FUNDAMENTALS & APPLICATIONS OF VEDIC MATHEMATICS

2014

सर्व शिक्षा अभियान

सब पढ़ें सब बढ़ें

राज्य शैक्षिक अनुसंधान एवं प्रशिक्षण परिषद दिल्ली

DELHI

SCERT

स्वाध्यायान्मा प्रमदः

State Council of Educational Research & Training,

Varun Marg, Defence Colony, New Delhi-110024

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State Council of Educational Research & Training
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Preface

Vedic Mathematics introduces the wonderful applications to Arithmetical computations, theory of numbers, compound multiplications, algebraic operations, factorisations, simple quadratic and higher order equations, simultaneous quadratic equations, partial fractions, calculus, squaring, cubing, square root, cube root and coordinate geometry etc.

Uses of Vedic Mathematics:

- It helps a person to solve mathematical problems 10-15 times faster
- It helps m Intelligent Guessing
- It reduces burden (need to learn tables up to 9 only)
- It is a magical tool to reduce scratch work and finger counting
- It increases concentration.
- It helps in reducing silly mistakes

"Vedic Mathematics" is a system of reasoning and mathematical working based on ancient Indian teachings called Veda. It is fast, efficient and easy to learn and use. Vedic mathematics, which simplifies arithmetic and algebraic operations, has increasingly found acceptance the world over. Experts suggest that it could be a handy tool for those who need to solve mathematical problems faster by the day.

Vedic Mathematics provides answer in one line where as conventional method requires several steps. It is an ancient technique, which simplifies multiplication, divisibility, complex numbers, squaring, cubing, square and cube roots. Even recurring decimals and auxiliary fractions can be handled by Vedic Mathematics. Vedic Mathematics forms part of Jyotish Shastra which is one of the six parts of Vedangas. The Jyotish Shastra or Astronomy is made up of three parts called Skandas. A Skanda means the big branch of a tree shooting out of the trunk.

The basis of Vedic mathematics, are the 16 sutras, which attribute a set of qualities to a number or a group of numbers. The ancient Hindu scientists (Rishis) of Bharat in 16 Sutras (Phrases) and 120 words laid down simple steps for solving all mathematical problems in easy to follow 2 or 3 steps. Vedic Mathematicsor one or two line methods can be used effectively for solving divisions, reciprocals, factorisation, HCF, squares and square roots, cubes and cube roots, algebraic equations, multiple simultaneous equations, quadratic equations, cubic equations, biquadratic equations, higher degree equations, differential calculus, Partial fractions, Integrations, Pythogorus theoram, Apollonius Theoram, Analytical Conics and so on.

How fast your can solve a problem is very important. There is a race against time in all the competitions. Only those people having fast calculation ability will be able to win the race. Time saved can be used to solve more problems or used for difficult problems.

This Manual is designed for Mathematics teachers of to understand Vedic System of Mathematics. The Chapters developed in this Manual will give teachers the depth of understanding of the Vedic methods for doing basic operations in Arithmetic and Algebra. Some important basic devices like **Digit Sum**, the **Vinculum**, are also explained along with independent **Checking Methods**.

All the techniques are explained with examples. Also the relevant Sutras are indicated along with the problems. In Vedic System a manual approach is preferred. The simplicity of Vedic Mathematics encourages most calculations to be carried out without the use of paper and pen. The content developed in this manual will be applicable in the curriculum of VI-X classes. Methods like Shudh Method is applicable in statistics. This mental approach sharpens the mind, improves memory and concentration and also encourages innovation.

Since the Vedic Mathematics approach encourages flexibility, the mathematics teachers encourage their students to device his/her own method and not remain limited to the same rigid approach, which is boring as well as tedious. Once the mind of the student develops an understanding of system of mental mathematics it begins to work more closely with the numbers and become more creative. The students understand the numbers better. Vedic Mathematics is very flexible and creative and appeals to all group of people. It is very easy to understand and practice.

I acknowledge a deep sense of gratitude to all the subject experts for their sincere efforts and expert advice in developing this manual which lead to qualitative and quantitative improvement in mathematics education and may this subject an interesting, joyful and effective.

Suggestions for further improvements are welcome so that in future this manual become more useful.

—Anita Satia

Contents

Introduction

The "Vedic Mathematics" is called so because of its origin from Vedas. To be more specific, it has originated from "Atharva Vedas" the fourth Veda. "Atharva Veda" deals with the branches like Engineering, Mathematics, sculpture, Medicine, and all other sciences with which we are today aware of.

The Sanskrit word Veda is derived from the root Vid, meaning to know without limit. The word Veda covers all Veda-Sakhas known to humanity. The Veda is a repository of all knowledge, fathomless, ever revealing as it is delved deeper.

Vedic mathematics, which simplifies arithmetic and algebraic operations, has increasingly found acceptance the world over. Experts suggest that it could be a handy tool for those who need to solve mathematical problems faster by the day.

It is an ancient technique, which simplifies multiplication, divisibility, complex numbers, squaring, cubing, square roots and cube roots. Even recurring decimals and auxiliary fractions can be handled by Vedic mathematics. Vedic Mathematics forms part of Jyotish Shastra which is one of the six parts of Vedangas. The Jyotish Shastra or Astronomy is made up of three parts called Skandas. A Skanda means the big branch of a tree shooting out of the trunk.

This subject was revived largely due to the efforts of Jagadguru Swami Bharathi Krishna Tirtha Ji of Govardhan Peeth, Puri Jaganath (1884-1960). Having researched the subject for years, even his efforts would have gone in vain but for the enterprise of some disciples who took down notes during his last days. The basis of Vedic mathematics, are the 16 sutras, which attribute a set of qualities to a number or a group of numbers. The ancient Hindu scientists (Rishis) of Bharat in 16 Sutras (Phrases) and 120 words laid down simple steps for solving all mathematical problems in easy to follow 2 or 3 steps.

Vedic Mental or one or two line methods can be used effectively for solving divisions, reciprocals, factorisation, HCF, squares and square roots, cubes and cube roots, algebraic equations, multiple simultaneous equations, quadratic equations, cubic equations, biquadratic equations, higher degree equations, differential calculus, Partial fractions, Integrations, Pythogorus Theoram, Apollonius Theoram, Analytical Conics and so on.

Vedic scholars did not use figures for big numbers in their numerical notation. Instead, they preferred to use the Sanskrit alphabets, with each alphabet constituting a number. Several mantras, in fact, denote numbers; that includes the famed Gayatri Mantra, which adds to 108 when decoded. How fast you can solve a problem is very important. There is a race against time in all the competitions. Only those people having fast calculation ability will be able to win the race. Time saved can be used to solve more problems or used for difficult problems.

Given the initial training in modern maths in today's schools, students will be able to comprehend the logic of Vedic mathematics after they have reached the 8th standard. It will be of interest to everyone but more so to younger students keen to make their mark in competitive entrance exams. India's past could well help them make it in today's world. It is amazing how with the help of 16 Sutras and 13 sub-sutras, the Vedic seers were able to mentally calculate complex mathematical problems.

Sixteen Sutras

S.N.	Sutras	Meaning
1.	एकाधिकेन पूर्वेण *Ekadhikena Purvena (also a corollary)*	One more than the previous one
2.	निखिलं नवतश्चरमं दशतः *Nikhilam Navatascaramam Dasatah*	All from 9 and last from 10
3.	ऊर्ध्वतिर्यग्भ्याम् *Urdhva-tiryagbhyam*	Criss-cross (Vertically and cross-wise)
4.	परावर्त्य योजयेत् *Paravartya Yojayet*	Transpose and adjust (Transpose and apply)
5.	शून्यं साम्यसमुच्चये *Sunyam Samyasamuccaye*	When the samuchchaya is the same, the samuch-chaya is zero, i.e it should be equated to zero
6.	(आनुरूप्ये) शून्यमन्यत् *(Anurupye) Sunyamanyat*	If one is in ratio, the other one is zero
7.	संकलनव्यवकलनाभ्याम् *Sankalana-vyavakalanabhyam* (also a corollary)	By addition and by subtraction
8.	पूरणापूरणाभ्याम् *Puranapuranabhyam*	By the completion or non-completion
9.	चलनकलनाभ्याम् *Calana-Kalanabhyam*	By Calculus
10.	यावदूनम् *Yavadunam*	By the deficiency
11.	व्यष्टिसमष्टिः *Vyastisamastih*	Specific and General (Use the average)
12.	शेषाण्यकेन चरमेण *Sesanyankena Caramena*	The remainders by the last digit
13.	सोपान्त्यद्वयमन्त्यम् *Sopantyadvayamantyam*	The ultimate & twice the penultimate
14.	एकन्यूनेन पूर्वेण *Ekanyunena Purvena*	By one less than the previous one
15.	गुणितसमुच्चयः *Gunitasamuccdyah*	The product of the sum of coefficients in the factors (The whole product)
16.	गुणकसमुच्चयः *Gunakasamuccayah*	Set of Multipliers

Thirteen Sub-Sutras

S.N.	Sutras	Meaning
1.	आनुरूपेण *Anurupyena*	Proportionately
2.	शिष्यते शेषसंज्ञः *Sityate Sesasanfitah*	The remainder remains constant
3.	आद्यमाद्येनान्त्यमन्त्येन *Adyamadyenantyainantyena*	The first by the first and last by the last
4.	केवलैः सप्तकं गुण्यात् *Kevalalh Saptakan Gunyat*	In case of 7 our multiplicand should be 143
5.	वेष्टनम् *Vestanam*	By osculation
6.	यावदूतं तावदूनम् *Yavadunam Tavadunam*	Lessen by the Deficiency
7.	यावदूनं तावदूनीकृत्यवर्ग च योजयेत् *Yavadunam Taradunikrtya Varganca Yojayet*	Whatever the extent of its deficiency, lessen it still to that very extent; and also set up the square of that deficiency.
8.	अत्ययोर्दशकेऽपि *Antyayordasake'pt*	Whose last digits together total 10 and whose previous part is exactly the same
9.	अन्त्ययोरेद *Antyayoteva*	Only the last terms
10.	समुच्चयगुणितः *Samuccayaguaitah*	The sum of the coefficients in the product
11.	लोपस्थापनाभ्याम् *Lopanasthapandbhyam*	By alternate elimination and retention
12.	विलोकनम् *Vilokanam*	By observation
13.	गुणितसमुच्चयः समुच्चयगुणितः *Gunitasamuccayah Samuccayagunitah*	The product of sum of the coefficients in the factors is equal to the sum of the coefficients in the product.

In the text, the words Sutra, aphorism, formula is used synonymously. So are also the words Upa-sutra, Sub-sutra, Sub-formula, corollary used.

The Sutras apply to and cover almost every branch of Mathematics. They apply even to complex problems involving a large number of mathematical operations. Application of the Sutras saves a lot of time and effort in solving the problems, compared to the formal methods presently in vogue. Though the solutions appear like magic, the application of the Sutras is perfectly logical and rational. The computation made on the computers follows, in a way, the principles underlying the Sutras. The Sutras provide not only methods of calculation, but also ways of thinking for their application.

This course on Vedic Mathematics seeks to present an integrated approach to learning Mathematics with keenness of observation and inquisitiveness, avoiding the monotony of accepting theories and working from them mechanically. The explanations offered make the processes clear to the learners. The logical proof of the Sutras is detailed, which eliminates the misconception that the Sutras are a jugglery.

Application of the Sutras improves the computational skills of the learners in a wide area of problems, ensuring both speed and accuracy, strictly based on rational and logical reasoning. The knowledge of such methods enables the teachers to be more resourceful to mould the students and improve their talent and creativity. Application of the Sutras to specific problems involves rational thinking, which, in the process, helps improve intuition that is the bottom - line of the mastery of the mathematical geniuses of the past and the present such as Aryabhatta, Bhaskaracharya, Srinivasa Ramanujan, etc.

This course makes use of the Sutras and Sub-Sutras stated above for presentation of their application for learning Mathematics at the secondary school level in a way different from what is taught at present, but strictly embodying the principles of algebra for empirical accuracy. The innovation in the presentation is the algebraic proof for every elucidation of the Sutra or the Sub-Sutra concerned.

Terms and Operations

(a) **Ekadhika** means 'one more'

e.g: Ekadhika of 0 is 1 Ekadhika of 1 is 2

Ekadhika of 8 is 9 Ekadhika of 23 is 24

Ekadhika of 364 is 365

(b) **Ekanyuna** means 'one less'

e.g: Ekanyuna of 1, 2, 3 ..... 8 ..... 14 .....69 ......is 0, 1, 2, ... 7 ......13 .... 68 ......

(c) **Purak** means 'complement' e.g: Purak of 1, 2, 3 ..... 8, 9 from 10 is 9, 8, 7,..... 2, 1

(d) **Rekhank** means 'a digit with a bar on its top'. In other words it is a negative number.

e.g: A bar on 7 is written as $\overline{7}$. It is called Rekhank 7 or bar 7. We treat Purak as a Rekhank.

e.g: $\overline{7}$ is 3 and $\overline{3}$ is 7

At some instances we write negative numbers also with a bar on the top of the numbers as

–4 can be shown as $\overline{4}$.

–21 can be shown as $\overline{21}$.

(e) Beejank: The Sum of the digits of a number is called Beejank. If the addition is a two digit number, then these two digits are also to be added up to get a single digit.

e.g: Beejank of 27 is 2 + 7 = 9.

Beejank of 348 is 3 + 4 + 8 = 15, further 1 + 5 = 6. i.e. 6 is Beejank.

Easy way of finding Beejank:

Beejank is unaffected if 9 is added to or subtracted from the number. This nature of 9 helps in finding Beejank very quickly, by cancelling 9 or the digits adding to 9 from the number.

e.g. 1: Find the Beejank of 632174.

As above we have to follow

$632174 \longrightarrow 6 + 3 + 2 + 1 + 7 + 4 \longrightarrow 23 \longrightarrow 2 + 3 \longrightarrow 5$

But a quick look gives 6 & 3 ; 2 & 7 are to be ignored because 6 + 3 = 9, 2 + 7 = 9. Hence remaining $1 + 4 \longrightarrow 5$ is the beejank of 632174.

(f) Vinculum: The numbers which by presentation contain both positive and negative digits are called vinculum numbers.

Conversion of general numbers into vinculum numbers

We obtain them by converting the digits which are 5 and above 5 or less than 5 without changing the value of that number.

Consider a number say 8. (Note that it is greater than 5). Use it complement (purak- rekhank) from 10. It is 2 in this case and add 1 to the left (i.e. tens place) of 8.

Thus, $8 = 08 = 1\bar{2}$

The number 1 contains both positive and negative digits.

i.e.1 and $\bar{2}$. Here $\bar{2}$ is in unit place hence it is –2 and value of 1 at tens place is 10.

Thus $1\bar{2} = 10 - 2 = 8$

Conveniently we can think and write in the following way

General Number	Conversion	Vinculum number
6	10 – 4	$1\bar{4}$
97	100 – 3	$10\bar{3}$
289	300 – 11	$3\bar{1}\bar{1}$ etc.,,

Chapter -1 || Addition and subtraction

Addition is the most basic operation and adding number 1 to the previous number generates all the numbers. The Sutra "By one more than the previous one describes the generation of numbers from unity.

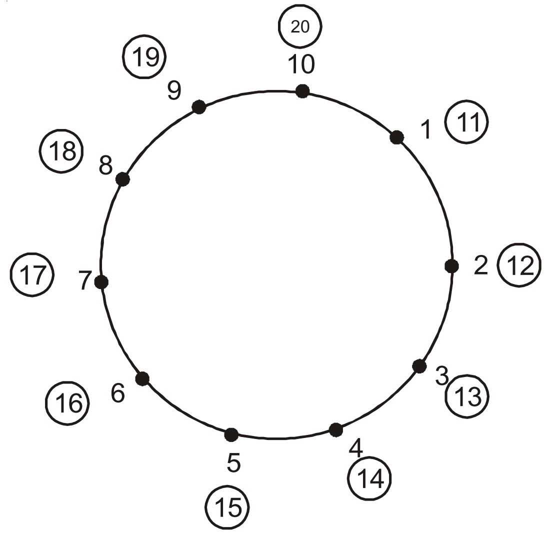
0 + 1 = 1	1 + 1 = 2	2 + 1 = 3	
3 + 1 = 4	4 + 1 = 5	5 + 1 = 6	
6 + 1 = 7	7 + 1 = 8	8 + 1 = 9	9 + 1 = 10......

Completing the whole method

The VEDIC Sutra 'By the Deficiency' relates our natural ability to see how much something differs from wholeness.

7 close to 10
8 close to 10
9 close to 10
17,18,19, are close to 20
27, 28, 29, are close to 30
37, 38, 39, are close to 40
47, 48, 49, are close to 50
57, 58, 59, are close to 60
67, 68, 69, are close to 70
77, 78, 79, are close to 80
87, 88, 89, are close to 90
97, 98, 99, are close to 100 ....
and so on

The ten Point Circle



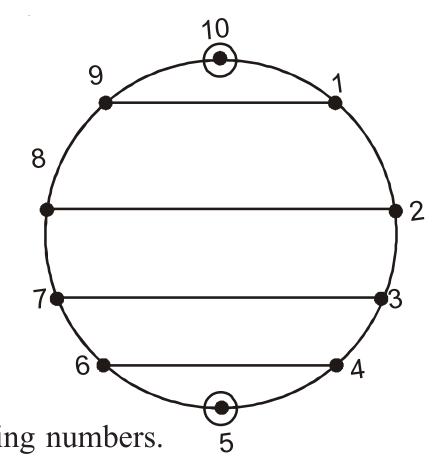
We can easily say that 9 is close to 10,
19 is close to 20 etc.

We can use this closeness to find addition and subtraction.

The ten Point Circle

Rule : By completion non-completion

Five number pairs
1+ 9
2 + 8
3 + 7
4 + 6
5 + 5



Use these number pairs to make groups of '10' when adding numbers.

Example : 24 + 26 = 20 + 4 + 20 + 6 = 20 + 20 + 10 = 50

Below a multiple of ten **Rule :** By the deficiency

49 is close to 50 and is 1 short.

38 is close to 40 and is 2 short.

Example : 59 + 4 = 59 + 1 + 3 = 60 + 3 = 63 {59 is close to 60 and 1 short 50, 59 + 4 is 60}

Example : 38 + 24 = 38 + 2 + 22 = 40 + 22 = 62

or

38 + 24 = 40 + 24 – 2 = 64 – 2 = 62 {38 is close and is 2 sheet so, 38 + 24 is 2 short from 40 + 24 hence 38 + 24 = 40 + 24 – 2 = 64 – 2 = 62

Example

Add 39 + 6 = ?

39 is close to 40 and is 1 less then it.

So we take 1 from the 6 to make up 40 and then we have 5 more to add on which gives 45

Add

29 + 18 + 3

29 + 18 + 1 + 2 [As 3 = 1 + 2 and 29 + 1 = 30, 18 + 2 = 20]

30 + 20 = 50 Note we break 3 into 1 + 2 because 29 need 1 to become 30 and 18 need 2 become 20]

Add

39 + 8 + 1 + 4

39 + 8 + 1 + 2 + 2

40 + 10 + 2 = 52

Sum of Ten

The ten point circle illustrates the pairs of numbers whose sum is 10.

Remember : There are eight unique groups of three number that sum to 10, for example 1 + 2 + 7 = 10

[1] + [2] + [7] = [10]

Can you find the other seven groups of three number summing to 10 as one example given for you?

[2] + [3] + [5] = [10]

Adding a list of numbers

Rule : By completion or non-completion

Look for number pairs that make a multiple of 10

7 + 6 + 3 + 4

The list can be sequentially added as follows :

7 + 6 = 13 then 13 + 3 = 16 then 16 + 4 = 20

Or

You could look for number pairs that make multiples of 10.

7 + 3 is 10 and 6 + 4 is 10

hence 10 + 10 is 20.

Similarily : 48 + 16 + 61 + 32

= (48 + 32) + (16 + 1 + 60)

= 80 + 77 = 157

10 10 10

or 7 + 8 + 9 + 2 + 3 + 5 + 3 + 1 + 2 + 3 + 7 + 9

10 10

= 10 + 10 + 10 + 10 + 10 + 9 = 59

PRACTICE PROBLEMS

Add by using completing the whole method

1. 39 + 8 + 1 + 5 =
2. 18 + 3 +2 + 17 =
3. 9 + 41 + 11 +2 =
4. 47 + 7 + 33 23 =
5. 23 + 26 + 27 + 34=
6. 22 + 36 + 44 + 18=
7. 33 + 35 + 27 + 25=
8. 18 + 13 + 14 + 23=
9. 3 + 9 + 8 + 5 + 7 + 1+ 2=
10. 37 + 25 + 33 =
11. 43 + 8 + 19 + 11=
12. 42 + 15 + 8 +4=
13. 24 + 7 + 8 + 6 +13=
14. 16 +43 + 14 +7 =
15. 13 + 38 +27=

ADDITION

Completing the whole method (class VI commutative & associative property)

1. 39 + 17 + 11 + 13 =
2. 16 + 23 + 24 + 7 =
3. 12 + 51 + 9 + 18 =
4. 35 + 12 +55 =

5. 123 + 118 + 27 =

6. 35 + 15 + 16 + 25=

7. 58 + 41 + 12 + 9=

8. 223 + 112 + 27 =

9. 24 + 106 + 508 + 12=

10. 506 + 222 + 278 =

Adding from left to right

The conventional methods of mathematics teachers use to do calculation from right and working towards the left.

In Vedic mathematics we can do addition from left to right which is more, useful, easier and sometimes quicker.

Add from left to right

1.
```
   23
 + 15
 ----
   38
```

2.
```
   2 3 4
 + 5 2 4
 -------
   7 5 8
```

3.
```
 1 5
 3 8
 ---
 4 3
 Add 1
 = 53
```

4.
```
 2 3 5
 5 2 6
 -----
 7 5 1
 Add 1
 =  761
```

The method: This is easy enough to do mentally, we add the first column and increase this by 1 if there is carry coming over from the second column. Then we tag the last figure of the second column onto this

Mental math

Add from left to right

(1) 6 6 + 5 5

(2) 5 4 6 + 6 7 1

(3) 5 3 4 + 7 1 7

(4) 1 4 5 7 + 2 8 5 7

(5) 4 5 + 7 6

(6) 3 1 2 4 6 5 + 7 6 1 2 4 6

(7) 7 4 5 + 2 7

(8) 1 4 3 2 + 8 6 6 8

(9) 8 5 + 2 3

(10) 5 3 7 + 7 1 8

(11) 4 5 6 + 1 2 7

(12) 2 6 4 8 + 8 3 6 5

(13) 1 3 4 5 + 5 8 3 6

(14) 5 4 6 + 4 5 6 1

(15) 7 8 8 5 + 1 5 4 3

(16) 3 7 8 + 4 8

(17) 3 5 6 7 1 + 1 2 3 4 5

(18) 2 4 6 8 + 1 2 3

Shudh method for a list of number

Shudh means pure. The pure numbers are the single digit numbers i.e. 0, 1, 2, 3…9. In Shudh method of addition we drop the 1 at the tens place and carry only the single digit forward.

Example: Find 2 + 7 + 8 + 9 + 6 + 4

```
  2
• 7
• 8
  9
• 6
  4
 ---
 36
```

We start adding from bottom to top because that is how our eyes naturally move but it is not necessary we can start from top to bottom. As soon as we come across a two-digit number, we put a dot instead of one and carry only the single digit forward for further addition. We put down the single digit (6 in this case) that we get in the end. For the first digit, we add all the dots (3 in this case) and write it.

Adding two or three digit numbers list

```
. 23.4
  6.5.8
  .81.8
    46
  ----
  1756
```

We start from the bottom of the right most columns and get a single digit 6 at the unit place. There are two dots so we add two to the first number (4) of the second column and proceed as before. The one dot of this column is added to the next and in the end we just put 1 down (for one dot) as the first digit of the answer.

(Shudh method)

```
•  5            2 6
•  9          • 4•5
   4            3 4
•  6          • 8 1
   7            5 2
•  8          -----
   4           23 8
 ---
  43
```

Add the following by (Shudh method)

```
1.     5        2.   37        3.   345
       7             64             367
       6             89             289
       8             26           + 167
       4           + 71
     + 9
```

4. 3126	5. 468	6. 235
1245	937	579
4682	386	864
+ 5193	654	+ 179

7. 59	8. 49	9. 98
63	63	83
75	78	78
82	85	62
+ 91	+ 97	+ 44

10. 37	11. 2461	12. 9721
79	4685	2135
52	6203	5678
88	1234	207
+ 91	+ 5432	+ 1237

Number Spliting Method

Quick mental calculations can be performed more easily if the numbers are 'split into more manageable parts.

For example : Split into two more manageable sums

$$\begin{array}{r} +\ 3642 \\ \underline{2439} \end{array} \qquad \begin{array}{r|l} 36 & 42 \\ +\ 24 & 39 \\ \hline 60 & 81 \end{array}$$

Note : The split allows us to add 36 + 24 and 42 + 39 both of which can be done mentally

Remember : Think about where to place the split line. It's often best to avoid number 'carries' over the line.

For example :

$$\begin{array}{r} 342 \\ +\ 587 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r|l} 3 & 42 \\ 5 & 87 \\ \hline 2 & 29 \end{array} \qquad \begin{array}{r|l} 34 & 2 \\ 58 & 7 \\ \hline 92 & 9 \end{array}$$

carry (1) No carry is required

A carry of '1' over the line is required

SUBTRACTION

Sutra: All from 9 and the Last from 10

The Concept of Base

Numbers made up of only 1's and 0's are known as a Base.

Examples of a Base are

10, 100, 1000, 1, .01….etc

The base method is used for subtracting, multiplying or dividing numbers. Like 98, 898, 78999 etc that are close to base.

Applying the formula "All form 9 and Last form 10" to any number especially the big one's reduces it to its smaller Counterpart that can be easily used for calculations involving the big digits like 7, 8, and 9.

Applying the formula "All from 9 and the last from 10"

Example: Apply 'All from 9 Last from 10' to

Subtract 789 from 1000

7 8 9

↓ ↓ ↓ [Here all from 9 last from 10 means subtract 78 8 from 9 and 9 from 10, so weget 211]

2 1 1

We get 211, because we take 7 and 8 from 9 and 9 from 10.

from 10000	from 100	from 100	from 100000
2772	54	97	10804
↓↓↓↓	↓↓	↓↓	↓↓↓↓↓
7228	46	03	89196

If you look carefully at the pairs of numbers in the above numbers you may notice that in every case the total of two numbers is a base number 10, 100, 1000 etc.

This gives us an easy way to subtract from base numbers like 10, 100, 1000…….

Subtracting from a Base

Example: - 1000 – 784 = 216

Just apply 'All from 9 and the Last from 10' to 784, difference of 7 from 9 is 2, 8 from 9 is 1, 4 from 10 is 6 so we get 216 after subtraction.

When subtracting a number from a power of 10 subtract all digits from 9 and last from 10.

$$\begin{array}{r} 1000 \\ -\ 276 \\ \hline 724 \\ \hline \end{array} \longrightarrow$$

from 9 | from 10

276

↓↓↓

724

Subtracting from a Multiple of a Base

Sutra: 'All from 9 and the last from 10'

and

'One less than the one before'

Example: 600 – 87

We have 600 instead of 100. The 6 is reduced by one to 5, and the All from 9 and last from 10 is applied to 87 to give 13. Infact, 87 will come from one of those six hundred, so that 500 will be left.

$\therefore$ 600 – 87 = 513 [***Note :*** First subtract form 100 then add 500, as 500 + 13 = 513]

Example: Find 5000 – 234

5, is reduced to one to get 4 and the formula converts 234 to 766

$\therefore$ 5000-234=4766

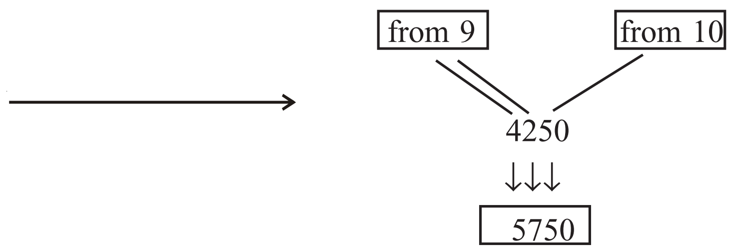
Example: 1000 – 408 = 592

Example:100 – 89 = 11

Example:1000 – 470 = 530 [Remember apply the formula just to 47 here.]

If the number ends in zero, use the last non-zero number non-zero number as the last number for example.

$$\begin{array}{r} 10000 \\ -\ 4250 \\ \hline 5750 \\ \hline \end{array}$$



Hence 1000 – 4250 = 5750

Adding Zeroes

In all the above sums you may have noticed that the number of zeros in the first number is the same as the numbers of digits in the number being subtracted.

Example: 1000 – 53 here 1000 has 3 zeros and 53 has two digits.

We can solve this by writing

$$\begin{array}{r} 1000 \\ -\ \ 053 \\ \hline 947 \end{array}$$

We put on the extra zero in front of 53 and then apply the formula to 053.

Example: 10000 – 68, Here we need to add two zeros.

10000 – 0068 = 9932

Practice Problems

Subtract from left to right

(1) 86 – 27 =

(2) 71 – 34 =

(3) 93 – 36 =

(4) 55 – 37 =

(5) 874 – 567 =

(6) 804 – 438 =

(7) 793 – 627 =

(8) 5495 – 3887 =

9) 9275 – 1627 =

(10) 874– 579 =

(11) 926 – 624 =

(12) 854– 57 =

(13) 8476 – 6278 =

(14) 9436 – 3438 =

Subtract the following mentally

(1) 55 – 29 =

(2) 82 – 558 =

(3) 1000 – 909 =

(4) 10000 – 9987 =

(5) 10000 – 72 =

(6) 50000 – 5445 =

(7) 70000 – 9023 =

(8) 30000 – 387 =

(9) 46678 – 22939 =

(10) 555 – 294 =

(11) 8118 – 1771 =

12) 61016 – 27896 =

Example: Find 9000 – 5432

Sutra: ‘One more than the previous one’ and ‘all from 9 and the Last from the 10’

Considering the thousands 9 will be reduced by 6 (one more than 5) because we are taking more than 5 thousand away

‘All from 9 and the last from 10’ is than applied to 432 to give 568

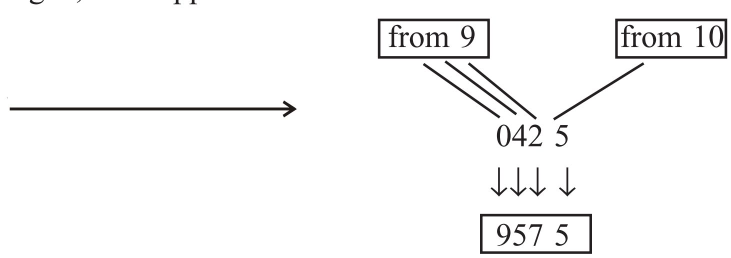
9000 – 5432 = 3568

Similary—7000 – 3884

= 3116 {3 = 7 – 4, 4 is one more than 3 and 116 = 4000 – 3884} by all from a and the last from 10}

If the number is less digits, then append zero the start :

$$\begin{array}{r} 1000 \\ -\ 425 \\ \hline 9575 \\ \hline \end{array}$$



When subtracting form a multiple ofa power of 10, just decrement the first digit by 1, then subtract remaining digits :

$$\begin{array}{r} 4000 \\ -\ 257 \\ \hline 3743 \end{array} \longrightarrow$$

from 9   from 10

257

↓↓↓

4 – 1 → 3 753

Look at one more example :

Money: A great application of "all from 9 and last from 10" is money. Change can be calculated by applying this sutra mentally for example :

$$\begin{array}{r} 10.00 \\ -\ 4.25 \\ \hline 5.75 \end{array} \longrightarrow$$

from 9   from 10

4.25

5.75

This is helpful because most our rupee notes are multiple of 10's.

PRACTICE PROBLEMS

Subtract (base method)

(1) 1000 – 666

(2) 10000 – 3632

(3) 100 – 54

(4) 100000 – 16134

(5) 1000000 – 123456

(6) 1000 – 840

(7) 1000 – 88

(8) 10000 – 568

(9) 1000 – 61

(10) 100000 – 5542

(11) 10000 – 561

(12) 10000 – 670

Subtract (multiple of base)

(1) 600 – 72 =

(2) 90000 – 8479 =

(3) 9000 – 758 =

(4) 4000 – 2543 =

(5) 7000 – 89 =

(6) 300000 – 239 =

(7) 1 – 0.6081 =

(8) 5 – 0.99 =

Subtracting Near a base

Rule : By completion or non completion.

when subtracting a number close to a multiple of 10. Just subtract from the multiple of 10 and correct the answer accordingly.

Example : 53 – 29

29 is just close to 30, just 1 short, so subtaract 30 from 53 making 23, then add 1 to make 24.

$$53 - 29 = 53 - 30 + 1$$
$$= 23 + 1$$
$$= 24$$

Similarily

$$45 - 18$$
$$= 45 - 20 + 2$$
$$= 25 + 2$$
$$= 27 \quad \{18 \text{ is near to 20, just 2 short})$$

Use the base method of calculating

To find balance

Q. Suppose you buy a vegetable for Rs. 8.53 and you buy with a Rs. 10 note. How much change would you expect to get?

Ans. You just apply "All from 9 and the last from 10" to 853 to get 1.47.

Q. What change would expect from Rs. 20 when paying Rs. 2.56?

Ans. The change you expect to get is Rs. 17.44 because Rs. 2.56 from Rs.10 is Rs. 7.44 and there is Rs. 10 to add to this.

Practice Problem

Q1. Rs. 10 – Rs. 3.45

Q2. Rs. 10 – Rs. 7.61

Q3. Rs. 1000 – Rs. 436.82

Q4. Rs. 100 – Rs. 39.08

Subtracting number just below the base

Example: find 55 – 29

Subtraction of numbers using "complete the whole"

Step 1: 20 is the sub base close to 19

19 is 1 below 20

Step 2: take 20 from 55 (to get 35)

Step 3: Add 1 back on 55 – 19 = 36

Example

61 – 38

38 is near to 40 = 40 – 38 = 2

61 – 40 = 21

61 – 38 = 21 + 2 = 23

Example

44 – 19

19 + 1 = 20

44 – 20 = 24

44 – 19 = 24 – 1 = 23

Example 88 – 49

49+1=50

88 – 50 = 38

88 – 49 = 38 + 1 = 39

Example

55 – 17

17 + 3 = 20

55 – 20 = 35

55 – 17 = 35 + 3 = 38

Number spliting Method

As you have use this method in addtion the same can be done for subtraction also :

$$\begin{array}{r} +\,3642 \\ \underline{2439} \end{array} \longrightarrow \begin{array}{r|l} 36 & 42 \\ +\,24 & 39 \\ \hline 12 & 03 \end{array}$$

Note : The split allows on to add '36 – 24' and 42 – 39 both of which can be done mentally

General Method of subtraction

Subtraction from left to right

In this section we show a very easy method of subtracting numbers from left to right that we have probably not seen before. We start from the left, subtract, and write it down if the subtraction in the next column can be done. If it cannot be done you put down one less and carry 1, and then subtract in the second column.

Subtraction from left to right.

Example: Find 83 – 37

$$\begin{array}{r} 8\ 3 \\ \underline{-\,3\ 7} \\ \underline{4\ 6} \end{array}$$

Find 78 – 56

$$\begin{array}{r} 7\ 8 \\ \underline{-\,5\ 6} \\ \underline{2\ 2} \end{array}$$

Left to right

(3)

$$\begin{array}{r|l} 5 & {}^{1}1 \\ -\,4 & 9 \\ \hline 0 & 2 \end{array}$$

(4)

$$\begin{array}{r|r|l} 3 & {}^{1}2 & {}^{1}1 \\ -\;2 & 8 & 9 \\ \hline 0 & 3 & 2 \end{array}$$

(5)

$$\begin{array}{r|c|l} 3 & 0 & {}^{1}1 \\ -2 & 0 & 4 \\ \hline 1 & 9 & 7 \end{array}$$

(6)

$$\begin{array}{r|c|l} 3 & 0 & 1 \\ -2 & 0 & 1 \\ \hline 1 & 0 & 0 \end{array}$$

(7)

$$\begin{array}{r|c|r|c|l} 3 & 5 & {}^{1}5 & 6 & {}^{1}7 \\ -1 & 1 & 8 & 2 & 8 \\ \hline 2 & 3 & 7 & 3 & 9 \end{array}$$

Starting from the left we subtract in each column 3-1=2 but before we put 2 down we check that in next column the top number is larger. In this case 5 is larger than 1 so we put 2 down

In the next column we have 5-1=4, but looking in the third column we see the top number is not larger than the bottom(5 is less than 8) so instead putting 4 down we put 3 and the other 1 is placed as the flag, as shown so that 5 becomes 15, so now we have 15-8=7. Checking in the next column we can put this down because 6 is greater than 2. In the fourth column we have 6-2=4, but looking at the next column (7 is smaller than 8) we put down only 3 and put the other flag with 7 as shown finally in the last column 17-8=9.

Chapter 2 ‖ Digit sums, casting out 9's and 9' check method

The word digit means a single figure number: The numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 are all digits. Big numbers can be reduced to single digit by adding the constituents.

Digit Sums

A digit sum is the sum of all the digits of a number and is found by adding all of the digits of a number

The digit sum of 35 is 3 + 5 = 8

The digit sum of 142 is 1 + 4 + 2 = 7

Note : If the sum of the digits is greater than 9, then sum the digits of the result again until the result is less than 10.

The digit of 57 is 5 + 7 = 12 → 1 + 2 = 3 (greater than 9, so need to add again)

Hence the digit sum of 57 is 3.

The digit sum of 687 is 6 + 8 + 7 = 21 → 2 + 4 = 3

Hence the digit sum of 687 is 3.

- Keep findig the digit sum of the result + unitl it's less then 10
- 0 and 9 are requivalent

Look and undevstand some more example :

To find the digit sum of 18, for the example we just add 1 and 8, i.e.1 + 8 =9 so the digit sum of 18 is 9. And the digit sum of 234 is 9 because 2 + 3+ 4 = 9

Following table shows how to get the digit sum of the following members

15	6
12	3
42	6
17	8
21	3
45	9
300	3
1412	8
23	5
22	4

Sometimes two steps are needed to find a digit sum.

So for the digit sum of 29 we add 2 + 9 = 11 but since 11 is a 2-digit number we add again 1+1=1 So for the digit sum of 29 we can write

29 = 2 + 9 = 11 = 1 + 1 = 1

Similarity for 49 = 4 + 9 = 13 = 1 + 3 = 4

So the digit sum of 49 is 4.

Number 14	Digit sum 1 + 4 = 5	Single digit 5
19	1 + 9 = 10	1
39	3+ 9 = 12	3
58	5 + 8 = 13	4
407	4 + 0 + 7 = 11	2

CASTING OUT NINE

Adding 9 to a number does not affect its digit sum

So 5, 59, 95, 959 all have digit sum of 5.

For example to find out the digit sum of 4939 we can cast out nines and just add up the 3 and 4 so digit sum is 7 or using the longer method we add all digit 4 + 9 + 3+ 9 =25 = 2 + 5=7

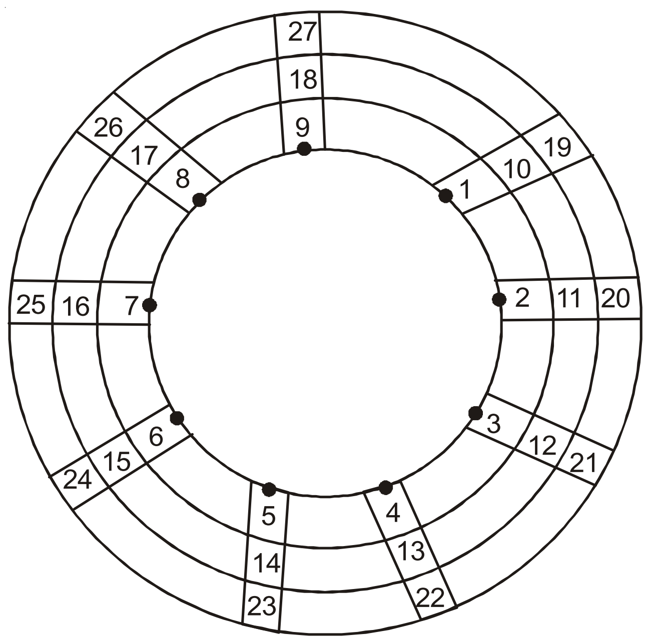
There is another way of casting out the nines from number when you are finding its digit sum.

Casting out of 9's and digit totalling 9 comes under the Sutra when the samuccaya is the same it is zero.

So in 465 as 4 and 5 total nine, they are cast out and the digit sum is 6: when the total is the same (as 9) it is zero (can be cast out) cancelling a common factor in a fraction is another example.

Number	Digit sum
1326	3
25271	8
9643	4
23674	4
128541	3
1275	6
6317892	9 or 0

Nine Point Circle



Number at each point on the circle have the same digit sum.

By casting out 9's, finding a digit sum can be done more quickly and mentally.

9. Check Method

Digit sum can be used to check that the answers are correct.

Example: Find 23 + 21 and check the answer using the digit sums

23 = digit sum of 23 is 2 + 3 = 5

+21 = digit sum of 21 is 2 + 1 = 3

44 = digit sum of 44 is 4 + 4 = 8

If the sum has been done correctly, the digit sum of the answer should also be 8

Digit sum of 44=8 so according to this check the answer is probably correct.

There are four steps to use digit sum to check the answers:

1. Do the sum.
2. Write down the digit sums of the numbers being added.
3. Add the digit sums.
4. Check whether the two answers are same in digit sums.

Add 278 and 119 and check the answer

278

+119

397

1. We get 397 for the answer
2. We find the digit sum of 278 and 119 which are, 8 and 2 respectively
3. Adding 8 and 2 gives 10, digits sum of 10=1+0=1
4. Digit sum of 397 is

   3 + 9 + 7 = 19 =1 + 9 = 10 = 1 + 0 = 1

   Which confirm the answer?

CAUTION!

Check the following sum:

279 9

121 4

490 4

Here an estimation can help you to find the result more accurate if by mistage you write 400 in place of 490 then it will show the result is correct.

The check is 9 + 4 = 13 = 4 which is same as the digit sum of the answer which confirms the answer.

However if we check the addition of the original number we will find that it is incorrect! This shows that the digit sum does not always find errors. It usually works but not always. We will be looking at another checking device i.e. 11 - check method.

Note : The difference of 9 and its multiples in the answer make errors. So, keep in mind a rough estimation.

Practice Problems

Digit sum Puzzles

1. **The digit sums of a two digit number is 8 and figures are the same, what is the number?**
2. **The digit sum of a two digit number is 9 and the first figure is twice the second, what is it?**
3. **Give three two digit numbers that have a digit sum of 3.**
4. **A two digit number has a digit sum of 5 and the figures are the same. What is the number?**
5. **Use casting out 9's to find the digit sums of the numbers below.**

Number	
465	
274	
3456	
7819	
86753	
4017	
59	

6. **Add the following and check your answer using digit sum check**

(1) 66 + 77 =
(2) 57 + 34 =
(3) 94 + 89 =
(4) 304 + 233 =
(5) 787 + 132 =
(6) 389 + 414 =
(7) 5131 + 5432 =
(8) 456 + 654 =

Chapter 3 ‖ Eleven Check Method

We have already used the digit sum check that helps to show if a calculation is correct. This method works because adding the digit in a number gives the remainder of the number after division by 9.

A similar method works by using remainders of numbers after division by 11 rather than 9

Alternate digit sum or Eleven-check Method

Suppose we want another check for 2434 × 23 = 55982 it can be done in the following steps

Step1: Alternately add and subtract (starting from right moving towards left) the digits of each numbers as described below

Number	Alternating signs	Digit sum
2434	–2 + 4 – 3 + 4	3
23	–2 + 3	1
55982	+5 – 5 + 9 – 8 + 2	3

Step 2: Now multiply the Digit Sum to get the product 3 × 1 = 3 Since the Digit Sum of the product and the two numbers is the same, the answer is correct as per 11 check method.

Two digit and Negative number in the digit sum checking the sum of addition

4364 + 1616

Left to right

```
 4364
 1916
 ----
 6280
```

Number	Alternating signs	Digit sum	Single digit
4364	–4 + 3 – 6 + 4	–3 (11-3)	8
1916	–1 + 9 – 1 +6	13(11+2)	2
6280	–6 + 2 – 8 + 0	– 12	10
		11 –12 = –1 11 – 1 = 10	

Step2: Apply the following rules to get a single positive digit for the number

- Subtract the negative numbers below 11 from 11 to get its positive counterpart so – 3 =11 –3 = 8
  And –12 = –12 +11 = –1 = 11 – 1 =10
- For the two digit number above 11, divide the number by 11 and get the remainder as the positive digit sum so 13 ÷ 13 gives remainder 2. Alternately, adding and subtracting digit of 13 starting from right can obtain this same result.

Step 3 : now add the Digit sums to get the sum 8 + 2 = 10, the answer is correct as per 11 check method.

Two digits in the digit sum

Check subtraction problem

2819174 – 839472

2819174

<u>839472</u>

1979702

Step 1: Alternatively add and subtract (staring from right moving towards left) the digit of each numbers as described below

Number	Alternating signs	Digit sum	Single digit
2819174	+2–8+1–9+1–7+4	–16(–16+11= –5)	11–5=6
839472	–8+3–9+4–7+2	–15(–15+11= –4)	11–4=7
1979702	+1–9+7–9+7–0+2	–1	11–1=10

Step 2: Apply the following rules to get a single positive digit for the number

- The negative numbers below –11 are to be first divided by 11 to get the remainder. Than subtract the remainder from 11 to get its positive counterpart. So –16/11 Remainder is –5 and –5 = 11 – 5 = 6 similarly –15/11 Remainder –4 = 11 –4 = 7.
- The negative number –1 = 11 – 1 = 10

Step3: Now subtract the Digit sums to get the answer 6 – 7 = –1 = 10, the answer is correct as per 11- checked method.

Practice Problems

Get the digit sum and single digit for the following numbers.

Numbers	Alternative signs	Digit sums	Single digit
567			
1536			
93823			
1978712			
849391			
82918			
5949393			
176780			

Using 11 check method check the following Addition problems:

(1) 37 + 47 = 84

(2) 55 + 28 = 83

(3) 47 + 25 = 72

(4) 29 + 36 = 65

(5) 526 + 125 = 651

(6) 1328 + 2326 = 3654

(7) 129 + 35644 = 35773

(8) 3425 + 7491 + 8834 = 19750

(9) 1423178 + 5467 + 123 + 34 = 1428802

(10) 1314 + 5345 + 65 +781 = 7505

Check the following subtraction problems:

(1) 63 – 28 = 35

(2) 813 – 345 = 468

(3) 695 – 368 = 372

(4) 3456 – 281 = 3175

(5) 7117 – 1771 = 5346

(6) 8008 – 3839 = 4165

(7) 6363 – 3388 = 2795

(8) 51015 – 27986 = 23029

(9) 14285 – 7148 = 7137

(10) 9630369 – 3690963 = 5939406

Chapter- 4 ‖ Special Multiplication Methods

Multiplication in considered as one of the most difficult of the four mathematical operations. Students are scared of multiplication as well as tables. Just by knowing tables up to 5 students can multiply bigger numbers easily by some special multiplication methods of Vedic Mathematics. We should learn and encourage children to look at the special properties of each problem in order to understand it and decide the best way to solve the problem. In this way we also enhance the analytical ability of a child. Various methods of solving the questions /problems keep away the monotonous and charge up student's mind to try new ways and in turn sharpen their brains.

Easy way for multiplication

Sutra:Vertically and Cross wise :

For speed and accuracy tables are considered to be very important. Also students think why to do lengthy calculations manually when we can do them faster by calculators. So friends/ teachers we have to take up this challenge and give our students something which is more interesting and also faster than a calculator. Of course it's us (the teachers/parents) who do understand that more we use our brain, more alert and active we will be for, that is the only exercise we have for our brain.

Example 1: 7 x8

Step 1: Here base is 10,

7 – 3 (7 is 3 below 10) also called deficiencies

× 8 – 2 (8 is 2 below 10) also called deficiencies

Step 2: Cross subtract to get first figure (or digit) of the answer: 7 – 2 = 5 or 8 – 3 = 5 , the two difference are always same.

Step 3 : Multiply vertically *i.e.* –3 × –2 = 6 which is second part of the answer.

So, 7 – 3

8 – 2 *i.e.* 7 × 8 = 56

5 / 6

Example 2: To find 6 × 7

Step 1 : Here base is 10,

6 – 4 (6 is 4 less than 10) *i.e.* deficiencies

7 – 3 (7 is 3 less than 10) *i.e.* deficiencies

Step 2: Cross subtraction : 6 – 3 = 3 or 7 – 4 = 3 (both same)

Step 3: – 3 × – 4 = + 12 , but 12 is 2 digit number so we carry this 1 over to 3 (obtained in 2 step)

6– 4

7 – 3

3 / (1) 2 *i.e.* 6 × 7 = 42

Try these : (1) 9 × 7 (ii) 8 × 9 (iii) 6 × 9 (iv) 8 × 6 (v) 7 × 7

Second Method:

Same Base Method :

When both the numbers are more than the same base. This method is extension of the above method i.e. we are going to use same sutra here and applying it to larger numbers.

Example 1: 12 × 14

Step 1: Here base is 10

12 + 2 [12 is 2 more than 10 also called surplus]

14 + 4 [14 is 4 more than 10also called surplus]

Step 2: Cross add: 12 + 4 =16 or 14 + 2 = 16,(both same) which gives first part of answer = 16

Step 3: Vertical multiplication: 2 × 4 = 8

So, 12 + 2

14 +4

16 / 8So, 12 × 14 = 168

(14 + 2 = 12 + 4)

Example 2:105x 107

Step1: Here base is 100

105 + 05 [105 is 5 more than 100 or 5 is surplus]

107 + 07 [107 is 7 more than 100 or 7 is surplus]

Base here is 100 so we will write 05 in place of 5and 07 in place of 7

Step 2: Cross add: 105 + 7 = 112 or 107 + 5 = 112 which gives first part of the answer = 112

Step 3: Vertical multiplication: 05 × 07 = 35 (two digits are allowed)

As the base in this problem is 100 so two digits are allowed in the second part.

So, 105 × 107 = 11235

Example 3: 112 x 115

Step 1: Here base is 100

112 + 12 [2 more than 100 i.e. 12 is surplus]

115 + 15 [15 more than 100 i.e. 15 is surplus]

Step 2: Cross add: 112 + 15 = 127 = 115 + 12 to get first part of answer

*i.e.*127

Step 3: Vertical multiplication 12 × 15 = ? Oh, my god!It's such a big number. How to get product of this? Again use the same method to get the product.

12 + 2

15 + 5

12 + 5 = 15 + 2 = 17/ (1) 0, 17 + 1 / 0 = 180 *i.e.* 12 × 15 = 180

But only two digits are allowed here, so 1 is added to 127 and we get (127 + 1) = 128

So, 112 × 115 = 128, 80

Try these: (i)12 × 14 (ii) 14 × 17 (iii) 17 × 19 (iv) 19 × 11 (v) 11 × 16 (vi) 112 × 113 (vii) 113 × 117 (viii) 117 × 111 (ix) 105 × 109 (x) 109 × 102 (xi) 105 × 108 (xii) 108 × 102 (xiii) 102 × 112 (xiv) 112 × 119 (xv) 102 × 115

Both numbers less than the same base:

Same sutra applied to bigger numbers which are less than the same base.

Example1: 99 × 98

Step 1: Check the base: Here base is 100 so we are allowed to have two digits on the right hand side.

∴ 99 – 01 (1 less than 100) i.e. 01 deficiency

98 – 02 (2 less than 100) i.e. 0 2 deficiency

Step 2: Cross – subtract: 99 – 02 = 97 = 98 – 01 both same so first part of answer is 97

Step3: Multiply vertically – 01 × – 02 = 02 (As base is 100 so two digits are allowed in second part

So, 99 × 98 = 9702

Example 2 : 89 × 88

Step1: Here base is 100

So, 89 – 11 (i.e. deficiency = 11)

88 – 12 (i.e. deficiency = 12)

Step2: Cross subtract: 89 – 12 = 77 = 88 – 11(**both same)**

So, first part of answer can be 77

Step 3: Multiply vertically – 11 × – 12

Again to multiply 11 × 12 apply same rule

11 + 1 (10 + 1)

<u>12 + 2</u> (10 + 2)

11 + 2 = 13 = 12 + 1 / 1 × 2 = 12 so, 11 × 12 = (1) 32 as only two digits are allowed on right hand side so add 1to L.H.S.

So, L.H.S. = 77 + 1 = 78

Hence 89 × 88 = 7832

Example 3: 988 × 999

Step 1: As the numbers are near 1000 so the base here is 1000 and hence three digits allowed on the right hand side

988 – 012 (012 less than 1000) i.e. deficiency = 0 12

999 – 001 (001 less than 1000) i.e. deficiency = 00 1

Step 2: Cross – subtraction: 988 – 001 = 987 = 999 – 012 = 987

So first part of answer can be 987

Step 3: Multiply vertically: –012 xs – 001 = 012 (three digits allowed)

∴ 988 × 999 = 987012

How to check whether the solution is correct or not by 9 – check method.

Example 1: 99 × 98 = 9702 Using 9 – check method.

As, $\not{9}\not{9} = 0$ Product (L.H.S.) = 0 × 8 = 0 [taking 9 = 0]

$\not{9}8 = 8$

R.H.S. = $\not{9}702 = 7 + 2 = \not{9} = 0$ $\not{9}702 = 9$ both are same

As both the sides are equal answer may be correct.

Example 2: 89 × 88 = 7832

$8\not{9} = 8$

88 = 8 + 8 = 16 = 1 + 6 = 7 (add the digits)

L.H.S. = 8 × 7 = 56 = 5 + 6 = 11 = 2 (1 + 1)

R.H.S. = $\not{7}83\not{2}$ = 8 + 3 = 11 = 1 + 1 = 2

As both the sides are equal, so answer is correct

Example 3: 988 × 999 = 987012

$\not{9}88$ = 8 + 8 = 16 = 1 + 6 =7

$\not{9}\not{9}\not{9}$ = 0

As 0 × 7 =0 = LHS

$\not{9}\not{8}\not{7}0\not{1}\not{2}$ = 0 (As 7 + 2 = 9 = 0 , 8 + 1 = 9 = 0 also 9 = 0)

∴ RHS = 0

As LHS = RHS So, answer is correct.

Try These*:*

(i) 97 × 99 (ii) 89 × 89 (iii) 94 × 97 (iv) 89 × 92 (v) 93 × 95 (vi) 987 × 998 (vii) 997 × 988 (viii) 988 × 996 (ix) 983 × 998 (x) 877 × 996 (xi) 993 × 994 (xii) 789 × 993 (xiii) 9999 × 998 (xiv) 7897 × 9997 (xv) 8987 × 9996.

Multiplying bigger numbers close to a base: (number less than base)

Example 1: 87798 x 99995

Step1: Base here is 100000 so five digits are allowed in R.H.S.

87798 – 12202 (12202 less than 100000) deficiency is 12202

<u>99995 – 00005</u> (00005 less than100000) deficiency is 5

Step 2: Cross – subtraction: 87798 -00005 =87793

Also 99995 – 12202 = 87793 (both same)

So first part of answer can be 87793

Step 2 : Multiply vertically: –12202 × – 00005 = + 61010

∴ 87798 × 99995 = 8779361010

Checking:

87798 total 8 + 7 + 7 + 8 = 30 = 3 (single digit)

99995 total = 5

LHS = 3 x 5 =15 total = 1 + 5 = 6

RHS = product = 8779361010 total = 15 = 1 + 5 = 6

L.H.S = R.H.S. So, correct answer

Example 2 : 88777 × 99997

Step 1: Base have is 100000 so five digits are allowed in R.H.S.

88777 – 11223 i.e. deficiency is 11223

<u>99997 – 00003</u> i.e. deficiency is 3

Step 2: Cross subtraction: 88777 – 00003 = 88774 = 99997 – 11223

So first part of answer is 88774

Step 3: Multiply vertically: – 11223 × – 00003 = + 33669

∴ 88777 × 99997 = 8877433669

Checking:

88777 total 8 + 8 + 7 + 7 + 7 = 37 = + 10 = 1

99997 total = 7

∴ LHS = 1 × 7 = 7

RHS = 8877433669 =8 + 8 + 7 + 7 + 4 = 34 = 3 + 4 = 7

i.e. LHS = RHS So, correct answer

Try These:

(i) 999995 × 739984 (ii) 99837 × 99995 (iii) 99998 × 77338 (iv) 98456 × 99993 (v) 99994 × 84321

Multiply bigger number close to base (numbers more than base)

Example 1: 10021 × 10003

Step 1: Here base is 10000 so four digits are allowed

10021 + 0021 (Surplus)

<u>10003 + 0003</u> (Surplus)

Step 2: Cross – addition 10021 + 0003 = 10024 = 10003 + 0021 (both same)

∴ First part of the answer may be 10024

Step 3: Multiply vertically: 10021 × 0003 = 0063 which form second part of the answer

∴ 10021 × 10002 = 100240063

Checking:

10021 = 1+ 2 + 1 + 1 = 4

10003 = 1 + 3 = 4

∴ LHS = 4 × 4 = 16 = 1 + 6 = 7

RHS = 1002400~~63~~ = 1 + 2 + 4 = 7

As LHS = RHS So, answer is correct

Example 2: 11123 × 10003

Step 1: Here base is 10000 so four digits are allowed in RHS

11123 + 1123 (surplus)

<u>10003 + 0003</u> (surplus)

Step 2: Cross – addition: 11123 + 0003 = 11126 = 10003 + 1123 (both equal)

∴ First part of answer is 11126

Step 3: Multiply vertically: 1123 × 0003 = 3369 which form second part of answer

∴ 11123 × 10003 = 111263369

Checking:

11123 = 1 + 1 + 1 + 2 + 3 = 8

10003 = 1 + 3 = 4 and 4 × 8 = 32 = 3 + 2 = 5

∴ LHS = 5

R.H.S = 1112~~63369~~ = 1 + 1 + 1 + 2 = 5

As L.H.S = R.H.S So, answer is correct

Try These:

(i) 10004 × 11113 (ii) 12345 × 111523 (iii) 11237 × 10002 (iv) 100002 × 111523 (v) 10233 × 10005

Numbers near different base: **(Both numbers below base)**

Example 1: 98 × 9

Step 1: 98 Here base is 100 deficiency = 02

9 Base is 10 deficiency = 1

∴ 98 – 02 Numbers of digits permitted on R.H.S is 1 (digits in lower base)

Step 2: Cross subtraction: 98

<u>-1</u>

88

It is important to line the numbers as shown because 1 is not subtracted from 8 as usual but from 9 so as to get 88 as first part of answer.

Step 3: Vertical multiplication: (-02) x (-1) = 2 (one digits allowed)

∴ Second part = 2

∴ 98 × 9 = 882

Checking:

(Through 9 – check method)

$\not{9}8 = 8$, $\not{9} = 0$, LHS = 98 × 9 = 8 × 0 = 0

RHS = 882 = 8 + 8 + 2 = 18 = 1 + 8 = $\not{9}$ = 0

As LHS = RHS So, correct answer

Example 2: 993 × 97

Step 1: 993 base is 1000 and deficiency is 007

97 base is 100 and deficiency is 03

∴ 993 – 007 (digits in lower base = 2 So, 2 digits are permitted on

× 97 – 03 RHS or second part of answer)

Step 2: Cross subtraction:

993

– 03

963

Again line the number as shown because 03 is subtracted from 99 and not from 93 so as to get 963 which from first part of the answer.

Step 3: Vertical multiplication: (–007) – (–03) = 21 only two digits are allowed in the second part of answer So, second part = 21

∴ 993 × 97 = 96321

Checking: (through 9 – check method)

$\not{9}\not{9}3 = 3$ $\not{9}7 = 7$

∴ L.H.S. = 3 × 7 = 21 = 2 + 1 = 3

R.H.S. = $\not{9}\not{6}\not{3}21$ = 2 + 1 = 3

As LHS =RHS so, answer is correct

Example 3 : 9996 base is 10000 and deficiency is 0004

988 base is 1000 and deficiency is 012

∴ 9996 – 0004 (digits in the lower base are 3 so,3digits

× 988 – 012 permitted on RHS or second part of answer)

Step 2 : Cross – subtraction:

9996

– 012

9876

Well, again take care to line the numbers while subtraction so as to get 9876 as the first part of the answer.

Step3 : Vertical multiplication: (–0004) × (–012) = 048

(Remember, three digits are permitted in the second part i.e. second part of answer = 048

∴ 9996 × 988 = 9876048

Checking:(9 – check method)

9̸9̸9̸6 = 6, 9̸88 = 8 + 8 + = 16 = 1 + 6 = 7

∴ LHS = 6 × 7 = 42 = 4 + 2 = 6

RHS = 9̸8̸76̸04̸5 = 8 + 7 = 15 = 1 + 5 = 6

As, LHS =RHS so, answer is correct

When both the numbers are above base

Example 1: 105 × 12

Step 1: 105 base is 100 and surplus is 5

12 base is 10 and surplus is 2

∴ 105 + 05 (digits in the lower base is 1 so, 1 digit is permitted in the second part of answer)

12 + 2

Step 2: Cross – addition:

105

+ 2

125 (again take care to line the numbers properly so as to get 125)

∴ First part of answer may be 125

Step 3: Vertical multiplication : 05 × 2 = (1)0 but only 1 digit is permitted in the second part so 1 is shifted to first part and added to 125 so as to get 126

∴ 105 × 12 = 1260

Checking:

105 = 1 + 5 = 6 , 12 = 1 + 2 = 3

∴ LHS = 6 × 3 = 18 = 1 + 8 = 9 = 0

∴ RHS = 1260 = 1 + 2 + 6 = 9=0

Example 2: 1122 × 104

Step1: 1122 – base is 1000 and surplus is 122

104 – base is 100 and surplus is 4

∴ 1122 + 122

104 + 04 (digits in lower base are 2 so, 2-digits are permitted in the second part of answer)

Step 2: Cross – addition

1122

+ 04 (again take care to line the nos. properly so as to get 1162)

1162

∴ First part of answer may be 1162

Step 3: Vertical multiplication: 122 × 04 = 4, 88

But only 2 – digits are permitted in the second part, so, 4 is shifted to first part and added to 1162 to get 1166 (1162 + 4 = 1166)

∴ 1122 × 104 = 116688

Can be visualised as: 1122 + 122

<u>104 + 04</u>

1162 / ← (4) 88 = 116688

+ 4 /

Checking:

1122 = 1 + 1 + 2 + 2 + = 6, 104 = 1 + 4 =5

∴ LHS = 6 × 5 = 30 = 3

RHS = ~~11~~66~~88~~ = 6 + 6 = 12 = 1 + 2 = 3

As LHS = RHS So, answer is correct

Example 3: 10007 × 1003

Now doing the question directly

10007 + 0007 base = 10000

<u>× 1003 + 003</u> base = 1000

10037 / 021 (three digits per method in this part)

∴ 10007 × 10003 = 10037021

Checking : 10007 = 1 + 7 = 8 , 1003 = 1 + 3 = 4

∴ LHS = 8 × 4 = 32 = 3 + 2 = 5

RHS = 1003~~7~~ 0~~2~~1 = 1 + 3 + 1 = 5

As LHS = RHS so, answer is correct

Try These:

(i) 1015 × 103 (ii) 99888 × 91 (iii) 100034 × 102 (iv) 993 × 97 (v) 9988 × 98 (vi) 9995 × 96 (vii) 1005 × 103 (viii) 10025 × 1004 (ix) 102 × 10013 (x) 99994 × 95

VINCULUM: "Vinculum" is the minus sign put on top of a number e.g. $\overline{5}, 4\overline{1}, 6\overline{3}$ etc. which means (–5), (40 – 1) , (60 – 3) respectively

Advantages of using vinculum:

(1) It gives us flexibility, we use the vinculum when it suits us .

(2) Large numbers like 6, 7, 8, 9 can be avoided.

(3) Figures tend to cancel each other or can be made to cancel.

(4) 0 and 1 occur twice as frequently as they otherwise would.

Converting from positive to negative form or from normal to vinculum form:

Sutras: All from 9 the last from 10 and one more than the previous one

$9 = 1\bar{1}$ (i.e. 10 – 1), $8 = 1\bar{2}$, $7 = 1\bar{3}$, $6 = 1\bar{4}$, $19 = 2\bar{1}$, $29 = 3\bar{1}$

$28 = 3\bar{2}$, $36 = 4\bar{4}$ (40 – 4), $38 = 4\bar{2}$

Steps to convert from positive to vinculum form:

(1) Find out the digits that are to be converted i.e. 5 and above.

(2) Apply "all from 9 and last from 10" on those digits.

(3) To end the conversions "add one to the previous digit".

(4) Repeat this as many times in the same number as necessary.

Numbers with several conversions:

$159 = 2\overline{41}$ (i.e. 200 – 41)

$168 = 2\overline{32}$ (i.e. 200 – 32)

$237 = 2\overline{43}$ (i.e. 240 – 7)

$1286 = 13\overline{14}$ (i.e. 1300 – 14)

$2387129 = 24\overline{13}13\bar{1}$ (here, only the large digits are be changed)

From vinculum back to normal form:

Sutras: "All from 9 and last from ten" and "one less than then one before".

$1\bar{1} = 09$ (10 – 1), $1\bar{3} = 07$ (10 – 3) , $2\bar{4} = 16$ (20 – 4), $2\overline{41} = 200 - 41 = 159$, $16\bar{2} = 160 - 2 = 158$

$2\overline{22} = 200 - 22 = 178$ $13\overline{14} = 1300 - 14 = 1286$, $24\overline{13}131 = 2387129$ can be done in part as

$131 = 130 - 1 = 129$ and $24\overline{13} = 2400 - 13 = 2387$

$\therefore$ $24\overline{13}131 = 2387129$.

Steps to convert from vinculum to positive form:

(1) Find out the digits that are to be converted i.e. digits with a bar on top.

(2) Apply "all from 9 and the last from 10" on those digits

(3) To end the conversion apply "one less than the previous digit"

(4) Repeat this as many times in the same number as necessary

Try These: Convert the following to their vinculum form:

(i) 91 (ii) 4427 (iii) 183 (iv) 19326 (v) 2745 (vi) 7648 (vii) 81513 (viii) 763468 (ix) 73655167 (x) 83252327

Try These: From vinculum back to normal form.

(i) $\overline{14}$ (i) $\overline{21}$ (iii) $\overline{23}$ (iv) $2\overline{31}$ (v) $17\bar{2}$ (vi) $14\overline{13}$ (vii) $23\overline{12}13\bar{2}$ (viii) $24\overline{12}\overline{31}$

(ix) $6\overline{322}33\bar{1}$ (x) $14\overline{14}23\overline{23}$

When one number is above and the other below the base

Example1: 102 × 97

Step 1: Here, base is 100

102 + 02 (02 above base i.e. 2 surplus)

97 – 03 (03 below base i.e. 3 deficiency)

Step 2: Divide the answer in two parts as 102 / + 02

97 / – 03

Step 3: Right hand side of the answer is (+ 02) × (– 03) = – 06 = $\overline{06}$

Step 4: Left hand side of the answer is 102 – 3 = 99 = 97 + 02 (same both ways)

∴ 102 × 97 = 99$\overline{06}$ = 9894 (i.e. 9900 – 6 = 9894)

Checking: 102 = 1 + 2 = 3, 9 7 = 7

∴ L.H.S. = 3 × 7 = 21 = 1 + 2 = 3

∴ R.H.S = 9894 = 8 + 4 = 12 = 1 + 2 = 3

As L.H.S. = R.H.S. So, answer is correct

Example 2 : 1002 × 997

1002 / + 002 (006 = 1000 – 6 = 994 and 1 carried from 999 to 999 reduces to 998)
997 / – 003
999 / 006

∴ 1002 × 997 = 998 994

When base is not same:

Example1: 988 × 12

988 / – 012 base is 1000 deficiency 12
12 / + 2 base is 10 surplus is 2, 1 digit allowed in R.H.S.
1188 – 2 / 024
= 1186 / = (2)4

∴ 988 × 12 = 1186 4 = 11856 (because 4 = 10 – 4 = 6)

Checking: ~~9~~88 = 8 + 8 = 16 = 1 + 6 = 7, 12 = 1 + 2 = 3

∴ LHS = 7 × 3 = 21 = 2 + 1 = 3

R.H.S = 11856 = 1 + 5 + 6 = 12 = 1 + 2 = 3

As LHS = RHS So, answer is correct

Example 2: 1012 × 98

1012   1012 / + 012 (base is 1000, 12 surplus (+ve sign)
– 02   98 / – 02 (base is 100, 2 deficiency (–ve sign)
992   992 / $\overline{24}$ [As 012 × (– 02) = – 24] 2 digits allowed in RHS of

Answer

∴ 1012 × 98 = 99224 = 99176 [As 992200 – 24 = 99176]

Checking:1012 = 1 + 1 + 2 = 4, 98 = 8

LHS = 4 × 8 = 32 = 3 + 2 = 5

RHS = 99176 = 1 + 7 + 6 = 14 = 1 + 4 = 5

As RHS = LHS so, answer is correct

Try These:

(i) 1015 × 89 (ii) 103 × 97 (iii) 1005 × 96 (iv) 1234 × 92 (v) 1223 × 92 (vi) 1051 × 9 (vii) 9899 × 87 (viii) 9998 × 103 (ix) 998 × 96 (x) 1005 × 107

Sub – base method:

Till now we have all the numbers which are either less than or more than base numbers. (i.e.10, 100, 1000, 10000 etc. , now we will consider the numbers which are nearer to the multiple of 10, 100, 10000 etc. i.e. 50, 600, 7000 etc. these are called sub-base.

Example: 213 × 202

Step1: Here the sub base is 200 obtained by multiplying base 100 by 2

Step 2: R. H. S. and L.H.S. of answer is obtained using base- method.

213 / + 13

202 / + 02

215 13 × 02 = 26

Step 3: Multiply L.H.S. of answer by 2 to get 215 × 2 = 430

∴ 213 × 202 = 43026

∴

Example 2: 497 × 493

Step1: The Sub-base here is 500 obtained by multiplying base 100 by 5.

Step2: The right hand and left hand sides of the answer are obtained by using base method.

Step3: Multiplying the left hand side of the answer by 5.

497 / –03

493 / –07

Same 497–07 = 490 / 21

493 – 03 = 490

490 × 5

= 2450

∴ 497 × 493 = 245021

Example 3: 206 × 197

Sub-base here is 200 so, multiply L.H.S. by 2

$$\begin{array}{r|l} 206 & +06 \\ 197 & -03 \\ \hline 206-3=203 & -18 \\ 197+06=203\times 2 & =\overline{18} \\ =406 & \end{array}$$

∴ 206 × 197 = $406\overline{18}$ = 40582

Example 4: 212 × 188

Sub – base here is 200

$$\begin{array}{r|l} 212 & +12 \\ 188 & -12 \\ \hline 200-12=200 & (1)\overline{44} \\ 188+12=200 & \\ \times 2 & \\ \hline 400-1=399 & \end{array}$$

∴ 212 × 188 = 399 $\overline{44}$ = 39856

Checking:(11 – check method)

+ – +

2 1 2 = 2 + 2 – 1 = 3

+ – +

1 8 8 = 1 – 8 + 8 = 1

L.H.S. = 3 × 1 = 3

+ – + – +

R.H.S. = 3 9 8 5 6 = 3

As L.H.S = R.H.S. So, answer is correct.

Try these

(1) 42 × 43 (2) 61 × 63 (3) 8004 × 8012 (4) 397 × 398 (5) 583 × 593

(6) 7005 × 6998 (7) 499 × 502 (8) 3012 × 3001 (9) 3122 × 2997 (10) 2999 × 2998

Doubling and Making halves

Sometimes while doing calculations we observe that we can calculate easily by multiplying the number by 2 than the larger number (which is again a multiple of 2). This procedure in called **doubling:**

$35 \times 4 = 35 \times 2 + 2 \times 35 = 70 + 70 = 140$

$26 \times 8 = 26 \times 2 + 26 \times 2 + 26 \times 2 + 26 \times 2 = 52 + 52 + 52 + 52$

$= 52 \times 2 + 52 \times 2 = 104 \times 2 = 208$

$53 \times 4 = 53 \times 2 + 53 \times 2 = 106 \times 2 = 212$

Sometimes situation is reverse and we observe that it is easier to find half of the number than calculating 5 times or multiples of 5. This process is called

Making halves:

4. (1) $87 \times 5 = 87 \times 5 \times 2/2 = 870/2 = 435$

(2) $27 \times 50 = 27 \times 50 \times 2/2 = 2700/2 = 1350$

(3) $82 \times 25 = 82 \times 25 \times 4/4 = 8200/4 = 2050$

Try These:

(1) 18×4

(2) 14×18

(3) 16×7

(4) 16×12

(5) 52×8

(6) 68×5

(7) 36×5

(8) 46×50

(9) 85×25

(10) 223×50

(11) 1235×20

(12) 256×125

(13) 85×4

(14) 102×8

(15) 521×25

Multiplication of Complimentary numbers :

<u>Sutra:</u> By one more than the previous one.

This special type of multiplication is for multiplying numbers whose first digits(figure) are same and whose last digits(figures)add up to 10,100 etc.

Example 1: 45×45

Step I: $5 \times 5 = 25$ which form R.H.S. part of answer

Step II: $4 \times$ (next consecutive number) i.e. $4 \times 5 = 20$, which form L.H.S. part of answer

$\therefore$ $45 \times 45 = 2025$

Example 2: $95 \times 95 = 9 \times 10 = 90/25 \longrightarrow (5^2)$

i.e. $95 \times 95 = 9025$

Example 3: $42 \times 48 = 4 \times 5 = 20/16 \longrightarrow (8 \times 2)$

$\therefore$ $42 \times 48 = 2016$

Example 4: $304 \times 306 = 30 \times 31 = 930/24 \longrightarrow (4 \times 6)$

$\therefore$ $304 \times 306 = 93024$

Try These:

(1) 63 × 67

(2) 52 × 58

(3) 237 × 233

(4) 65 × 65

(5) 124 × 126

(6) 51 × 59

(7) 762 × 768

(8) 633 × 637

(9) 334 × 336

(10) 95 × 95

Multiplication by numbers consisting of all 9's :

Sutras: 'By one less than the previous one' and 'All from 9 and the last from 10'

When number of 9's in the multiplier is same as the number of digits in the multiplicand.

Example 1 : 765 × 999

Step I : The number being multiplied by 9's is first reduced by 1

i.e. 765 – 1 = 764 This is first part of the answer

Step II : "All from 9 and the last from 10" is applied to 765 to get 235, which is the second part of the answer.

$\therefore$ 765 × 999 = 764235

When 9's in the multiplier are more than multiplicand

Example II : 1863 × 99999

Step I : Here 1863 has 4 digits and 99999 have 5-digits, we suppose 1863 to be as 01863. Reduce this by one to get 1862 which form the first part of answer.

Step II: Apply 'All from 9 and last from 10' to 01863 gives 98137which form the last part of answer

∴ 1863 x 99999 = 186298137

When 9's in the multiplier are less than multiplicand

Example 3 : 537 x 99

Step I: Mark off two figures on the right of 537 as 5/37, one more than the L.H.S. of it i.e. (5+1) is to be subtracted from the whole number, 537 – 6 = 531this forms first part of the answer

Step II: Now applying "all from 9 last from 10" to R.H.S. part of 5/37 to get 63 (100 – 37 = 63)

∴ 537 x 99 = 53163

Try these

(1) 254 × 999	(2) 7654 × 9999	(3) 879 × 99	(4) 898 × 9999
(5) 423 × 9999	(6) 876 × 99	(7) 1768 × 999	(8) 4263 × 9999
(9) 30421 × 999	(10) 123 × 99999		

Multiplication by 11

Example 1: 23 × 11

Step 1 : Write the digit on L.H.S. of the number first. Here the number is 23 so, 2 is written first.

Step 2 : Add the two digits of the given number and write it in between. Here 2 + 3 = 5

Step 3 : Now write the second digit on extreme right. Here the digit is 3. So, 23 × 11 = 253

OR

23 × 11 = 2 / 2+3 / 3 = 253

(Here base is 10 so only 2 digits can be added at a time)

Example 2: 243 × 11

Step 1: Mark the first, second and last digit of given number

First digit = 2, second digit = 4, last digit = 3

Now first and last digits of the number 243 form the first and last digits of the answer.

Step 2: For second digit (from left) add first two digits of the number i.e. 2 + 4 = 6

Step 3: For third digit add second and last digits of the number i.e. 3 + 4 = 7

So, 243 × 11 = 2673

OR

243 × 11 = 2 / 2 + 4 / 4 + 3 / 3 = 2673

Similarly we can multiply any bigger number by 11 easily.

Example 3: 42431 × 11

42431 × 11 = 4 / 4 + 2 / 2 + 4 / 4 + 3 / 3 + 1 / 1 = 466741

If we have to multiply the given number by 111

Example 1: 189 × 111

Step 1: Mark the first, second and last digit of given number

First digit = 1, second digit = 8, last digit = 9

Now first and last digits of the number 189 may form the first and last digits of the answer

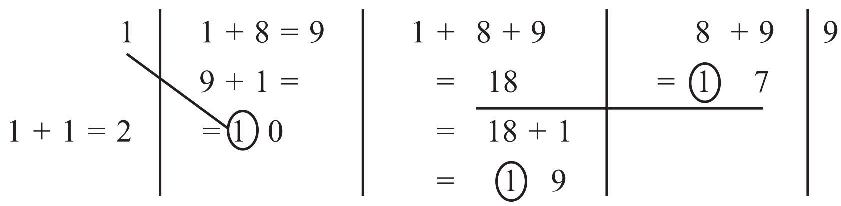
Step 2: For second digit (from left) add first two digits of the number i.e. 1 + 8 = 9

Step 3: For third digit add first, second and last digits of the number to get 1 + 8 + 9 = 18 (multiplying by 111, so three digits are added at a time)

Step 4: For fourth digit from left add second and last digit to get, 8 + 9 = 17

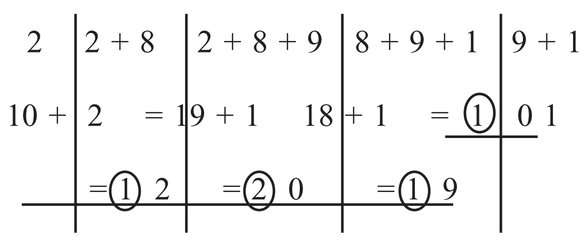
As we cannot have two digits at one place so 1 is shifted and added to the next digit so as to get 189 × 111 = 20979

OR



∴ 189 × 111 = 20979

Example 2 : 2891 × 111



2891 × 111 = 320901

Try These:

(1)	107 × 11	(2)	15 × 11	(3)	16 × 111	(4)	112 × 111
(5)	72 × 11	(6)	69 × 111	(7)	12345 × 11	(8)	2345 × 111
(9)	272 × 11	(10)	6231 × 111.				

Note: This method can be extended to number of any size and to multiplying by 1111, 11111 etc. This multiplication is useful in percentage also. If we want to increase a member by 10% we multiply it by 1.1

General Method of Multiplication.

<u>Sutra</u>: Vertically and cross-wise.

Till now we have learned various methods of multiplication but these are all special cases, wherenumbers should satisfy certain conditions like near base, or sub base, complimentary to each other etc. Now we are going to learn about a general method of multiplication, by which we can multiply any two numbers in a line. Vertically and cross-wise sutra can be used for multiplying any number.

For different figure numbers the sutra works as follows:

Two digit – multiplication

Example: Multiply 21 and 23

Step1: Vertical (one at a time) 2 [1], 2 [3] ↓ $1 \times 3 = 3$ | 3

Step2: Cross –wise (two at a time) 2 ✕ 1, 2 ✕ 3 $(2 \times 3 + 2 \times 1) = 8$ / 8 / 3

Step3: Vertical (one at a time) ↓ [2] 1, [2] 3 $2 \times 2 = 4$ 4 / 8 / 3

$\therefore$ $21 \times 23 = 483$

Multiplication with carry:

Example: Multiply 42 and 26

Step1: Vertical 42, <u>26</u> ↓ $2 \times 6 = 12$ / / 12

Step2: Cross-wise 4 ✕ 2, 2 ✕ 6 $4 \times 6 + 2 \times 2$, $24 + 4 = 28$ / 2_8 / $_1 2$

Step3: Vertical ↓ 42, <u>26</u> $4 \times 2 = 8$

8 / 8 / 2

+ 2 (1)

= 10 = (2) / 9

$\therefore$ $42 \times 26 = 1092$

Three digit multiplication:

Example: 212×112

Step1: Vertical (one at a time)

212 ↓ 2 × 2

112 = 4   /4

Step2: Cross-wise (two at a time)

2 1 2   2 × 1 + 2 × 1

1 1 2   = 2 + 2 = 4   /4 /4

Step3: Vertical and cross-wise (three at a time)

2 1 2

1 1 2

2 × 2 + 2 × 1 + 1 × 1 = 4 + 2 + 1 = 7   /7 /4 /4

Step4: cross wise (Two at a time)

2 1 2   2 × 1 + 1 × 1

1 1 2   = 2 + 1 = 3   /3 /7 /4 /4

Step 5: vertical (one at a time)

↓ 2 1 2   2 × 1 = 2   2 /3 /7 /4 /4

1 1 2

∴ 212 × 112 = 23744

Three digits Multiplication with carry:

Example: 816 × 223

8 1 6

2 2 3

8 × 2	8 × 2 + 2 × 1	8 × 3 + 6 × 2 + 1 × 2	3 × 1 + 2 × 6	6 × 3 = 18
	= 16 + 2 = 18	= 24 + 12 + 2	3 + 12 = 15	
		=38		
16				
16 + 2 = (2) 1	18 + 3 = (3) 9	38 + 1 = (1) 6	15 + 1 — (1) 8	

∴ 816 × 223 = 181968

Checking by 11 – check method

\+ – +   – +

8 1 6 = 14 – 1 = 1 3 = 3 – 1 = 2

\+ - +

2 2 3 =3

∴ L.H.S. = 3 × 2 = 6

- + - + - +   - +

1 8 1 9 6 8   =   1 7 = 7 – 1 = 6

As L.H.S. = R.H.S.

∴ Answer is correct

Try These:

(1) 342 × 514	(2) 1412 × 4235	(3) 321 × 53	(4) 2121 × 2112	(5) 302 × 415
(6) 1312 × 3112	(7) 5123 × 5012	(8) 20354 × 131	(9) 7232 × 125	(10) 3434 × 4321

Number Split Method

As you have earlier used this method for addition and subtraction, the same may be done for multiplication also.

For example :

$$\begin{array}{r} 263 \\ \times\ 2 \\ \hline \\ \hline \end{array} \longrightarrow \begin{array}{r|r} 26 & 3 \\ \times\ 2 & \times 2 \\ \hline 52 & 6 \end{array}$$

Note : The split allows us to add 36 + 24 and 42 + 39 both of which can be done mentally

Multiplication of algebraic expressions:

Sutra: Vertically and cross-wise

Example1 : $(x + 3)\ (x + 4)$

$$\begin{array}{c} x + 3 \\ x + 4 \\ \hline x^2 + 7x + 12 \end{array}$$

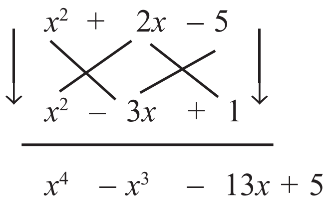
$x \times x$	$4x + 3x$	4×3
x^2	$= 7x$	$= 12$
x^2	7×12	

Example2: $(2x + 5)\ (3x + 2)$

$$\begin{array}{ccc} 2x & + & 5 \\ 3x & + & 2 \\ \hline 6x^2 & +19x\ + & 10 \end{array}$$

$(2x) \times (3x)$	$4x + 15x$	10
$=6x^2$	$= 19x$	

Example3: $(x^2 + 2x + 5)\ (x^2 - 3x + 1)$



x^4	$-3x^3 + 2x^3$	$x^2 + 5x^2 - 6x^2$	$2x - 15x$	5×1
	$= -x^3$	$= 0$	$= -13x$	$= 5$
x^4	$-x^3$	0	$-13x$	5

Try These:

(1) $(2x - 1)\ (3x + 2)$

(2) $(2x + 1)\ (x^2 + 3x - 5)$

(3) $(5x + 5)\ (7x - 6)$

(4) $(x + 5)\ (x^2 - 2x + 3)$

(5) $(x - 4)\ (x^2 + 2x + 3)$

(6) $(x^2 + 4x - 5)\ (x + 5)$

(7) $(x^3 - 5)\ (x^2 + 3)$

(8) $(x^2 - 2x + 8)\ (x^4 - 2)$

(9) $(x^2 - 7x + 4)\ (x^3 - 1)$

(10) $(x^3 - 5x^2 + 2)\ (x^2 + 1)$

Chapter 5 ‖ Squaring and square Roots

Square of numbers ending in 5 :

Sutra: 'By one more than previous one"

Example: 75 × 75 or 75^2

As explained earlier in the chapter of multiplication we simply multiply 7 by the next number i.e. 8 to get 56 which forms first part of answer and the last part is simply 25= $(5)^2$. So, 75 × 75 = 5625

This method is applicable to numbers of any size.

Example: 605^2

$60 \times 61 = 3660$ and $5^2 = 25$

$\therefore\ 605^2 = 366025$

Square of numbers with decimals ending in 5

Example : $(7.5)^2$

$7 \times 8 = 56$, $(0.5^2) = 0.25$

$(7.5)^2 = 56.25$ (Similar to above example but with decimal)

Squaring numbers above 50:

Example: 52^2

Step1: First part is calculated as $5^2 + 2 = 25 + 2 = 27$

Step2: Last part is calculated as $(2)^2 = 04$ (two digits)

$\therefore\ 52^2 = 2704$

Squaring numbers below 50

Example : 48^2

Step1: First part of answer calculated as: $5^2 - 2 = 25 - 2 = 23$

Step2: second part is calculated as : $2^2 = 04$

$\therefore\quad 48^2 = 2304$

Squaring numbers near base :

Example : 1004^2

Step1: For first part add 1004and 04 to get 1008

Step2: For second part$4^2 = 16 = 016$ (as,base is 1000 a three digit no.)

$\therefore\quad (1004)^2 = 1008016$

Squaring numbers near sub - base:

Example $(302)^2$

Step1: For first part = 3 (302 + 02) = 3 × 304 = 912 [Here sub – base is 300 so multiply by 3]
Step2: For second part = 2^2 = 04

∴ $(302)^2 = 91204$

General method of squaring:

The Duplex

Sutra: "Single digit square, pair multiply and double" we will use the term duplex,` D' as follows:

For <u>1 figure</u>(or digit) Duplex is its squaree.g. D(4) = 4^2 = 16

For<u>2 digits</u>Duplex is twice of the product e.g. D(34) = 2 (3 x 4) = 24

For <u>3 digit number</u>: e.g. $(341)^2$

D(3) = 3^2 = 9

D (34) = 2 (3 × 4) = 24

D (341) = 2 (3 × 1) + 4^2 = 6 + 16 = 22

D (41) = 2 (4 × 1) = 8

D (1) = 1^2 = 1

∴ $(341)^2 = 116281$

9 / 4 /2 /8 /1
/ 2/ 2/ /
=116281

Algebraic Squaring :

Above method is applicable for squaring algebraic expressions:

Example: $(x + 5)^2$

D $(x) = x^2$

D$(x + 5) = 2\ (x \times 5) = 10x$

D $(5) = 5^2 = 25$

∴ $(x + 5)^2 = x^2 + 10x + 25$

Example: $(x – 3y)^2$

D $(x)= x^2$

D$(x – 3y) = 2\ (x) \times – 3y) = – 6xy$

D$(–3y\ ) = (–3y)^2 = 9y^2$

∴ $(x – 3y)^2 = x^2 – 6xy + 9y^2$

Try these:

(I)	85^2	(II)	$(8_2^1)^2$	(III)	$(10.5)^2$	(IV)	8050^2
(V)	58^2	(VI)	52^2	(VII)	42^2	(VIII)	46^2
(IX)	98^2	(X)	106^2	(XI)	118^2	(XII)	$(x + 2\ )^2$
(XIII)	$(y – 3)^2$	(XIV)	$(2x – 3)^2$	(XV)	$(3y – 5)^2$		

SQUARE ROOTS:

General method:

As $1^2 = 1$ $2^2 = 4$ $3^2 = 9$ $4^2 = 1[6]$ $5^2 = 2[5]$ $6^2 = 3[6]$

$7^2 = 4[9]$ $8^2 = 6[4]$ $9^2 = 8[1]$ i.e. square numbers only have digits 1,4,5,6,9,0 at the units place (or at the end)

Also in 16, digit sum = 1 + 6 = 7, 25 = 2 + 5 =7 , 36 = 3 + 6 = 9 , 49 = 4 + 9 = 13

13 = 1 + 3 =4, 64 = 6 + 4 = 10 = 1 + 0 = 1 , 81 = 8 + 1 = 9 i.e. square number only have digit sums of 1, 4, 7 and 9.

This means that square numbers cannot have certain digit sums and they cannot end with certain figures (or digits) using above information which of the following are not square numbers:

(1) 4539 (2) 6889 (3) 104976 (4) 27478 (5) 12345

Note: If a number has a valid digit sum and a valid last figure that does not mean that it is a square number. If 75379 is not a perfect square in spite of the fact that its digit sum is 4 and last figure is 9.

Square Root of Perfect Squares:

Example1: $\sqrt{5184}$

Step 1: Pair the numbers from right to left <u>5184</u> two pairs

Therefore answer is 2 digit numbers

$7^2 = 49$ and $8^2 = 64$

49 is less than 51

Therefore first digit of square root is 7.

Look at last digit which is 4

As $2^2 = 4$ and $8^2 = 64$ both end with 4

Therefore the answer could be 72 or 78

As we know $75^2 = 5625$ greater than 5184

Therefore $\sqrt{5184}$ is below 75

Therefore $\sqrt{5184} = 72$

Example 2: $\sqrt{9216}$

Step 1: Pair the numbers from right to left <u>9216</u>two pairs

Therefore answer is 2 digit numbers

$9^2 = 81$ and $10^2 = 100$

81 is less than 92

Therefore first digit of square root is 9.

Look at last digit which is 6

As $4^2 = 16$ and $6^2 = 36$ both end with 6

Therefore the answer could be 94 or 96

As we know $95^2 = 9025$ less than 9216

Therefore $\sqrt{9216}$ is above 95

Therefore $\sqrt{9216} = 96$

General method

Example 1 : $\sqrt{2809}$

Step1: Form the pairs from right to left which decide the number of digits in the square root. Here 2 pairs therefore 2 - digits in thesquare root

Step 2: Now $\sqrt{28}$, nearest squares is = 25

So first digit is 5 (from left)

Step3: As 28 – 25 = 3 is reminder which forms 30 with the next digit 0.

Step 4: Multiply 2 with 5 to get 10 which is divisor 10 $\sqrt{\underline{2809}}$

30

Now 3 × 10 = 30 $\frac{30}{10}$ = Q R

3 0

Step 5: As $3^2 = 9$ and 9 – 9 (last digit of the number) = 0

∴ 2809 is a perfect square and $\sqrt{2809} = 53$

Example 2:3249

Step1: Form the pairs form right to left which decided the number of digits in the square root. Here 2 pairs therefore 2 digits in the square root.

Step2: Now $32 > 25 = 5^2$ so the first digit in 5 (from left)

Step 3: 32 – 25 = 7 is remainder which form 74 with the next digit 4

5 7

Step 4: Multiply 2 with 5 to get 10 which is divisor 10$\sqrt{\underline{3249}}$

Now $\frac{74}{10}$ = Q R 7 4

7 4

Step5: $7^2 = 49$ and 49 – 49 =0 (remainder is 4 which together with9 form49)

∴ 3249 is a perfect square and $\sqrt{3249} = 57$

Example 3: $\sqrt{54\ 75\ 6}$

Step1: Form the pairs from right to left therefore the square root of 54756 has 3-digits.

Step2: $5 > 4 = 2^2$ i.e. nearest square is $2^2 = 4$

So first digit is 2 (from left)

Step3: As 5 – 4 = 1 is remainder which form 14 with the next digit 4.

Step4: Multiply 2 with 2 to get 4, which is divisor

2

4 $\underline{5_1 4\ _2 75\ 6}$ Now $\underline{14}$ = Q R

4 3 2

Step 5: Start with remainder and next digit, we get 27.

Find 27 – 3^2 = 27 – 9 = 18 [square of quotient]

234

Step 6: $\underline{18}$ = Q R 4 $\underline{5_1 4_2\ 7_2 5_1\ 6}$

4 4 2

Now 25 – (3 × 4 × 2) = 25 – 24 = 1

$\underline{1}$ = Q R

4 0 1

16 – 4^2 = 16 – 16 = 0

∴ 54756 is a perfect square and so $\sqrt{5\ 4\ 7\ 5\ 6}$ = 234

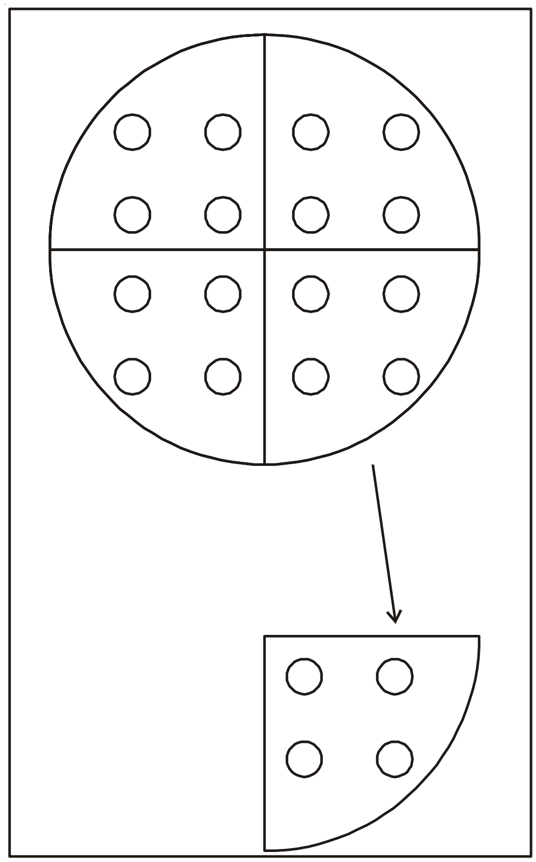
Try These:

1. 2116
2. 784
3. 6724
4. 4489
5. 9604
6. 3249
7. 34856
8. 1444
9. 103041
10. 97344

CHAPTER-6 DIVISION

Defining the Division terms

There are 16 balls to be distributed among 4 people How much each one will get is a problems of division. Let us use this example to understand the terms used in division.



Divisor: —Represent number of people we want to distribute them or the number that we want to divide by. Here the divisor is 4.

Dividend: -Represents number of balls to be divided 16 in this case.

Quotient: Represents the number of balls in each part, 4 is this case.

Remainder: What remains after dividing in equal parts, 0 in this case?

The remainder theorem follows from the division example above and is expressed mathematically as follows.

Divided = Divisor × Quotient + Remainder

The remainder theorem can be used to check the Division sums in Vedic Mathematics as described in the following sections.

Different methods are used for dividing numbers based on whether the divisor is single digit numbers below a base, above a base or no special case.

Special methods of Division.

Number splitting

Simple Division of Divisor with single digits can be done using this method.

Example: The number 682 can be split into

6/82 and we get 3/41 because

6 and 82 are both easy to halve

Therefore 682/2 = 341

Example : 3648/2 becomes

36/48/2 = 18/24 = 1824

Example:1599/3 we notice that 15 and 99 can be separately by 3 so

15/99/3 = 5/33 = 533

Example: 618/6 can also be mentally done

6/18/6 = 103 note the 0 here

Because the 18 takes up two places

Example: 1435/7

14/35/7 = 2/05 = 205

Example: 27483/3 becomes

27/48/3/3 = 9/16/1 = 9161

Practice Problem

Divided mentally (Numbers Splitting)

(1) 2)656

(2) 2)726

(3) 3)1899

(4) 6)1266

(5) 3)2139

(6) 2)2636

(7) 4)812

(8) 6)4818

(9) 8)40168

(10) 5)103545

Division by 9

As we have seen before that the number 9 is special and there is very easy way to divide by 9.

Example : Find 25 ÷ 9

25/9 gives 2 remainder 7

The first figure of 25 is the answer?

And adding the figures of 25 gives the remainders 2 + 5 = 7 so 25 ÷ 9 = 2 remainder 7. It is easy to see why this works because every 10 contains 9 with 1 left over, so 2 tens contains 2 times with 2 left over. The answer is the same as the remainders 2. And that is why we add 2 to 5 to get remainder. It can happen that there is another nine in the remainder like in the next example

Example: Find 66 ÷ 9

66/9 gives 6 + 6 = 12 or 7 or 3

We get 6 as quotient and remainder 12 and there is another nine in the remainder of 12, so we add the one extra nine to the 6 which becomes 7 and remainder is reduced to 3 (take 9 from 12) We can also get the final remainder 3, by adding the digits in 12. The unique property of number nine that it is one unit below ten leads to many of the very easy Vedic Methods.

This method can easily be extended to longer numbers.

Example: 3401 ÷ 9 = 377 remainder 8

Step 1: The 3 at the beginning of 3401 is brought straight into the answer.

9)3401

<u>3</u>

Step 2: This 3 is add to 4 in 3401 and 7 is put down

9)3401

<u>37</u>

Step 3: This 7 is then added to the 0 in 3401 and 7 is put down.

9)3401

<u>377</u>

Step 4: This 7 is then added to give the remainder

9) 340/1

377/8

Divided the following by 9

(1) 9)51

(2) 9)34

(3) 9)17

(4) 9)44

(5) 9)60

(6) 9)26

(7) 9)46

(8) 9)64

(9) 9)88

(10) 9)96

Longer numbers in the divisor

The method can be easily extended to longer numbers. Suppose we want to divide the number 21 3423 by 99. This is very similar to division by 9 but because 99 has two 9's we can get the answer in two digits at a time. Think of the number split into pairs.

21/34/23 where the last pair is part of the remainder.

Step 1: Then put down 21 as the first part of the answer

99)21/34/23

21

Step 2: Then add 21 to the 34 and put down 55 as next part

99)21/34/23

21/55

Step 3: Finally add the 55 to the last pair and put down 78 as the remainder

99)21/34/23

21/55/78

So the answer is 2155 remainder 78

Example: 12/314 ÷ 98 = 1237

Step 1: This is the same as before but because 98 is 2 below 100 we double the last part of the answer before adding it to the next part of the sum. So we begin as before by bringing 12 down into the answer.

98) 12/13/14

12

Step 2: Then we double 12 add 24 to 13 to get 37

98) 12/13/14

12/37

Step 3: Finally double 37 added 37 × 2 = 74 to 14

98)12/13/14

12/37/88 =1237 remainder 88.

It is similarly easy to divide by numbers near other base numbers 100, 1000 etc.

Example: Suppose we want to divide 236 by 88 (which is close to 100). We need to know how many times 88 can be taken from 235 and what the remainder is

Step 1: We separate the two figures on the right because 88 is close to

100 (Which has 2 zeros)

88) 2/36

Step 2: Then since 88 is 12 below 100 we put 12 below 88, as shown

88) 2/36

Step 3: We bring down the initial 2 into the answer

88) 2/36

12

2

Step 4: This 2 is multiplies Haggled 12 and the 22 is placed under the 36 as Shown

88) 2/36

12 2 / 24

Step 5: We then simply add up the last two columns.

88) 2/36

12 2 r 60

In a similar way we can divide by numbers like 97 and 999.

Practice problems

Divide the following using base method

(1) 121416 by 99

(2) 213141 by 99

(3) 332211 by 99

(4) 282828 by 99

(5) 363432 by 99

(6) 11221122 by 98

(7) 3456 by 98

Sutra: Transpose and Apply

A very similar method, allows us to divide numbers, which are close to but above a base number.

Example: 1479 ÷ 123 = 12 remainder 13

Step 1: 123 is 23 more than base 100

Step 2: Divide 1479 in two columns therefore of 2digit each

Step 3: Write 14 down

Step 4: Multiply 1 by $\overline{23}$ and write it below next two digits. Add in the Second column and put down 2.

Step 5: Add multiply this $\overline{2}$ the $\overline{2}$, $\overline{3}$ and put $\overline{46}$ then add up last two Columns

123) 14 78

23 23

46

12/02

Straight Division

The general division method, also called Straight division, allows us to divide numbers of any size by numbers of any sine, in one line, Sri BharatiKrsnaTirthaji called this "the cowing gem of Vedic Mathematics"

Sutra: - 'vertically and crosswise' and 'on the flag'

Example: Divide 234 by 54

The division, 54 is written with 4 raised up, on the flag, and a vertical line is drawn one figure from the right hand end to separate the answer, 4, from the remainder 28

23	$_3$4
$5^4$20	16
4	28

Step 1: 5 into 20 goes 4 remained 3 as shown

Step 2: Answer 4 multiplied by the flagged 4 gives 16 and this 16 taken from 34 leaves the remainder 28 as shown

Example: Divide: 507 by 72

50	$_1$7
7 2	
49	14
7	3

Step 1: 7 into 50 goes 7 remainder 1 as shown

Step 2: 7 times the flagged 2 gives 14 which we take from 17 to have remainder of 3

Split Method

Split method can be done for division also. For example :

6234 ÷ 2

62	34
÷ 2	÷2
31	17

The 'split' may require more 'parts'.

30155 ÷ 5

30	15	5	
÷5	÷5	÷5	
6	03	1	6031

244506 ÷ 3

24	45	06	
÷3	÷3	÷3	
8	15	02	81502

Practice Question

Divide the following using straight division

(1) 209 ÷ s52	(2) 621 ÷ 63
(3) 503 ÷ 72	(4) 103 ÷ 43
(5) 74 ÷ 23	(6) 504 ÷ 72
(7) 444 ÷ 63	(8) 543 ÷ 82
(9) 567 ÷ 93	(10) 97 ÷ 28
(11) 184 ÷ 47	(12) 210 ÷ 53
(13) 373 ÷ 63	(14) 353 ÷ 52
(15) 333 ÷ 44	(16) 267 ÷ 37
(17) 357 ÷ 59	(18) 353 ÷ 59
(19) 12233 ÷ 53	

Books for Reference

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