to prevent its repetition or continuation. Psychologically curious, incidentally, is the tendency to judge what is contrary to what ought to be more leniently, if not to accept it as right, once it has lasted long enough: a kind of transfer of consent to the facts to what, according to a habitual judgement — provided we do nothing to prevent it — will "remain so". Perhaps there is a trace of justification for such behaviour in the experience that "it works that way too", which, of course, can only result in a relative ought to be for **the future**.

§ 12. THE WILL AND THE VALUE

I. Dependence of ought on value

We do not want anything that does not have value for us in itself or for the sake of something else (valuable), and we should want something if it actually has value and should decide what is most valuable under the given circumstances. In fact, it is primarily feelings of value and considerations of value that determine our desires; Thoughts about the possibility and probability of realisation play only a secondary role and seem to come into play only when it is a question of the possibility of realising a value. Something worthless or unworthy may be easy to realise, but that does not mean it should be done. Our reduction of formal correctness to maximum reliability therefore owes its apparent justification to the fact that the fact that when we think of reliability, we somehow think of values. Only if value itself is based on reliability and not the other way around is our statement of the necessary and sufficient condition of formal correctness correct.

As every relative target value corresponds to a value, it is - in
Section 11, 3 — already discussed, the relationship between value and decisive obligation still needs to be defined more precisely. The formula that one should realise the greatest possible value in a given case needs to be clarified. Obviously, neither the purely external probability of achieving the individual goal nor the value of this goal alone is decisive for this decisive obligation. If values are equal, the more probable goal should be chosen; if probabilities are equal, the more valuable goal should be chosen. Thus, the decisive factor for the choice is what is called the hope *value* or *expected value* in probability theory; if value and probability are expressed numerically, it is measured by the product of their measures; if *w* is the measure of the probability of achievement and *a* is the value of the goal, then *w.a* is its expected value. One should choose the goal whose expected value is the greatest. For a high goal, one will risk a lot and should risk a lot; for a low goal, one will risk little and should risk little. The validity of this approach can also be demonstrated by statistical consideration: "Among a large number *n* of cases" — where one attempts to realise the value *a* with the probability w — "approximately nw will yield the profit *a*, the rest nothing: thus, the average profit is *anw:n ——aw."*** The larger the number of cases, the closer the observed average amount of the realised value will approach the amount *aw*; so "on the whole" of the event, the greatest value will be realised if the greatest expected value is chosen in each case — at least, the probability of this can be approximated to certainty as desired, provided that a sufficiently large number of cases are taken into account.

Of course, in the vast majority of our actions, neither the value of the goal nor the probability are given in quantitative terms, but rather are somehow given to us by our sense of value or our values, while the probability is given **by** the strength of our prior assumption that the goal can be achieved. Together, these two factors provide a kind of estimate or "impression" of the feasibility, which, in the best case, characterised by the mathematical expected value, but not directly, so that it would give the *measure* of this quantity. The expected value itself cannot be this measure, because feasibility is a possibility and has an upper limit of 1, but it can become arbitrarily large with increasing values *of a* for any given w.

To find the measure of certainty, let us consider a hypothetical example whose simple circumstances are easy to grasp. In a shooting competition, different prizes *a*, a2, a3, etc. are awarded for different levels of performance in ascending order, the highest of which *is az — A.* Each shooter may only compete for *one* prize.

Someone who has the probabilities *w, w2,* Wg of winning the individual prizes will decide to compete for the prize that, according to their ability, has the highest expected value for them.

offers, let us say *a* with the corresponding probability w. However, the greatest certainty would obviously be that of a shooter who was sure of winning the highest prize *A*; the certainty of our less perfect contestant's endeavour must be measured against this, and the measure we assign to it must indicate the degree to which it approximates that highest certainty. The reliability to be measured corresponds to an expected value *w3az,* the highest possible expected value being *A* – since w= l here – and so we have the measure *w3•3lA* for the reliability under consideration· This indicates the degree of approximation of the existing expected value •3a to the highest possible value here, *A,* and at the same time the approximation of the existing reliability •3a3/d to the highest, i.e. *A*|*A* or 1.

The result of this consideration is easy to summarise in general terms. If, in the given case, n possible behaviours are possible with the values to be realised *a, a, ... a,* among which the largest is *A*, and with the corresponding achievement possibilities wt. • w, then the provenability of a particular one of these behaviours measured by the ratio of their expected value *w;a;* to the highest value that could be considered here, 1. *A* or *A,* which would exist if the highest value could be realised with certainty. The requirement of formal correctness demands that one choose the behaviour with the highest expected value; then the will also has the greatest probability of success that it can have according to the circumstances and the possibilities of achievement resulting from them. The probability of success also depends on the ability that a will has at its disposal, for this is a determining factor in the probability of achievement. A will that had the highest ability at its disposal would always have to choose the highest value and, since it would certainly realise this, would have the highest probability of success 1, which ensures material correctness.

If the highest value offers the greatest probability, then the correct choice is *wa|A ——wA|A* -- *w*, and the probability is equal to the possibility of achieving the individual desired goal. Otherwise, however, the value of the goal is always a determining factor in the probability. The question arises as to how this fact is compatible with our earlier findings, according to which formal correctness is simply given by the maximum probability of realising one's will.

2. *Value and feasibility*

Where the solution to the question just posed is to be found has already been noted in § 11, 5. The material correctness of volition requires that not only should the individual volition achieve its individual, explicitly formulated goal, but that it should also be a link in a system of entirely correct acts of volition, for only then will everything that is intended be achieved. An act of will must take this requirement of material correctness into account in order to be formally correct. The individual acts of will must be such that they appear to be maximally suitable for combining into a consistent system of acts of will, into a unified, harmonious overall will: for this inner harmony is the formally required partial condition of material consistency. Now I can certainly say that something has value for me to the extent that it is in accordance with my overall will, corresponding to my lasting and essential interest. If the object presents itself to me as a possible goal of my will, the greatest strength°° with which my will, i.e. my overall will – if it is sufficient – would still strive for its realisation, would be a measure of the value that object has for me. A measurement of value that is possible in principle would result if

two values *a* and an with the possibilities of realisation m and •2, between which well-considered volition without recourse to external motives — because one must decide anyway — finds no solution

. Then the expected values are to be regarded as equal,

w a *w a2* and this results in the ratio of values *a :a* ——

= w,: **w,.**[38] Of course, the respective desirability is only a fluctuating measure of value, but in this respect the sense of value is not better, but rather worse; moreover, it proves even less amenable to any measurable comparison than the former.

In this sense, the (potential) participation of my total will in a desired goal, as value factor *a*, is represented in the expected value *wa*, and the feasibility of the desire, which is proportional to this *wa*, is therefore also directly proportional to the participation of my total will. The greater this participation, the more my total will is fulfilled in the event of achievement, and we are thus justified in seeing in the *value a (a ——wa|A*, in the case *of w*= l, *A* ——l) that *validity* of our volition which it would have by virtue of the participation of the total will given in it, if the possibility of achieving the (individual) goal l, i.e. if it were certain to be achieved, and if the greatest value that could be strived for in that case (therefore the greatest possible participation of the total will here) is assumed as a unit (of the participation of the will). Accordingly, the value — the value "for me" or the personal value — of a fact can be understood in the manner indicated as a chance or possibility of proving oneself, i.e. here simply of asserting my total will.

The simple idea that we have thought through to its precise formulation finds its natural expression in the entirely plausible statement that whatever has value for me, to the extent that it has value for me, is in accordance with my overall will and, precisely because of this, represents an opportunity for the fulfilment of this overall will when I realise it through my volition. For *A* —- l, the verifiability *wa* is to be understood as a composite probability: the probability that I will achieve my individual goal and at the same time satisfy my overall will. Of course, *A* would then have to represent the highest value that could be considered a possible goal for me, not only in this case, but in general; that would be the case in a decision where, in the strictest sense, everything is at stake. Here, too, it is not necessarily the highest value that is to be desired, but the value *a* that, together with the corresponding ir, provides the greatest expected value, and one sees that, for example, those whose powers are not sufficient for the highest goal to give them a sufficient chance of achieving it are faced with the harsh necessity of striving with all their might for a lesser value. His only consolation is that he has fulfilled his duty and thus the enthaving attained decisive moral value. This is what is both terrible and redeeming about tragedy.

In place of consideration for the value for me, the share of my total will, consideration for the value for all, the share of the total will of society, must now take its place in very many cases — actually in all cases, except that it does not become modifying in all cases — for the person who lives with others. Of course, my will is also a constituent of this total will, and like mine, it is subject to the regulative principle that it must strive for a harmonious whole, and is ultimately compelled by facts to strive for it: a duty and a compulsion that has a concrete effect on the individual will and ultimately on the individual acts of will.

3. *The objective value*

The value for the general public is still a personal value, except that it refers to a collective of subjects rather than to a single subject. This raises the question of whether the value that something has for someone in terms of desirability or worth does not also correspond, in favourable cases, to an objective determination that would then deserve to be called actual value or value per se. Without doubt, the justification for valuing something, if it exists, must be based on such a reality — just as the justification for desiring something is based on the reality of oughtness. The circumstances of oughtness, which we are now familiar with, provide useful guidance in examining this question.

What ought to be has value, insofar as it ought to be. No justification is required for this statement; it follows as an axiom directly from the nature of ought and value. Just as what ought to be in the sense of a desire has value in the sense of desirability, so what ought to be in reality must also have actual value. What ought to be in the true sense and unconditionally is the formal correctness of the will and everything that it demands: it therefore also has unconditional value; because it has this value, it ought to be. The necessary and sufficient condition of formal correctness is now maximum provability, i.e. the predominant possibility that the will, or more precisely the total will exercised in the will, will achieve its goal. This is the objective reason for oughtness and, apparently, also for value: the purely intellectually grasped

Equivalent to both unconditional obligation and unconditional value. Of course, neither of these terms captures the actual essence.
- otherwise they would have to be identical and not merely equivalent to each other and to those purely "objective" facts. The essence of value can only be "perceived" through the mediation of the feeling called the sense of value, the essence of oughtness only through the mediation of the will; but not by thinking about them and considering something that stands in a certain relation to them, but more simply by the feeling or the will — without having to be grasped itself — bringing to mind, presenting or offering our comprehension the corresponding object — as its "meaning" — similar to how an idea presents its object. Just as the shape of a circle can only be grasped through intuition itself and in its own essence, so value and oughtness can only be grasped through this emotional presentation. Without feeling and wanting, we would never know what value is and what oughtness is, just as without a sense of colour we would never know the essence of red and blue, and without a vivid grasp of form we would never know the essence of the circular shape. However, this does not prevent us from assigning an equivalent that cannot be grasped intuitively to the intuitive shape of a circle, for example in a mathematical, in its purest form in the analytical version of the concept of a circle, which contains the necessary and sufficient objective conditions for the existence of such a shape. Similarly, our analysis of the ought has provided the purely intellectually comprehensible, "objective" equivalent, the necessary and sufficient objective basis for the existence of this object, which can only be "perceived" emotionally, i.e. directly comprehended. It is at the same time that of unconditional value. Such theoretical endeavour therefore in no way deserves the name of intellectualism, if this is to be taken as a reproach. For it respects the uniqueness of objects, even those that cannot be grasped purely intellectually. It does not seek to dissolve them through thought. But in the end, cognition is still thinking, even if it uses non-mental and even non-intellectual means of comprehension (as presenters), and for every object that appears, there must be conditions that can be grasped by thought and that are sufficient and necessary for its appearance. It is the task of science to seek them out: in this respect, science, like any other, is necessarily intellectualistic and rationalistic.

The unconditional value of formal correctness is therefore based on proven

. A bad, i.e. formally incorrect, individual will can prove itself, "by chance" and externally, insofar as the necessarily co-willed remains unconsidered. But a bad overall will cannot prove itself at all, for in the overall will everything that is required is represented; it is the epitome of all my will and co-will. If it is formally incorrect, it will act against some conditions of this correctness, but it will also want the conditions of this correctness, because these are necessarily co-willed in every will, according to its meaning. It therefore wants contradictory things and can never be fulfilled as an overall will.

The relative values that correspond to the relative and incomplete target determinations — §11, 3 — will be accurate insofar as these target determinations actually exist, i.e. insofar as they are based on proven facts. Now, the values of the objects to which primary — cf. § 11, 2 — will is directed have been described as the share of our total will in these objects. Of course, this description initially applies to personal value. However, one can derive a characteristic, i.e. actual objective value, from it if, instead of the share of a given personal — individual or "collective" — will, one allows the share of a formally correct, i.e. verifiable, total will to be decisive. Since the will still has a great deal of leeway within correctness in choosing its concrete goals, this seems to result in a relativity of values – with the exception of ethical values – in the sense that not only, as a matter of course, something real has value insofar as it carries a valuable determination in itself – alongside which it can also have the worthlessness of worthless determinations – but also that a determination is only valuable insofar as its applicability is conducive to the realisation of a correct overall will. The right will, however, would be the only absolute good.[43] Now there are certainly purposes of such a kind that a will can commit itself to them in circumstances of varying intensity within broad limits without thereby becoming incorrect: these are the purposes of things whose personal evaluation cannot really be measured by the standard of objective correctness; among them, one may prefer this, another may prefer that, as it suits him, because they are indifferent to the correctness of the will. But there are also determinations in which every will in certain circumstances

To take part in order to want correctly. We do justice to this fact when we explain: the provisions which, in a certain proportion to the strength of a will (overall will), give it the greatest probability of fulfilment, are *objectively* valuable in precisely this proportion (strength of participation within the meaning of § 12, 2).

This value formula does not define the value of individual provisions in a circular manner by the validity of a will and this in turn by the value of its goals, but rather it names those that are valuable to the extent that they must be desired by a will so that it, as a total will, is most valid. Under all circumstances, therefore, formal correctness itself is to be desired most strongly: its value is the highest, absolutely decisive one. But this does not mean that all other values spring from this value, but rather that it includes them all. It is not because a right will advocates for the objects in this relationship that they are so valuable, but rather the will is right and valuable because it wills the objects in the relationship of their values, because it does justice to their values. As an objective — intellectually comprehensible — basis for their values, the determinations have a certain validity in a consistent system, the realisation of which the right will strives for. From this point of view, it becomes understandable that truthfulness, reliability, justice and goodwill are valuable qualities, and a developed ethic should have gained clarity about the relationships between their values on the basis of the value formula. At the same time, the reason becomes clear why — apart from errors in values — different things are considered morally good at different times by different people: among unequal individuals, under unequal conditions of the existence and emergence of a society, very unequal primary demands have the best proven reliability.

One should not argue that a consistent system would be easiest and safest to achieve if one did not want anything at all and simply let things take their course: our desire is a fact, and our will is a real factor in reality and, by its very nature, seeks to shape it. Evidently, not all formations are equally viable in the sense indicated, and events necessarily strive towards increasingly viable formations: comprehensive real wholes, such as the development of organisms, supra-individual formations shaped by real connections

, as shown by the development of society. Where we ourselves consciously experience our participation in this process, there we want. But the laws that are binding for the will lie in the meaning and essence of the will itself: every will seeks to prove itself and imposes on itself the requirement to strive for the greatest possible validity.

4. *Good will*

The consideration of what lies in the sense of a judgement or a will immediately led us to the co-judged, the co-willed: this, in its entirety, to which the explicit is only a borderline case, results in what is *meant* in the act, its meaning. Depending on how precisely we mean it, the judgement or the will for the meaning of the current experience is applied in varying degrees of completeness and, so to speak, with varying degrees of weight. However, this commitment means that one is not only bound to the implicit in a logical and deontic sense, i.e. by demands of correctness, but that this ideal bond also corresponds to a psychological-real one. Judging or willing a state of affairs results in an increased disposition or readiness to judge or will its implications — including the deontic ones — and all that is needed is an occasion, but not a new reason, to trigger them. This empirical fact is only understandable if there is something real that forms the basis of these increased possibilities — disposition is possibility — and it can be concluded that the basis of disposition, which is expressed in a judgement or in a volition, is at the same time the basis for judging or volition of the implications and, through the positing of that act, enters into a state of readiness for the explicit positing of what is implicitly posited. Its entry into an act gives it its essential (real and psychological) content, which corresponds to an object as the intended meaning: it thus determines the objective direction or the "meaning" of the experience.[44] In the case of volition, we call this real basis the will; it is the permanent psychological representation of what has presented itself to us in meaningful certainty as the total volition of a human being. The total volition of a society cannot be found in the form of an individual will; it is represented by a majority of individuals with wills, who but at least it is not merely a collective, but is connected by real interrelationships of manifold mutual influence, and through these finds an admittedly imperfect reflection and representation in the individual will. Thus, by pursuing the collective will, the individual will ultimately reflect on his own — socially determined — will.

A consideration that aims at right will is always a reflection on one's own will and often presents itself in this form. One asks oneself: what do I actually want? Is what I want there actually what I want? Since the facts are judged everywhere and what should be is wanted everywhere, all convictions and all wills have a core in which they agree and are right. The criminal out of weakness and even the villain basically wants what should be; he only fails in that he also wants what should not be. Thus, his will contradicts itself and the law it gives itself by simply being will. Every evil will is also a kind of stupidity, no matter how clever and astute it may be. Convictions which, because they are correct, are included in every thought and lie ready somewhere in every mind. They have all already been expressed, but it is only through insight into the laws of oughtness that they acquire their clear scientific meaning and justification.4°

When deliberation precedes a decision, judgements take into account the facts of the case and the expected outcome, resulting in a desire that is actually motivated by judgement. But very often, in unimportant matters and also in important ones where there is no time for deliberation, we act without thinking, without necessarily acting "rashly". Here, too, the whole situation is grasped in a quick, comprehensive judgement, only without analysis; most of it remains only implied, which would also be explicitly thought in further analysis, both on the part of the given facts and on the part of the intended facts. With such rapid comprehension, most of it remains unconscious and is only represented dispositionally, but it is nevertheless truly represented, namely by the dispositional foundations that come into play in such actions. Thus, we can also specify and explain more precisely afterwards what was meant.

For the assessment of the correctness and evaluation of an intention is precisely *what* was meant. Only in this and not

In most cases, a given guiding principle contains what can be called the maxim of this desire. But the requirement to will to the best of one's knowledge also demands that one should not simply be determined by the aspects at hand, but should seek out the most complete ones that can be attained. Here, a principle that is little known in its essence guides us to understand the circumstances of the case that are precisely the most decisive for the possibility of a certain success, in this case the viability of the will.

The judgements that determine our will have little to do with the chances of success. The probability of achieving an individual goal is considered—and even this is usually only reflected in the strength of our expectations—but the viability that our desires bring to our overall will is not grasped intellectually, but rather as a value in an emotionally "vivid" form. Given the infinite complexity of the circumstances that determine this, the ability to grasp this possibility is a task that—perhaps with a few rare exceptions—exceeds the powers of our intellect, and where it fails, our sense of value provides us with a means of comprehension that is certainly not infallible, but nevertheless astonishingly effective in view of the magnitude of the task. If, in order to determine whether a line drawn on a plane is circular, one had to first examine whether there is a point from which all its points are equidistant, one would indeed strive for very precise knowledge, but would not achieve it, since one would have to carry out an infinite number of measurements (tracing with a compass already made use of the visual perception of curves); but intuition gives us the shape at a glance, albeit imprecisely and with the shortcomings of visual perception. Feeling accomplishes something similar — incidentally, it is not only a means of comprehension, but also has its own significance for psychological life. Where we experience evidence for the correctness of our volition, it has its basis in a special quality of the sense of value.[4®] It is only through this evidence-based quality that formally correct volition also becomes *internally correct and justified.* Furthermore, a correct desire can only be considered truly good if it does justice to felt values, not just their intellectually grasped equivalents. Only feeling has access to the essence of value, and it is through emotional presentation that the comprehends what constitutes an essential characteristic of good overall will as a harmonious unity of individual goals; what can be comprehended purely intellectually is nothing more than the consistency of a coherent system.

We attribute individual will to the will and evaluate it accordingly. A basis for disposition is, of course, only partially and usually very incompletely characterised by a single achievement. If I see only the external effect, I can say: it was such that it was able to achieve this, and therefore also: there are cases where it achieves such things — knowing this can also be important, in particular because one can infer the disposition for smaller achievements from the larger achievement. I know much more when I recognise the additional circumstances under which the basis has been activated, because now I can assume that under such or similar circumstances it will also achieve such or similar things elsewhere. I not only know that there are cases of such behaviour, but also something about the conditions under which it occurs. The more special external conditions are required for the performance to come about, the less characteristic it is of the disposition; the less, the more the nature of the basis comes into play in the performance. According to these general points of view, we judge a will on the basis of its actions. And this is how we measure the positive or negative contribution to the evaluation of the will that one of its actions provides.

The fact that a will behaves in a given way ff under the given conditions *A B* C no longer depends in any way on the conditions, but is solely a matter of the will: a characteristic that distinguishes it. If this applies to all human beings, we attribute this behaviour to human nature and, among human beings, do not attribute it to the individual; we are inclined to say that, as a human being, he must behave in this way. But if such a law of behaviour applies — properties are always laws of behaviour or state
- only for some people, then it is all the more characteristic for each of them, the fewer people share it with them, and finally there will be individual behaviours of the individual will. These constitute the peculiarity of the individual will: it is up to them and them alone that they behave in this way and not differently under these circumstances. One then says: a person can behave in the same way under the same circumstances.

circumstances; this amounts to saying that there are people who actually do so. The mere possibility that "a person will behave in such and such a way under circumstances *A, B, and C*" is applied to the person in question and results in a certain relative possibility for his behaviour (cf. § 9, 3). But the given person is completely determined beyond this incomplete determination, and the fact that his will behaves in this way and not differently under given circumstances is precisely due to this individual will and is attributable to him. The phrase that the given will is completely determined as a reality is easily misunderstood to mean that all its determinations are given by the rest of reality. But if every element of reality were to be completely determined by the totality of the rest, without determining itself, i.e., without contributing to its own determination, then in the end everything would remain indeterminate. Determination must not be understood purely passively, as being determined by something else. An element of reality must be determined in something simply in itself and from itself. One can also see what this irreducible core of determination consists of: in the fact that this reality behaves in this way and not differently under the completely given conditions of the environment.

Of course, a composite reality owes its self-determination to its elements. It came into being through their coming together, it constantly takes on new elements and loses old ones. This is true of human beings as living creatures. That is why, mindful of their origins and history, we do not hold them equally responsible for every behaviour. But we do hold them responsible for every behaviour insofar as their actual will is manifested in it. Thus, the past also belongs to the completely given conditions of the environment. I am not simply responsible for having acted in this way, nor am I unreservedly responsible for having acted in this way under the present circumstances. Rather, what belongs to my innermost will and my very essence is *the* unchangeable: *I am this way, i.e. I want it this way,* that I have become this way (as a result) under the conditions of my external history, that I now act this way under such present circumstances. All changeable characteristics are an external, prehistorically conditioned form of activity of this permanence. To the exact extent that this core of will comes to the fore in my behaviour, I am responsible for it.

Responsibility. Attribution and responsibility presuppose the will—at least an innermost core of will—as something that is absolutely self-responsible. It can only be so if it is a genuine element 4°; for only that which is not composite and has not been created would find nothing else in the world onto which it could shift responsibility for its essence. However, it is not the purpose of the present investigation to decide on this matter.

NOTES

'The term 'fact' here refers to what Meinong first recognised in its objective nature under the name of the objective and subjected to explicit consideration. See in particular this author's book *Über Annahmen (On Assumptions),* 1st edition, Leipzig 1902,
2nd edition 1910, then my works: 'Gegenstandstheoretische Grundlagen der Logik und Logistik' (Object-theoretical Foundations of Logic and Logistics), Leipzig 1912, 'Studien zur Theorie der Möglichkeit und Ähnlichkeit, Allgemeine Theorie der Verwandtschaft gegenständlicher Bestimmungen' (Studies on the Theory of Possibility and Similarity, General Theory of the Relationship of Objective Determinations), *Proceedings of the Academy of Sciences in Vienna, Philosophy and History Class,* vol. 194,
1st treatise, Vienna 1922.
The main thing is that the present duality is recognised and brought to bear in the practice of logic. Attempts in this direction are presented in my 'Gegen-standstheoretische Grundlagen der Logik und Logistik' (Supplementary booklet to Volume 148 of *the Journal of Philosophy and Philosophical Criticism, Leipzig* 1912, and M. Honecker, Gegenstandslogik und Denklogik, Berlin 1921. See also my article 'Uber Wesen und Aufgabe der modernen Gegenstandstheorie', in: *Zeitschrift für Philosophie und philosophische Kritik, Leipzig 1912, and M. Honecker, Gegenstandslogik und Denklogik, Berlin 1921. See also my article 'Uber Wesen und Aufgabe* der modernen Gegenstandstheorie', in: Zeitschrift *für Philosophie und philosophische Kritik, Leipzig 1912, and M. Honecker, Gegenstandslogik* und Denklogik, Berlin 1921. *Critique1* Leipzig 1912, and M. Honecker, *Object Logic and Thought Logic,* Berlin 1921. See also my article 'On the Nature and Task of Modern Object Theory', *Die Geisteswissenschaften,* Vol. I, 1913/4,
pp. 616—19.
° For more details, see my *studies on the theory of possibility and probability,* op. cit., esp. chap. I.
^ "Propositional functions" as opposed to "propositions" in the designation of logic. See in particular Whitehead and Russell, *Principia Mathematica,* Vol. I, Cambridge, 1910.
Whitehead and Russell, op. cit. p. 18.
® What modern exact logic deals with under this name is, according to the words, a relationship between propositions or propositional functions, i.e. between judgements or assumptions; but very often what is actually meant is the relationship between facts, which gives them their objective meaning. This is what we are concerned with here. A point where "object logic" and "thought logic" should be clearly distinguished.
° This idea is represented and developed, based on F. Brentano, by K. Wolff in his *Grundlehre des Sollens (Basic Theory of Obligation),* Innsbruck, 1924. Here, the essential significance of the dispositional for the fact of subjective obligation is brought to bear in a commendable manner.
^ Cf. A. Meinong, *Zur Grundlegung der allgemeinen Werttheorie* (published by E. Mally), Graz, 1923, p. l45f.
• Meinong's "desiderative". Cf. his fundamental work Über emotionale Präsentation, *Sitzungsberichte der kais. Akad. d. Wiss. in Wien,* vol. 183, 2nd treatise.

lung, Vienna, 1917, esp. §§ 5, 11, 14, 15 and p. 43, reference to F. Weber. Cf. now also this author's *Etika,* Ljubljana 1923.

1° Not entirely, insofar as a system can also establish principles that are not immediately evident but prove themselves in all their consequences. In a system of definitions, the principles are not judgements at all, but free assumptions; but this is not what is sought here, rather a system that does justice to the existing facts of oughtness in correct judgements.

This applies to the (absolute) ought-to-be considered here, but the relative ought-to-be is consistent with the can-be or possible-to-be. See below, § 11, 3, and further.

1• ! *AB* naturally stands for !*{AB).*

'° *M f A* v *B* stands for *M f{A* V *B),* analogous to 3f^ *A v B.*

• It is irrelevant whether *B* is desired because *A* is desired or because *A* is, in fact, because *A* is believed to be true, judged to be true. "If *A* is, then & should be" and "If A should be, then *B* should be" are equivalent relationships, and the fact that this equivalence is completely and immediately obvious is another sign that our extension of the natural concept of ought is not exactly unnatural, because here the ought-to-be of A and the actual being of A can stand in for one another. The concept of imaginary desire (and that of imaginary feeling) was introduced by Meinong. See this author's book: *Über Annahmen,* op. cit., index.

• Cf. Meinong, *Über Möglithkeit und Wahrscheinlichkeit,* Leipzig 1915 (index); also my *Studien zur Theorie der Möglichkeit und Ahnlichkeit,* op. cit., § 37.

l' Cf. Meinong, op. cit. (index); my "Studies", op. cit., § 39.

1® The sentence does not correspond exactly to Principle I - § 2 - but to a subsequent sentence - 6., § 3, § 5 - which, when applied to volition, allows for a simpler version than the former.

1° Naturally, time specifications remain unaffected by this summary. If Tst requires that n (z) applies today and *Box)* applies tomorrow, then *A{x)* must apply today and *B{x)* must apply tomorrow, and not both at the same time. The "simultaneous" existence of the (specific) facts does not mean that the determinations apply at the same time. As obvious as this is, it is sometimes overlooked, just as the conditional applicability of a determination is occasionally taken to mean the conditional existence of the fact that it applies, and then one speaks of the conditional truth or "validity" of a judgement – which is absurd. (So 0. Külpe, *Vorlesungen ilber Logik,* Leipzig, 1923. What is treated here as a "judgement" is not a judgement at all, but only a linguistic form of judgement.)

°0 Here, a principle comes into play whose systematic significance only becomes clear later (§ 11, 4). It is only used here to avoid repetition.

°' If a given event I is called the (full) cause of the immediately following event II, the opinion seems to be that II is completely determined by I in an irreversible manner, i.e. that for every determination *N{x)* applicable in II there is a determination *M y)* in I, such that *M(y) N[x),* but not necessarily vice versa. This may be true if II is not the "entire" (immediate) effect of I; however, it is questionable whether a partial effect can be singled out without violating the assumption of the complete determinacy of II, and then it is questionable whether something other than two successive "world situations" can be set for I and II, which of course imply each other again. — These difficulties indicate that the idea of causal connection only captures something that, as explained in the text, is relative to real events

as a relative: insofar as they are cases of certain incomplete determinations.
°° Of course, several variables may also occur in the provisions; for reasons of simplicity, the presentation only selects the case of one variable.
°° Kant *(Groundwork of the Metaphysics of Morals):* "A maxim is the subjective principle of action."
One could argue that the correctness of the law is irrelevant, as it is binding simply by virtue of being law. However, in this case, sufficient correct motivation for its application is not provided by the condition "that I have an income of m crowns", but only by this condition together with the determination that the law applies in the state and is binding on me as a citizen of that state. In this case, it no longer depends on any desire that would have to link the requirement to comply with the law to these conditions.
•5 Cf. the remark on the vague concept of the "conditional validity" of a judgement,
§ 10, 1.
°° Cf. § 9, 3 above.
°' Developing the specific requirements of formal correctness that arise from the individual laws of material correctness is omitted here, although it would not be without interest.
°® See § 12 below.
°® See § 11, 2 above.
°° The statement "if A(x) is true, then *B(x)* is possible" is, as can be seen here once again, a misleading expression. *A(x) does not imply* this possibility for the applicability of *B[x),* because otherwise it would have to exist in every case where *A(x)* applies, and that is only the case if *A(x)* itself implies *B(x),* and therefore only in a meaningless sense also the possibility of the applicability of *B{x).* One can only say: if *A(z)* applies, then relative to this circumstance, there is the possibility of *B(x)* applying – but this relative possibility can cease at any time through the emergence of other decisive determinations that are decisive – resulting – possibilities for our expectation of *B(x)* applying.
°' The claim relationships of the form *A AB,* which are dealt with in our deontic laws (in chapters T and II), are either those between certain facts
- A implies material ! *B* - or they are demand relationships between determinations - *A{x)* implies (formally) that *Box)* should apply -; then, if they actually exist, they are always demands of a higher order, otherwise merely unrealised, "subjective" demands that appear as the meaning of a desire. These include the laws and maxims of a primary nature given in the form of "if — then"; they correspond only — in the best case — to a *relative* actual ought.
If C(x) *B{x),* then we obtain the determination that contains all determination elements of *C‹x)* that are not implied in *Box)* in the form *B’{x)* v *C(x),* which is equivalent to the determination "that *if* (in that) *B(x)* applies, then *C{x)* applies". Tch performs a dutiful gait, for example, stepping out with the left foot, and thus sets a determination *C(x)* which, although not required itself, includes the required *C{x)* — to perform the gait. Then the partial determination of *B(x)* not contained in *B(x)* is that "when I perform the walk, I step out with my left foot". For more on elements of determination and the difference between determination and implication considered here, see my 'Studies on the Theory of Possibility and Similarity',
loc. cit. §§ 20, 23.
®° The sentence, which, incidentally, will be clarified further, corresponds

the general concept of moral goodness, in particular the view of F. Brentano *(Vom Ursprung sittlichen Erkenntnis,* 2nd edition, published by 0. Kraus, Leipzig, 1921), but this does not correspond to the conviction regarding the nature of value expressed here.

°^ On the sense of value and its relationship to value, see Meinong, Zur *Grindle-gttng der allgemeinen Werttheorie,* edited by E. Mally, Graz, 1923.

°^ F. Hack, *Wahrscheinlichkeitsrechnung,* Berlin and Leipzig, 1914, p. 18 (the pointers at *a* and have been omitted from the quotation as irrelevant).

However, the assumption that, among the n cases, after deducting the n+ favourable cases, "the rest will yield nothing" is not strictly accurate in our consideration, because they can and will yield a wide variety of positive and negative values, about which nothing can be said from our assumption that the expected value *aw* existed each time, and therefore the overall result of the expected failures is not to be taken into account.

°' Not act strength, but content strength. For the concept of content as it is considered here, cf. Meinong, *Über emotionale* Präsentation, op. cit., §§ 6, 7.

° In principle, the same idea can already be found in F. Brentano, *Vom Ursprung sittlichen Erkenntnis,* 2nd ed., op. cit., p. 22f, insofar as he too equates higher value not with a stronger act of desire, but with a justified preference.

°® Cf. Meinong, *Zur Grundlegung der allgemeinen Werttheorie,* op. cit., IV, § 6.

^° This constraint of facts in human coexistence provides the basis for a possible and entirely probable naturalistic conception of the origin of ethical will and thought. Such naturalism can guide our development towards ethics, but it cannot do justice to the essence of ethical laws, which are not created or invented through development, but only *discovered* in the course of it.

4' Even the improper oughtness of facts presupposes a value: the unactual value of simple factuality. In this sense, everything real, as the bearer of actual facts, as that to which determinations are actualised, has a value that exists alongside other, actual values and non-values.

^° This relation could only be that of objective correspondence, and what corresponds to something psychological in this sense is given to us directly through it. The idea of the relation therefore presupposes this givenness, i.e. the direct apprehension of the objective.

** The beginning of Kant's *Groundwork of the Metaphysics of Morals* expresses this idea. "There is nothing in the world, indeed nothing at all outside of it, that can be thought of as unconditionally good except *good will.* The talents of the mind, qualities of temperament, gifts of fortune, happiness, even qualities that are conducive to good will, nevertheless have no intrinsic, unconditional value, but still presuppose good will (in order to be good).

4^ See my article 'Über Begriffsbildung' in *Beiträge zur Pädagogik und Dispositionstheorie,* edited by A. Meinong, Prague, Vienna, Leipzig, 1919, p. 94ff. On the concept of disposition, see Meinong, 'Allgemeines zur Lehre von den Dispositionen', ibid., p. 33tf.

45 See also H. Pichler, ‘Zur Logik der Gemeinschaft’ (On the Logic of Community), Tübingen, 1924. A work which agrees with the present one in that it reveals essential and not merely accidental analogies between judgement and volition, truth and goodness,

and also takes into consideration aesthetic feeling and the beauty that corresponds to it. This is a "love characterised as right," according to F. Brentano, op. cit. But the evidence of correctness is itself a property of judgement.

• A similar view of the will – echoes of which can of course be found elsewhere – has recently been put forward by K. Sapper, based on natural philosophical considerations, in *Das Element der Wirklichkeit und die Welt der Erfahrung. Grundlinien einer anthropozentrischen Naturphilosophie,* Munich, 1924.

BI BU OG RA PHI E

Books

1. *Grundgesetze des Sollens, Elemente der Logik des Willens.* Verlag Leuschner und Lubensky, Graz, 1926, 85 pp.
2. *Experience and Reality, Introduction to the Philosophy of the Natural World View*, Julius Klinkhardt Publishers, Leipzig, 1935, 135 pp.
3. *Probability and Law. A contribution to the probabilistic-theoretical foundation of natural science.* Yerlag Walter de Gruyter, Berlin, 1938, 72 pp.
4. *Fundamentals of Philosophy. Guide for Introductory Philosophy Classes at Secondary Schools.* Hölder-Pichler-Tempsky, Vienna-Leipzig, 1938, 31 pp.

Essays

1. 'Abstraction and Similarity Recognition' in *Archive for Systematic Philosophy* (ed. Paul Natorp), New Series of Philosophical Monthly Journal, Volume VI, Georg Reimer Publishers, Berlin, 1900, Issue 3, pp. 291—320.
2. (With Rudolf Ameseder) 'On the experimental basis of the method of teaching spelling' (from the psychological laboratory of the University of Graz) in *Zeitschri t f'ur Pädagogische Psychologie, Pathologie und Hygiene* (ed. Ferd. Kemies and Leo Hirschlaff), IV. Year, Yerlagsbuchhandlung H. Walther, Berlin, 1902, issue 5J6, pp. 381—441.
3. 'Investigations into the Object Theory of Measurement' in Alexius von Meinong (ed.): *Investigations into Object Theory and Psychology,* Volume **III**, Verlag J. A. Barth, Leipzig, 1904, pp. 121–262.
4. 'Das Maß der Verschiedenheit' (The Measure of Diversity) in *Zeitschrift für Philosophie und philosophische Kritik (Journal of Philosophy and Philosophical Criticism),* formerly *Fichte-Ulricische Zeitschrift* (ed. Hermann Schwarz), vol. 131, Verlag J. A. Barth, Leipzig, 1907, pp. 33–50.

5. ‘Grundgesetze der Determination’ in *Proceedings of the Third International Congress of Philosophy,* Heidelberg 1908, pp. 862–66.
6. ‘Object Theory and Mathematics’, ibid., pp. 881–85.
7. ‘On the Question of the Significance of Phenomena for the Comprehension of the Non-Phenomenal’ in *VIII. Annual Report of the Imperial and Royal II* (formerly *III) State Gymnasium in* ***Graz,*** Graz 1910, pp. 3–15.
8. ‘On the Concept of Time in the Theory of Relativity’ in *IX. Annual Report of the Imperial and Royal II State Grammar School in Graz,* Graz 1911, pp. 3–17.
9. ‘The fundamental relationships and connections between objects’ in *X. Annual Report of the Imperial and Royal II State Grammar School in Graz,* Graz 1912, pp. 3–51.
10. ‘Borderline Questions of Logic, Psychology and Epistemology’ in *Deutsche Literaturzeitung* (ed. Paul Hinneberg), XXXIII. Year, Weidmannsche Buchhandlung, Berlin, 1912, No. 7 of 17 February. 1912, columns 389–403.
11. ‘Object-theoretical foundations of logic and logistics’ in *Journal for Philosophy and Philosophical Criticism,* vol. 148, supplement, Leipzig 1912, 87 pp.
12. ‘On the Concept of the Object in Meinong’s Theory of Objects’ (lecture given on 10 February 1913 at the Philosophical Society at the University of Vienna) in *Yearbook of the Philosophical Society at the University of Vienna,* 1913 (scientific supplement to the 26th annual report), J. A. Barth Publishers, Leipzig, 1913, pp. 61–75.
13. ‘On the nature and task of modern object theory’ in *Die Geisteswissenschaften* (weekly journal, ed. 0. Buck and P. Herre), Vol. 1, 1913/14, Verlag Veit u.Co, Leipzig, 1914, issue 23, 5. March 1914, pp. 616–19.
14. ‘On Minimal Determinations’ in *XII. Annual Report of the Imperial and Royal II. State Gymnasium in Graz,* Graz 1914, pp. 3–19.
15. ‘On the Independence of Objects from Thought’ in *Journal for Philosophy and Philosophical Criticism,* Volume 155, Leipzig 1914, pp. 37–52.
16. ‘**Additions** to Treatise I’ (Hume Studies II) in Alexius Meinong, *Collected Treatises,* Volume II, Verlag J. A. Barth, Leipzig, 1913, pp. 173–83.

17. ‘Additions to Treatise V’ (On Object Theory), ibid., pp. 531–35.
18. ‘Additions to Treatise V’ (On the Psychology of Complexions and Relations) ibid., Volume I, Leipzig 1914, pp. 301–03.
19. ‘Additions to Treatise VIII’ (Abstracting and Comparing), ibid., pp. 493–94.
20. ‘Die neuere Syllogistik im Logikunterricht’ (Modern Syllogistics in Logic Teaching) in *Zeitschrift für die österreichischen Gymnasien* (editors J. Huemer, E. Hauler and L. Radermacher), 65th year, Verlag Gerolds Sohn, Vienna, 1914, issue 10, pp. 939–49.
21. ‘Stefan Witasek’ (obituary) in *Zeitschrift für Philosophie und philosophische Kritik*, vol. 158, Leipzig 1915, pp. 1–3.
22. ‘Vorstellungen und Begriffe’ in *Der Friede, weekly magazine for politics, economics and literature* (ed. Benno Karpeles), Volume II, Vienna 1918, double issue 48/49 of 23 December 1918, pp. 534f.
23. ‘Logisches über Orakel’ in *Der Friede,* Volume II, Vienna 1919, issue no. 51 of 10 January 1919, pp. 587–89.
24. ‘Sprachphilosophische Probleme’ in *Der **Friede,*** Volume III, Vienna 1919, issue no. 59, 7 March 1919, pp. 157–59.
Issue No. 61, 21 March **1919,** pp. **203–06.**
Issue No. 63, 4 April 1919, pp. 253–54.
25. ‘Über Begriffsbildung’ in *Beiträge zur Pädagogik und Dispositions-theorie* (Eduard Martinak zur Feier seines 60. Geburtstages dar-gebracht von Fachgenossen, Schülern und Freunden, ed. by Alexius Meinong), Schulwissenschaftlicher Verlag A. Haase, Prague, Vienna, Leipzig, 1919, pp. 94–115.
26. ‘On the involvement of schools in career guidance in Styria’ (Preliminary communications) in *Volkserziehung,* Nachrichten des österreichischen Unterrichtsamtes, Pädagogischer Teil, 1921 edition, published by the Austrian Education Authority, Vienna, 1922, Part IX, issued on 1 May 1921, pp. 217–25. See also Ernst Mally's draft career guidance forms:
27. Observation Form for Career Guidance. Edition for Secondary Schools. Published and issued by the Styrian Employment Agency, Graz Office, Graz 1920.
28. Observation form for career counselling. Edition for primary schools. With information sheet. Ibid., Graz 1921.

29. Drafts of career guidance questionnaires. I. Observation questionnaire for career guidance. Edition for primary schools. In *Volkserziehung*, ibid., pp. 254–57.
30. ‘Alexius Meinong’s Philosophical Work’ (lecture given on
6. December 1920 at the German Philosophical Society in Graz) in *Beiträge zur Philosophie des deutschen Idealismus* (ed. Arthur Hoffmann), vol. 2, Verlag K. Stenger, Erfurt, 1921, issue 2, pp. 31–37.
31. ‘On the implementation of psychological student observation' (lecture given at the official Reich Conference of Provincial and District Inspectors in Vienna in July 1921) in *Volkserziehung*, Pedagogical Section, 1921, Vienna 1922, XXII. Piece, published on 15 November 1921, pp. 507–15.
32. ‘On the significance of the Bravais-Pearson correlation coefficient’ in *Archive for General Psychology* (ed. Wilh. Wirth), vol. XLII, Akademische Verlagsgesellschaft, Leipzig, 1922, pp. 221–34.
issue, pp. 221–34.
33. ‘Linear Regressions and Mean Ratio’ ibid., XLIII. Volume, Leipzig 1922, 1st issue, pp. 64–71.
34. Four contributions as a transition from logic to logistics in Alois Höfler, ***Logik*, ** 2nd **edition,** Verlag Hölder-Pichler-Tempsky, Vienna, Leipzig, 1922:
Appendix to § 20: ‘The starting points of command theory’, pp. 213–15.
Appendix to § 57: ‘The axiom groups in two systems of contemporary logistics’, pp. 577–92.
Appendix to § 70: ‘On informal conclusions’, pp. 658–68.
Appendix to § 96: ‘General remarks on the axiomatics of contemporary logic’, pp. 886–92.
35. ‘Studies on the Theory of Possibility and Similarity. General Theory of the Relationship between Similar Things’ in *Academy of Sciences in Vienna, Philosophical-Historical Class, Session Reports,*
Volume 194, I. Treatise, Alfred Hölder Publishing House, Vienna, 1922, 131 pp. Reprinted in the *complete edition of the Proceedings*, Volume 194, Hölder-Pichler-Tempsky Publishing House, Vienna, Leipzig, 1923, pp. 1–131.
36. ‘Logik und Erkenntnistheorie’ (Collective Report) in *Literarische Berichte aus dem Gebiet der Philosophie* (ed. Arthur Hoffmann), Kurt Stenger Publishing House, Erfurt, 1924, 3rd issue, pp. 3–14.
37. ‘On Driesch’s main argument against psychophysical

Parallelism' in *Archive for the Entire Field of Psychology,* L. Volume, Leipzig 1925, pp. 525–27.

38. ‘Logic and Epistemology’ (collective report) in *Literary Reports from the Field of Philosophy,* Erfurt 1926, issue 9J10, pp. 5–12, issue l 1/12, pp. 5–10.
39. ‘Logic and Epistemology’ (collective report, with the collaboration of H. Mokre) in *Literary Reports from the Field of Philosophy,* Erfurt 1930, issue 21J22, pp. 5–42.
40. ‘On Subjectivities and Their Objective Significance’ (commemorative publication for Liljequist) in *Studies Dedicated to Efraim Liljequist* (published by Gunnar Aspelin and Elof Akesson), Lund 1930, pp. 125–52.

4t. ‘Content, Form and Evaluation of the Work of Art’ in *Quarterly Journal for Youth Studies* (ed. Otto Tumlirz), 1st year, Verlag J. Klinkhardt, Leipzig, 1931, issue 2, pp. 81–98.

42. ‘Wesen und Dasein des Volkes’ in *Volksspiegel, Zeitschrift f'ur deutsche Soziologie und Volkswissenschaft* (ed. Max H. Boehm, Hans Freyer and Max Rumpf), 2nd year, Verlag Kohlhammer, Stuttgart, 1935, 2nd issue, pp. 76–77.
43. ‘Alexius Meinong’ **in *Neue österreichische Biographien (***New Austrian Biographies***),*** Volume VIII, Amalthea-Verlag, Vienna, Zurich, Leipzig, 1935, pp. 90–100.
44. (With Krug and Pommer) ‘Guidelines for Conducting Introductory Philosophy Classes’ in *Verordnungsblatt f'ur den Dienst-bereich des Ministeriums f'ur innere und kulturelle Angelegenheiten,* Abteilung IV, Jahrgang 1938, Vienna 1938, 14th issue of 15 October 1938, pp. 143–48.
45. ‘Probability and Law’ in *Research and Progress,* Newsletter of German Science and Technology (ed. Karl Kerkhof), 15th year, J. A. Barth Publishers, Leipzig, Berlin, No. 31 of 1 November 1939, pp. 383–84.
46. ‘On the Question of “Objective Truth”’ in *Scientific Yearbook of the University of Graz,* Graz 1940, pp. 177–97.
47. ‘The Nature of Natural Laws’, in *Vienna Journal of Philosophy,* ***Education,*** *Psychology* (ed. Alois Dempf, Theodor Erismann et al.), Vol. II, A. Sexl Publishers, Vienna, 1948, Issue 1, pp. 1–17.

Book reviews on:

1. Harald Höffding, *Philosophical Problems,* Leipzig 1903. In *German*

Literaturzeitung, Volume 26, Leipzig 1905, No. 18 of 6 May 1905, columns 1102–03.

2. Richard Hönigswald, *On Hume's Doctrine of the Reality of External Objects,* Berlin 1904. Ibid., No. 23, 10 June 1905, column 1419.
3. Richard Wahle, *On the Mechanism of Intellectual Life,* Vienna 1906. In *Deutsche Literaturzeitung,* vol. 28, Leipzig 1907, no. 34 of 24 August 1907, columns 2132–35.
4. Viktor Kraft, *Weltbegriff und Erkenntnisbegreif (Concept of the World and Concept of Knowledge),* Leipzig 1912. In *Deutsche Literaturzeitung,* Volume 34, Leipzig 1913, No. 36, 6 September September 1913, columns 2257–63.
5. Bernhard Hell, *Ernst Mach's Philosophy,* Stuttgart 1907, and Herbert Buzello, *Critical Examination of Ernst Mach's Theory of Knowledge,* Berlin 1911. In *Deutsche Literaturzeitung,* Volume 34, Leipzig 1913, No. 43, 25 October 1913, columns 2717–18.
6. August Gallinger, *Das Problem der objektiven Miiglichkeit (The Problem of Objective Possibility),* Leipzig 1912. In *Zeitschrift fiir Ästhetik und allgemeine Kunstwissenschaft* (ed. Max Dessoir), Volume IX, Verlag Ferdinand Enke, Stuttgart, 1914, Issue 1, pp. 117–19.
7. Oswald Külpe, *Die Realisierung (Realisation),* Volume I, Leipzig 1912. In *Deutsche Literaturzeitung (German Literary Journal),* Volume 35, Leipzig 1914, No. 13, 28 March 1914, Columns 788–93.
8. William Stanley Jevons, *Leitfaden der Logik* (translation by Hans Kleinpeter), Leipzig 1913. In *Die Geisteswissenschaften,* vol. I, 1913/14, issue 18, p. 496.
9. Fred Bon, *Is it true that* 2 x 2 = 4*? An experimental investigation,* Volume I, Leipzig 1913. In *Zeitschrift für Philosophie und philosophische Kritik,* Volume 159, Leipzig 1915, pp. 110–22.
10. Heinrich Lanz, *The Problem of Objectivity in Modern Logic,* Berlin 1912. In *Deutsche Literaturzeitung,* Volume XXXVI, Leipzig 1915, No. 48 of 27 November 1915, columns 2523–25.
11. Richard Meister, *The Educational Values of Antiquity and the Idea of a Unified School System.* In *Contributions to the Philosophy of German Idealism,* Volume 2, Erfurt 1921, Issue 2, pp. 42–44.
12. T. L. Kelley, *Statistical Method,* New York 1923. In *Archive for the Entire Field of Psychology,* **Volume** 50**,** Leipzig 1925, pp. 254–55.
13. Karl Sapper, *The Element of Reality and the World of Experience.* In *Journal of Psychology* (ed. Ebbinghaus and Schumann),

Volume 97, Section 1, Frankfurt am Main 1925, pp. 273–75.

14. Franz Brentano, *Essay on Knowledge*, Leipzig 1925. In *Archive for the Entire Field of Psychology*, Volume 56, Leipzig 1926, p. 549.
l5. Aurel Kolnai, *Der ethische Wert und die Wirklichkeit.* In *Blätter f'ür Deutsche Philosophie*, Journal of the German Philosophical Society (ed. Hugo Fischer-Leipzig and Gunther Ipsen), Volume III, Verlag Junker und Dünnhaupt, Berlin, 1929, pp. 134-35.
16. Karl Sapper, *Naturphilosophie. Philosophie des Organischen (Natural Philosophy: Philosophy of the Organic)*, Breslau 1928. In *Zeitschrift für Psychologie (Journal of Psychology)*, Volume 113, Section 1, Frankfurt am Main 1929, pp. 183–84.
17. Karl Bühler, *Sprachtheorie (Language Theory)*, Jena 1934. In *Kant-Studien, Philosophical Journal* (ed. Hans Heyse), Volume XL, Pan-Verlagsgesellschaft, Berlin, 1935, Issue 4, pp. 334–46.
18. J. Krug — 0. Pommer, *Textbook for Introductory Philosophy Classes in Austrian Secondary Schools*, Part 2, Issue 1, *Logic and Theory of Science*, Vienna 1936. In *Österreichs Höhere Schule*, 5th year, Vienna 1936, issue 6, pp. 36–31 (supplement to *Der Mittelschul-lehrer* 1936).

Publishing activities

1. Alexius von Meinong, *On the Foundations of General Value Theory*. Instead of a second edition of *Psychological-Ethical Investigations into Value Theory*, Leuschner und Lubensky Publishers, Graz, 1923, 176 pp.
2. (In collaboration with Otto Tumlirz) Eduard Martinak, *Psychological and Pedagogical Treatises, on the 70th Birthday of the Researcher*, Leykam Publishing House, Graz, 1929, 281 pp.

NA CH LA SS MALLY I N DER

U N I V E RSI TY LIBRARY GRAZ

It contains:

I. *Manuscripts:*

A: Manuscripts of published works (see list of publications).

B: Manuscripts of unpublished works. These include:

(1) A planned contribution to an Ehrenfels commemorative publication: "Fundamentals of Gestalt".

(2) Several notes on "Probability and Natural Law" (preliminary work on "Probability and Law")

(3) Notes in connection with the posthumous Lo-

.

C: Excerpts.

II. ***Lectures:***

A: Lecture notes with keyword-like information are available for the following lectures:

(1) Psychology: Lecture notes from the semesters: WS. 1925J26, WS. 1930J31, WS. 1931J32, WS. 1935/36, SS. 1938

(2) General Theory of Values and Norms, WS. 1926J27, SS. 1927

(3) Methodology of introductory philosophy courses, WS. 1927/28, 1929J30, 1931J32, 1933J34, 1935/36, 1937J38

(4) Epistemology, WS. 1927J28

(5) The Concepts of Mind and Spirit, SS. 1928

(6) Theory of experience, SS. 1930

(7) Form and Meaning, SS. 1930

(8) General Theory of Values, WS. 1930J31

(9) Theory of Value, SS. 1931

(10) Psychology of Personality, SS. 1932

(11) Theory and Criticism of Mythical, Magical and Mystical Thinking, SS. 1932
(12) Philosophy I: On the Forms of Objects and Thought, Part I, winter semester 1932/33, Part II, summer semester 1933
(13) Philosophy II: On Reality and Empirical Knowledge, Part I, Winter Semester 1933/34, Part II, Summer Semester 1934
(14) Language as Expression, summer semester 1934
(15) Systematic Philosophy: Theory of Values and Ethics, summer semester 1935
(16) Main Questions and Tasks of Philosophy, summer semester 1936
(17) Logic, winter semester 1936/37
(18) Epistemology: Methodology of Research, SS. 1937
(19) Philosophy of Cultural Life, WS. 1937/38
(20) Worldview and Philosophy, WS. 1938/39
(21) Theory of Values, SS. 1939
(22) Guiding Principles of Ethnic Psychology, 1st trimester 1940
(23) Logic, 2nd trimester 1941

B: *Lecture notes:*

(1) Theory of Experience, SS. 1930
(2) Theory and Critique of Mythical, Magical and Mystical Thinking, SS. 1932
(3) Systematic Philosophy I: On the Forms and Objects of Thought, WS. l932{33
(4) Systematic Philosophy II: On Reality and Empirical Knowledge, SS. 1934
(5) Language as Expression, SS. 1934
(6) Psychology, SS. 1938
(7) The Methods of Mathematics (based on the lecture: Methodology of Mathematics Teaching, WS. 1927/28?)
(8) Epistemology (winter semester 1927/28?)

III. *Miscellaneous:*

Noteworthy for the criticism of Meinong is the review of J.N. Findlay's dissertation "Meinong's Theory of Objects", 1933.

IV. Letters (collection in preparation).

N AM ENV E RZE I CHN IS

Ackermann, W. 21, 25, 190, 191, 195—97, 201—03, 208, 218
Ambrose, A. 187
Aristotle 23

Baldwin, J. M. 14
Barcan-Marcus, R. 24, 25
Becker, 0. 218
Behmann, H. 187
Benussi, V. 14
Bergson, H. 7, 10
Bernays, P. 23, 25, 185, 186
Böhm-Bawerk, E. v. 12 Bollnow, 0. F. 183
Bolzano, B. 23, 25
Brentano, F. 5, 6, 8, 23—25, 323, 324
Brower, L. E. J. 154, 186
Burkamp, W. 183

Carnap, R. 5, 181, 184, 186, 187, 204, 218
Chisholm, R. 14
Couturat, L. 13

Eaton, H. 0. 14
Ehrenfels, Ch. 9, 15

Findlay, J. N. 14
Frege, G. 23, 123, 136, 181

Gödel, K. 130

Hartmann, N. 8, 57, 182
Hilbert, D. 21, 23, 25, 119, 136, 151, 185, 187, 189, 190, 191, 195-97, 201—03, 208, 218
Hintikka, J. 25
Hoffmann, P. 182
Höfler, A. 13, 188
Honecker, M. 320
Husserl, E. 5, 53

Kant, I. 292, 303, 322, 323
Kastil, A. 5
Kern, F. 14
Kern, L. 14
Kindinger, R. 14
Kraus, 0. 5, 323
Kröner, F. 14, 15, 31
Krug, J. 13-15
Külpe, 0. 321
Küng, G. 25

Laurin, G. 14, 15 Leibniz, G. W. 23 Lesniewski, St. 24, 25
Liljeqvist, E. 13
Lukasiewicz, J. 23

Mach, E. 52
Mally, E. 4, 5, 11–17, 20, 23, 25, 182, 183, 185, 320, 323
Martinak, E. 4
Marty, A. 5
Meinong, A. 3-7, 9, 11-14, 55, 58, 182, 184, 320, 321, 323
Menger, K. 12
Mokre, J. 20, 181, 182
Morschet, E. 25
Müller, G. H. 15

Nelson, E. J. 174, 188
Nietzsche, F. 53

Pichler, F. 14, 323
Pommer, 0. 13, 15
Principia Mathematica 8, 22, 54, 116-15, 136-40, 151, l57f, 181, 182, 184—88, 203

Quine, W. V. 0. 24, 25

Reichenbach, H. 5

Rothacker, E. 13, 15
Russell, B. 13, 14, 16, 20, 22, 25, 136, 144, 146, 148, 174, 186-82, 188, 218, 320

Sapper, K. 324
Scheler, M. 182
Schlick, M. 5, 183
Scholz, H. 187
Schröder, E. 183
Slupecki, J. 25
Sobotka, M. 14
Strohal, R. 5 Urban,

W. M. 14

Weber, F. 14, 320
Weingartner, P. 25
Weiss, P. 185
Weyl, H. 186
Wendelin, H. 14, 28, 31
Whitehead, A. N. 7, 16, 20, 136, 181, 320
Wiener, N. 5
Wieser, F. v. 12
Wittgenstein, L. 5, 135, 186
Wolf, K. 15, 182
Wolff, K. 320

Pull, Th. 13, 15

SACHY E RZE ICHNIS

The subject index of the original edition of the Basic Laws of Duty, compiled by Johann Mokre, has been retained and arranged alphabetically.

Illustration theory 50, 54
Absolute demand and relative ought 300
Theory of Abstraction
62 Similarity
– through Implikatgemeinschaft 17
- between the concrete and assigned soetic element 50f
Act(s) 66
- Real 123
Universal
statement
- unrestricted l54f
– as conjunction of individual statements *I* s0ff
All domain 92f, 102
All determination
156 General
– the 115—18
– Moral law 296
Universal principle 156
Universal sign 203
Analogy
- of relative ought determinations and applied possibility 299
Assumptions
- Axiomatic 122
- mere 70
- free 91
approximation
– to the best 304
– to what is offered 304
– to highest reliability 308
Accept 237, 279 Clear
explanation
– of arithmetic 119-24
Visualisation 317
- artistic 74
- of digits l19ff, 185
- Significance for arithmetic 136
- Significance for geometry 132
– Significance for mathematics 119
Visual content 75f Proportion 309f
Applicability
- of a theorem to itself l12ff
- precise definitions 222
– logical theorems and concepts to itself 82
Application
– of sequential relationships
177 Principle of application
- of the requirement relationship 289
- of implication 289
Number
– Existence of 143—49
– Constitution of the concept 139—42
Concept of number 148
– of the Principia Mathematica 136-38
– Self-reference of the logical at 170
A priori 206
Apriori 157
Equivalence 240
- of provisions 94ff, 139
- Requirements 255, 263
- of debit and credit 312
Arithmetic l02f
– finite l l9f
- logically justified 172
– Fundamental questions of
34 Aspect 299, 302f
- Supplement to the 302
- most complete achievable 312
expression
- of an experience 47f

- of a meaning 38
- Setting of 38
– Understanding of 225
statement(s) 36, 44, 76ff, 86ff, l28ff, 187
- general 150ff
- Valid lllf
- of ordinary language 35f
- of Hilbert and Ackermann's logical calculus 19ff, 195ff
- mathematical 132f, 135
- primary and logical 157
– in the step theory 1l0f, 114
– Development of 177
– content to be assessed in the 36
– Rank of the 225
- Meaning of a 37, 78f
- fuzzy 161 form
of statement 89f
Statement logic 21, 189—99, 222f
Declarative sentences 35f
Meaning of statements 37, 133
- of the formula
115 External
world 51f
Separation
- Principle of 249
Axiomatisation
– of the vividly given 122, 185
Axiomatic
– fundamental laws of oughtness 18
– Theory 127
Axioms 190
- of formalism 2lf. 222
– by Hilbert and Ackermann **190,** 222
Axiomatic system
- of Euclidean geometry 189
Axiom freedom
- of logical theory 172
– of logically founded number theory and arithmetic 172

Mean 8, 37ff, 54f, 75, 80, 112, 185
Condition
– necessary as implication 290
condition, sufficient
– as implicans 290
of right will 289
– and necessary right intention 311
Conditional demand 248
- Truth and validity 321
Concept 44f, 68, 76f
- of the determination 171f
- of the geometrically unambiguous 184
- of objects and subject 66
– logical 82f
– "two things" 186
– Expressions for universals as precursors of 108
- logical and empirical scope of 159
Conceptual objects 98ff Justification
- Illustrative of number theory 119-24
Justification finitism 125-31, 150, 186
Assertion(s) 76, 182
- Applicable determination implication 92
– Validity of 70
Designations 217
Willingness
- to explicit positing 315
Reflection
- of one's own will 316
Certainty 38
– of will 319
Provisions **76f, 83, 91-97, l96f,** 198f, 236
- antinomical 168
– simple 183
– empirical, in the logical sense 225
- intentional 281
- in formal science
- empty 168
– mathematical statements l 33f, 138
- primary and logical l56f
- real through volition 284
- sharp 159
- in the doctrine of degrees l l0f, 114
- Independence from fulfilment l54ff
- fuzzy 158, 160f
– incomplete 68
- defined by scope 164
- specified 281
- Acceptance of an 279
- Concept of 172, 222
- Relationships between 162-73
– Determinate of 98f, 181

– "Existence" of 143
- Fulfillability and non-fulfillability l68ff
- Extension of 218
- Calculation of 191, 200
- Calculation of the fulfilment of 22, 20(t- 7
- Calculation of fulfilment of 22, 208-18
– as predicate 83
– Self-evidence of the concept 178
– Self-referentiality in formal theory 170
– Meaning of a 222
- Meaning contents of 165
- Carriers of 57, 63f
– Objectification of 57f
- Predetermination in Principia Mathematica 115
Determining element 17, 93ff, 183, 185
Determining content
– of number 139—42 Determinative implication 58f, 91, 103,
159f, 165, 273
– applied 273
Best shooting 308
Participation of the collective will 310
Provenability 301
- greatest achievable 304f
– its measure 308
– Maximum 311
- its estimation 307
- and value 307
– of volition 310
– as a composite probability 310
probation
- external 313
Probation possibility
- as a sufficient and necessary condition of actual oughtness 300
Proof
- mathematical 132—35
– Verification of 134f
– for completeness 197
Consciousness
- natural 6
– perfect 89
– Theorem of 65
Designation 8, 54f, 185
Designation 193 Relationships
– between determinations 162-73
Bond
- through the demand for correctness 315
Biologism 43 Evil
- as self-contradiction 316

Cogito 63, 66

Will to exist 291
Deduction 289 Definitions
208ff, 2l5f System of
definitions 321 Act of
thinking
- Sinneixes 37
Thinking 17f
– natural 51
- of meaning
69 Logic of
thinking 320
Deontics 17f, 232
Desiderative 320
Determinate 98—104, 181
- and Class 108f
Determinant Theory 8, 16,
181 Things
- Existence of 219
- Insufficient certainty of 160f
- lowest level 123
– Absence of individual 9f, 103
– real 221, 223f
Disjunction 114
Disposition 315
Basis for disposition 315, 318 Average
- of value 307
May 303f

property 124, 318
Enclosure
– between provisions 174-80, 205ff
Enclosure relationships l65ff Appointment
- Principle of universal 22
Rule of appointment 192,
200

Individual statements
– Conjunctive linking of 180
- Conjunctive linking of Material links between 164
- Conjunction and disjunction of l50ff, 180
Individual, the
– and the general 115-18
Element, genuine 320
Sensation 50fi
- Meaning as an ability of
37 Empirical
- Determinations 91
– Ways of knowing 157
- Scope 164
Decidability
– of fulfilment 8
- of statements 86f, 78f
Decisive oughtness 307
Decision judgement 300 Decision
- correct with the decision 300
Experience 219
Comprehension 96f, 113
Empirical knowledge
– Doctrine of reality and the 9-11
Fulfillability(ies)
– of provisions 105—07, 133, 144
- and their negations l66ff, 170f
- of number 122f
– Calculation of 191, 20s- 7, 219f
- Logic of 220fi
- Self-evidence of the concept 178 Proof
of satisfiability
- from fulfilment l46ff
Fulfillment operator 166
Fulfilment 170
- Approximate 11
- of provisions 157f
– of provisions by inclusion 178
– objective 65f
- of the meaning 40, 74f
- set meaning 64f
– valuable and illustrative 64, 74
- in reality 220ff
– incomplete definition of 68
- fulfilment of 99f
- Judgement of 116ff
- Calculus of 191, 208-18
- Predicate calculus of 22f
– Independence of 91, 139, 154f, 160ff, 165, 220ff
- Independence of the sequence relation from 177
Modes of fulfilment 161
– of determinations 91
- Original unity of
supplement
- of the aspect 302
Knowledge 50f
– philosophical 42
- perfect 117
cognitive
achievement
- Independence of meaning from 134f
subject of knowledge
- ideal 86 Theory
of knowledge
– psychological 55
Cognitive ability 132 Experience
50ff
- and reality 10f
Experience 12, 181
– of meaning 42, 73
- meaningful 47
- Irrelevance to meaning 76
Achievability 308
Probability of achievement 307
Judgement 151, 181
- of fulfilment 117f
- of objects 36
- a real one 176
Expected value 307
Ethics 232, 314
– Development and nature 323
Evidence
- for correctness 317, 324
Existence
- of numbers 143-49
- of definitions l43ff, 172
- of a concept's content 139
– of fulfilment 211
- of objects 171f, 224
– mathematical 106f
– of zero 107, 138, 184
- of meanings 171

Existence axiom
- of logic 9f, 22ff, 139, 161, 170, 203, 2l9f
Existence operator
- of formal implication 165f
Existence theorem
– on digits l26ff
Extensionals
– Conception of modern logic 109
- Functions 96f
Extensionality
– of the lowest-level determinations 159
– of setting 184

Case 236
– incomplete coverage 280
– Determination 279
Figure(s) 21f, 185
– Determination of handling of l89f
- Functional significance in Formalism I 193-99
- Functional significance in formalism II 201—07
– equivalent 194f
- interchangeable 192f
- composite l91f, 200
– composite, valid 196
- composite, invalid, unfulfillable 197
Finite
– meaning, absence of 155
– Point of view 119-
35 Consequence
- and reason 277 Consequential relationships l72f, 177—80
– Independence from truth and fulfilment 177, 205f
Consistency 244
– Principle of 246
– of volition 245
Consistency principle 287f
Consequential propositions 252ff
Consequential connection 278
Requirement 44
- of fulfilments 68
- of good sense
- Apply a 70
- rank differences of 89
- and ratings 11—13
- Meaning and expressiveness of a
37 Claim (relationship) 243
- Absolute and relative obligations 300
- general 236
– Principle of application of the 289
– conditional 248
– of evil: inconsistency 305f
– actual 277
– higher level 296
- primary 296
– Reflexivity of 267
- of meaninglessness 305
- subjective 322
Contents of demands 42
Meaning of demands
74f Form 100, 109
Formal
- Implication 164f, 180, 273
- Theory, axiom-free and axiomatic 172
- Theory, provisions 171
- Theory of Scholz 187 Formalism
21—24, 28, 189-218, 219
Formulas, derived 211—15
Rules of form 191f Form
science l02f Feeling l2f

Commandment 287, 300f
Thought 44f
Thought motivation 277
Counter-ground 303
Object 45, 74, 102
- in Euclidean geometry 103
– of thought 112
- ideal 8, 36, 55f, 59—62, 65, 68, 72, 89, 103, 148, 178
- immanent, of consciousness 55f
– of logic l54f, 161, 170f
– of nothingness **169**
- psychic and physical 61
- impossible 112
– determination of 202ff
– Determinate as l00ff
– Non-existence of 224
- through projection 51
- Spiritual as 181

- Meaning as 63—66
– Predetermination of the
35 Concept of object
- Mally's 7f, l6f
- Meinong's 6f
Object logic 320
Object theory 60, 233
- Meinong's 55f
– of meaning
65
Generalisation
- Principle of existential 22
Spirit 43
Applicability 45, 76-73, 89
– of a determination 220
– Unconditional, of a sense of purpose 175
- Subject independence of 183
Validity
- conditional
321 Geometry
- Axiom system of Euclidean 189
- illustrative 27
– Determination from the realm of meaning of Euclidean
Geometric determination
– unambiguous 101f, 184
– Approximate application of 223ff
- Independence from fulfilment 160, 170, 220f
Overall aspect 299, 302f, 304
Overall intention 316
- his participation 310
- of the company 311
- more harmonious 309
- worse 313
law 322 designer's concept
- illustrative 312
certainty
– in willing 283
Equivalence 255, 263
equality 140, 145 degrees
– of relative oughtness 256, 305
Reason
- and Episode 277
Fundamental principles of formalism II 215
Basic Figures 21, **191, 200**
Basic formulas 21, 190, 192, 210f, 215
- Introduction to the correct 200
Basic rules 217
Principles 243ff
- Overview of the
250 Good will 323
Good
- absolute 313

attitude 47
Hope value 307

I 64, 66
Idealism 66
Identical 95
Imperative 287f
– Categorical 292, 303
Implicans
- as a sufficient condition 290
Implicate(s) 9lf, 94ff, 121
- Actual 273, 274, 290
- Co-will of the 271
- as necessary condition 290
- inappropriate 274, 290
Implication community 17
Implication 114, 206f, 238
- formal 164f, 180
- general 91, 126
– material 162, 174
- Mathematical statements l32ff
– Final rule of 174
- applied 290
- Principle of application of the 289
- actual 274
– Means of detection of the 277
– Reflexivity of 261
- tautological 151, 205f
Indifferent
- Determination 304, 313
- Action 304
- Moral 301
Individual will 318
Individual sphere 203
Content(s) 315
- of a statement 36
- of a thought 112
- and scope of a determination 94ff

- Thought of a 99
- Truth relationship as a meaning relationship of the 180
Content finiteness 125-31
Strength of content 323
Inconsistency
- as a demand of evil 305f In this respect 298
Intellectualism 312 Intention
- towards vivid fulfilment 122
- on meaning 97
Interval 304

Calculation 134
- of fulfilment 2tD-07
- Fulfilment 208-18
– of the right thing 191-99
Categorical imperative 292, 303
Causal relationship 289f
Parentheses
- Rules for 201
Class(es) 92ff, 98, 102f, 108f, 1l0f, ll4ff, l36ff, l43ff, 156, 159, 178, 183
- Relationships between 162ff
Collocation 159
Conjunction 114, 240 Can 308f
Consistent system 314, 318
Contraposition 255, 264 Body
- of reality 117

Life
- meaningful 72f
- Purposes of daily life 34
Meaning of life 40
Ultimate fulfilment 179
Ultimate fulfilment 180
Logic 17, 21—24, 27f, 93f, 136, 172, 187, 231f
– General 222f, 220
- Aristotelian 86
- existential 27, 150—55, 157, 164, 203, **219f**
– Limits of existential logic 156-61
- and Doctrine of Meaning 5—9
- of Modalities 157—61
- more recent 77f, 109
- Application to reality 161
– Structure 174-80
– Formalism of 219
- Sentences and concepts of 114
- Language of 130
Logical
– Formulas 219f
– Foundations of Mathematics and Arithmetic 136
- Right of concepts 178
– Propositions 219f, 46
- Language 225
- Theory 172, 178, 187
- Self-reference of the 170f
- Essence of 154
Lies 296

Material implication 274
– Psychological correspondence of 278 Maxim 292, 303, 317
- Impossibility of a general 302
Ambiguity
– Signs of systematic 111, 113, 182
My 315, 316 Quantity 27, 120f, l23ff
Misinterpretation
- objectivist 54-d2
– ontological 178
Mitfordem
- Principle of 246
Co-constitution
- in volition 284
Co-being
- actual 277
means 289
co-willing
- of the Implicates 271 Modal
Determination 283
- incomplete 300
Modality
- deontic 298
Modality concepts 207
Modality logic 218, 220
Possibility 17, 19f, 207, 281, 302
- applied 282
– of a provision 282
- pure 282
– and relative ought **299**

motivation
– in the narrower and broader sense 290
– Formal correctness of 294
– Principle of 289
- of judgement 278
– through judgement 291, 316
– of volition 277
- by volition 289f
Mythical
– Thinking 10
- religious consciousness of a people 73
Myth 42

Approximation 10, 102, 222-25
Approximation formulas 303
Naturalism 323
- Misinterpretations of meaning
46-53 Natural system 243
Negat(e) 240
- of destiny 172
- Enclosure relationships and their 166
- Fulfillability and their 166
Necessity l66f, 207, 220, 229
Zero 136-38, 146
- Existence of 184

Object(s) 64f
- clearly presented arithmetic thinking 12lf
Objectivism
- Mally's 16
- Meinongs 55ff
Objectivity
- of the object 6
- of meaning 66

Paradoxes
- Cases of implication and corresponding cases of inclusion 175—77
Paradoxes 77-81
– Meinong's 7
- Lack of meaning in Russell's example of a 183
duty 254
- Self-evidence of 257
Imaginary desires 280
Philosophy 35f, 42f, 62, 73, l8lf
Physicalism 52, 181
- Sensory blindness of 43, 46–50,
52 Potentiality
– of propositions 69
Predicables 84f, 182
Predicate calculus 22, 191
- Hilbert-Bernays 203
– Reiner 208, 220
Pragmatism 87f
Premise
- second 187
Presentation
- emotional 12, 312
Primary requirement 296
Proposition 320
Propositional function 320

Revenge
305 Rank
- in the order of meanings 225
Ranking 70-73
Rank differences 89
Reflexivity
- of the demand 267
Rules 297, 303
Relative
- and applied possibility 282f, 299
- Data 300
– Determination of what ought to be 298f
– Target 322
– Target, its degrees 305
Relativism 88ff
Correct(ness)
- calculation of
191 correctness
- external 287
- formal 299, 300f
- of a formula 193
- inner 295
– material 287
– motivation 294 Figures of
equal correctness l94ff
Relationship of correctness 195
Direction
- towards the valid 71
- of intention 97
- in practice 74
- of better meaning 73
- to the structure of meaning 69

– of striving 88f
- of striving 225
- in theory 74
– in reality 74 lack of direction
– of false desire 293f

Practical sense 41, 49, 52
Facts 233, 17f
- specific 235
- theory of 233
– Indefinite 236
Sense of facts 41, 49, 74
Sentence 44, 133
Sentences
– Arithmetic 136
- logical 22
– of logic 114
– mathematical l32ff
- of everything real 158
- number theory 125
– Determination of 205
Meaning of the sentence 78
Estimation
– of validity 307f
Separation
- of meanings 60, 74—77
Apparent statement 77ff
Apparent determination 77, 79, 84
Apparent object 103 Conclusion
- of statements 177
Conclusion 174, l76f, 289
- Condition of a 180
Rule of inference 192, 201
Mental 50ff, 64, 181
Being able to be 247
oughtness 241
- and possibility 299f
- incomplete 298
Question of being 44f, 76f Types of being 55
Statement of being 150, l52ff
Questions of being and questions of meaning 44f, 56f, 221 Signs of being 203
Sejunction 260 Self-reference
- of concepts 178
- of the logical 170
Self-relation 110 Self-evidence
– of duty 257
Assumption
– of expression 88
- a provision 96f
- relating to recruitment 74
- fulfilment 117
- free, accepting 91
- self-evident 171
– of meaning 44f, 64ff, 72
– of meaning 63
- unrestricted 59f
- meaningful 70f
- content of a 59ff
- Meaning of intellectual 41
- unlimited potentiality 69
meaning 171, 315
– of a statement 133, 186
– Concepts 37—39
- good 38, 40, 45, 68, 71f
- of language 35f
- and reality 43
- general discussion of 42f
– Reflection on the 226
– uses of the word 37-39
– Validity of 70—73
- Logic and doctrine of 5—9
Acts of perception 40, 45
- of judgement 49
Meaning range 156f, 168f, 205f, 221
Meaning experience 47, 52, 73
Meaning development 72f
Form of meaning 7f, 9, 143
Structure of meaning 69
Meaning(s)
– of destiny 92f, 94ff
– logically unfulfillable 8
– practical 74f
– of the sentence 90
– of language 34
– theoretical 42, 74f, 76f
– statements of 36
- Divorce of the 74-75
Nonsense
- Demanding of 305
Lack of meaning 82f, 84f, 87f, 114, 183
Meaningful behaviour 177
Misjudgement of meaning 43-66

– From meaning blindness 44-53
– Objectivist 54—62
- Subjectivist 62—66
Contradiction of meaning
45f, 64, 78ff Context **67—69**
Moral law
– General 296
Moral evaluation
- Diversity of 314
Morality 73
Ought 241, 242f
- actual 280
- and wrong 305
– decisive 307
- fulfilled 276
- degree of 305
– relative 322
- of the contrary to duty 305f
- subjective 322
- inappropriate 276, 278
- and value 31l
- its essence 311
determination of
ought
- relative 298 doctrine
of ought l7ff contrary to
ought 250, 305
- permanent 306
- its relative oughtness 305f
– and volition 293
scope 304 language
40f, 224f, 75f
– formalised of arithmetic 130
- formalised of mathematics 134
– physical 46—50
- Theory of scientific 182
- Use and understanding 33-36
– Dangers 33f
- Misinterpretations due to 56f, 63f
- Syntax 204
Language use
– exacerbating determination of 172
manner of speaking
– Advantages and dangers of simplified 178
Aspiration 10
Expression of aspiration 225
Step teaching 110-14, 185
Subject 6A-66
- Meaningful activity or attitude 40
Subjective demands 322
Subjectivisation 43, 53
Subjectivity
- of meaning 64-66
Subject independence
- of the validity of a meaning 88, 183
System
- harmonious system of desires 301
- consistent 314, 318
- natural 243
- consistent 309

facts 72, 117, 135, 235, 239
- Consent to them 275
– explicit desire for them 275
- timeless and volition 284
actuality 220
– of oughtness, principle of 249
Transitivity
– of the demand 261
- of implication 261
Transcendentalism 5f, 53

supplement to conviction
– through volition
284 Scope
– of concepts 159
- and content 93-97
- Relationships between
empirical 164 transformation rules
190, 192f, 200 Independence
principle 298f Unconditional
claim
- Principle of 249
Unconditionally applicable meaning 175f
Indeterminate 280
Indeterminacy
– apparent 281
– subjective 283
Axiom of infinity 136 Impossibility
– of meaning 54f
Question of universals
l08f Impossibility
- of a general primary form (Maxim) 302
Untruth 235, 239

Incompatibility
- of a postulate 176f
Original intention
128
Original intuition 148
– of the number
148 Cause 289
- full 32lf
Judgement 44, 76,
182
- general 236
- general about numbers 125f
– certain 247, 286
- particular 236
– as a psychological prerequisite for feelings of value
Form of judgement 321

Variables 21, 193
- and formulas 197f
- Renaming rules for bound 201
variable range
- Universal proposition and existential proposition in finite
– recognition of the unlimited 155, 156
Variable 280
Responsibility 3I9f
Union
– Principle of 248
transgressions
- minor and serious 305
Objectification
- of number 143
- of types 101
- in Meinong's theory of objects 57f, 60ff
- of mental experiences 50ff
- through language 35
– of the subject 64ff
Retribution 305
behaviour 38, 40, 48f
behavioural psychology
- and its theory of language 34f,
180 Law of behaviour 318
Connection sentence 287
Assumption 247
Negation 114
Rule of negation 21, 193, 201 Diversity
– moral evaluation 314

Reason 96f
Understanding 48f
Actualisation
– of a provision 238
People
- 73
– Morality of a 73
Concept
- of psychologism 51
- objectivism 55, 60
prejudice
- intellectualist 75, 182
- of language 35

truth 45, 87ff, 183, 186
– conditional 321
- Independence of the consequential relationship from 177
truth relationships l62f, 174
– as meaning relationships of the
contents 180 Perception
- of psychologism 50
– Judgement of the 181
– judgemental value of 127
probability 11, 19, 158
– of achievement 307
– and relative determination of what
ought to be 299f Because 290
- as an expression of an applied determination implication 29lf
Worldview 73
If 298
If - so 238, 322
Value 19f, 51f, 298, 306
– and proven reliability 307
– Relevance 88f
– of formal correctness 314
– for me 309
– its (factual) basis 314
- Measurement (by wooliness) 310
– objective 311, 314
– personal 310
- Relative 313
- and target 311
– actual 311
- improper 323
- misjudgement 51ff
– its essence 311f
values 43, 45

Weighing up values 306
value formula **314**
Sense of value 19, 75, 306, 317 Values
74f
Value measurement (through woolability)
310 Value theory Hf, l7ff
Evaluation 68
– Validity of a 70
- Meaning of a 37
- Prerequisite 75
Evaluation principle 37
Methods of evaluation 73
Entities 58, 61, 108f
Absurdity 71f, 99
- of the negation of the unconditionally valid 175
Consistency 195, 199 Inconsistency
- Principle of 250
– as criterion of right will 287
– material 293
Will 31S
- good 323
Laws of the will 286
Logic of the will 286
Reality 225
– Binding formal theory to the 136, 144f
- Things of 98, 222f
- Fulfilment in 179, 22ff
- Objects of 102
- Doctrine of **9—11**
- Direction of 74
- Theory of 28
- confidently judged 117
- of setting 72
– and language 182 Statement of
reality 68, 165, 225
Determination of reality 170
Independence from reality of a inclusion relationship 165
Effect 289
Knowledge 50, 74
Desirability as a measure of value 310
Willingness 17ff, 245f
– justified 317
– actual 280, 291
- fulfilled 278
- co-constitutive 284
– Real function 284
- correct 286
- as absolutely good **313**
- formal 300
- internal 317
– material 287
– contrary to what ought to be 293
- Meaning of 74
Willingness and
ability 288 Word
- Meaning, 37, 67, 75f
Wording
- without meaning 79f

Number
– natural l02f, 121, 125
– logical definition of natural 123
- Set of natural numbers as variable range l50ff
Number system 149
Symbol 37, 279
– Meaning of a 80, 109
- for figures **192**
- meaningless 54
- of systematic ambiguity 113, 182
Time determination
- and counterposition 255
- and conjunction clause 287
Timelessness
- of facts 284 Goal
225f
– apparently objective 69, 224ff
- Anticipation of a 73
Number 119ff, 125-30,
185 Simultaneous
existence
– of facts 321
Classification 50, 61
Classification theory
- language theory 54
Attribution 320
Composition
– Principle of 248
Zustaadsgesetz 318

BERSERKER

BOOKS